

Universal quantum logic from Zeeman and anisotropic exchange interactions

Lian-Ao Wu and Daniel A. Lidar

Chemical Physics Theory Group, Chemistry Department, University of Toronto, 80 St. George Street, Toronto, Ontario, Canada M5S 3H6

(Received 24 February 2002; published 20 December 2002)

Some of the most promising proposals for scalable solid-state quantum computing, e.g., those using electron spins in quantum dots or donor electron or nuclear spins in Si, rely on a two-qubit quantum gate that is ideally generated by an isotropic exchange interaction. However, an anisotropic perturbation arising from spin-orbit coupling is inevitably present. Previous studies focused on removing the anisotropy. Here we introduce a new universal set of quantum logic gates that *takes advantage* of the anisotropic perturbation. The price is a constant but modest factor in additional pulses. The gain is a scheme that is compatible with the naturally available interactions in spin-based solid-state quantum computers.

DOI: 10.1103/PhysRevA.66.062314

PACS number(s): 03.67.Lx, 71.70.Ej, 85.35.Be

I. INTRODUCTION

A fundamental notion in quantum computing (QC) is *universality*: a set of quantum logic gates (unitary transformations) is said to be “universal for QC” if any unitary transformation can be approximated to arbitrary accuracy by a quantum circuit involving only those gates [1]. Mathematically, this means the ability to efficiently generate a dense subgroup of the group of unitary operations on N qubits, $U(2^N)$. Physically, this is accomplished by carefully manipulating single qubit-external field and (or only) qubit-qubit interactions, thus generating unitary gate operations. A *universal-gate set* that accomplishes this, may be continuous, discrete, or both. A well-known example is the set of all single-qubit gates plus a controlled-phase (CP) gate (that flips the phase of a target qubit depending on the state of a control qubit), but many other universal sets are known [1]. An important example of a universal gate set, of relevance to us, is the set generated by controlling only *isotropic* Heisenberg exchange interactions. This set was shown [2] to be universal in the context of research on decoherence-free subspaces (DFSs) [3], and requires that a logical qubit be encoded into at least three physical qubits [4]. Efficient gate sequences for universal QC in this case were subsequently presented in Ref. [5]. These results assume that all qubits have equal energies. However, this assumption may break down under magnetic field and (or) g -factor inhomogeneity [6]. When the resulting Zeeman splitting is taken into account, it can be shown that the isotropic Heisenberg interaction is universal for QC using an encoding of one logical qubit into only two physical qubits, and efficient gate sequences have been found [7,8]. We describe here a new universal gate set: that generated by the Zeeman splitting and the *anisotropic* Heisenberg interaction, defined more precisely below. This set is of particular importance to spin-based solid-state approaches to quantum computing [9,10], where anisotropy is inherently present [11].

II. ZEEMAN AND EXCHANGE INTERACTIONS

A single spin $\vec{S} = (S^x, S^y, S^z)$ with magnetic moment μ_B couples to a magnetic field $B(t)$ oriented along the z axis through the Zeeman splitting Hamiltonian $g\mu_B B(t)S^z$. This

interaction can be spatially controlled by making $B(t)$ inhomogeneous [9], or by modulating the g factor [10]. Conversely, inhomogeneities and (or) a nonuniform g factor may be naturally present [6]. The Zeeman splitting removes the degeneracy of the two-spin states and serves to define a physical qubit. Switching on the Zeeman term for the j th qubit causes a phase shift, i.e., it generates the single-qubit gate $e^{-i\eta S_j^z}$, where

$$\eta = \int dt g \mu_B B(t)$$

is a controllable parameter (we use units where $\hbar = 1$). For example, a useful gate is $Z_j = i \exp(-i\pi S_j^z)$, which is a 180° rotation about the z axis. The typical switching time of the Zeeman splitting is fast: it is similar to that of the Heisenberg interaction (GHz), which is the interaction assumed to govern the operation of two-qubit gates in some of the spin-based approaches to quantum computing [9,10]. These QC proposals, as well as schemes for universal QC using the Heisenberg interaction alone [2,5] rely on this interaction being perfectly isotropic. However, in a crystal environment that lacks inversion symmetry, the actual interaction between spins i and j is

$$H_{ij}(t) = J(t) [\vec{S}_i \cdot \vec{S}_j + \vec{\beta}(t) \cdot \vec{S}_i \times \vec{S}_j + \gamma(t) \vec{\beta}(t) \cdot \vec{S}_i \vec{\beta}(t) \cdot \vec{S}_j], \quad (1)$$

where only the exchange parameter $J(t)$ is directly controllable [11]. This means that the isotropic Heisenberg interaction $J(t)\vec{S}_i \cdot \vec{S}_j$ itself is *not* independently tunable. The anisotropic part arises from spin-orbit coupling, as a relativistic correction. As written, the anisotropy parameters $\vec{\beta}$ and γ are dimensionless; in systems like coupled GaAs quantum dots $|\vec{\beta}|$ is of the order of a few percent, while the last term is of the order of 10^{-4} [11]. $H_{ij}(t)$ given in Eq. (1) is the most general anisotropic exchange interaction that is symmetric about a *given axis*, here $\vec{\beta}$. Further corrections will be even smaller. The anisotropic perturbation has been considered a problem and strategies have been designed to cancel it. For example, it can be removed to first order by shaped pulses [12], or canceled in the absence of an external magnetic field

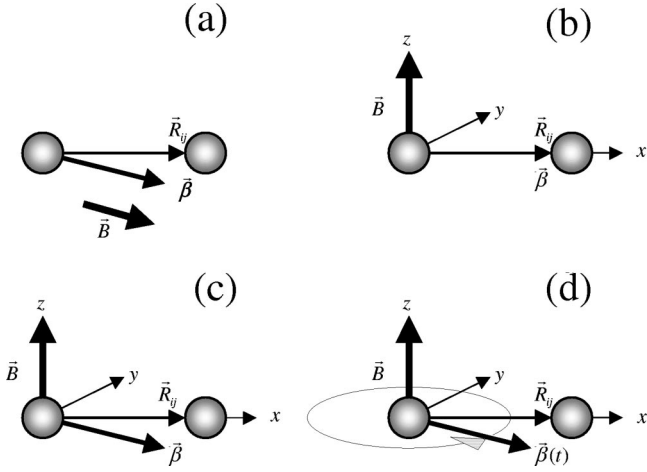


FIG. 1. Geometries of magnetic field \vec{B} , relative position of quantum dots \vec{R}_{ij} , and spin-orbit field $\vec{\beta}$, considered in the text. Quantum dots are indicated by shaded circles.

[13]. Instead of trying to cancel the anisotropy, we show here how to use it to our advantage in order to generate a universal gate set.

We first focus on the case of time-independent $\vec{\beta}$ and γ , which should be dominant as discussed recently in Ref. [13]. The corrections arising from the time-dependent anisotropic interaction are much weaker, sufficiently so that they are below the threshold for fault tolerant quantum computation [13,14]. Nevertheless, we also consider the time-dependent case below. Now, turning on the exchange term $H_{ij}(t)$ generates a unitary evolution

$$U_{ij}(\varphi) = \exp[-iH_{ij}(\varphi)]$$

through the Schrödinger equation, where $H_{ij}(\varphi) \propto \varphi$ and

$$\varphi = \int dt J(t)$$

is a second controllable parameter.

We assume that we can only use the two parameters η and φ to manipulate computational states and construct a universal gate set. Direct control of Hamiltonian terms that generate single-qubit rotations about the x and y axes causes device heating and other major technical problems, so that this type of control is best avoided [5,7]. We thus refer to H_{ij} and the Zeeman splitting as the “available Hamiltonians.” We now show that using control only over these available Hamiltonians suffices to generate universal gate sets for a variety of orientations of the vector $\vec{\beta}$.

Following Ref. [11], the orientation of $\vec{\beta}$ is expressed in terms of the vector \vec{R}_{ij} pointing from qubit i (e.g., the center of the i th quantum dot) to qubit j (Fig. 1). We can always choose the direction of the magnetic field as the z axis. Since \vec{R}_{ij} is a vector in the plane the quantum dots are lying on, if the magnetic field is applied parallel to $\vec{\beta}$, it too should be in the plane of the dots [Fig. 1(a)]. A more common case is

when the magnetic field is perpendicular to the plane of the dots [Figs. 1(b)–1(d)]. We proceed to analyze each of these four cases.

III. CASE 1: MAGNETIC FIELD PARALLEL TO $\vec{\beta}$

We first discuss the case in which the magnetic field \vec{B} is parallel to $\vec{\beta}$ [$=\beta\vec{e}_z$, Fig. 1(a)]. In this (and only this) case it was shown in Ref. [13] that the effect of the anisotropy may be made to cancel exactly. However, this approach requires precise alignment of \vec{B} along $\vec{\beta}$, and utilizes single-qubit S^x, S^y interactions for universality, which as discussed above, we seek to avoid here. Indeed, in the $\vec{\beta} \parallel \vec{B}$ case, the available Hamiltonians are not universal for QC because they have too much symmetry: H_{ij} and the Zeeman splitting both commute with $S_1^z + S_2^z$. There is a simple way to solve the problem: we encode a pair of physical qubit states into a logical qubit: $|0_L\rangle = |\uparrow\rangle|\downarrow\rangle$ and $|1_L\rangle = |\downarrow\rangle|\uparrow\rangle$ (see Refs. [7,8,15] for other cases where this encoding proved useful for universality). In this manner the first logical qubit is given by physical qubits 1,2, the second by physical qubit 3, 4, and so on. A calculation then shows that the *encoded* (denoted by a bar) single-qubit operations are $\bar{S}_i^x = \vec{S}_{2i-1} \cdot \vec{S}_{2i} - S_{2i-1}^z S_{2i}^z$, $\bar{S}_i^y = -(\vec{S}_{2i-1} \times \vec{S}_{2i})_z$ and $\bar{S}_i^z = (S_{2i-1}^z - S_{2i}^z)/2$, where the subscript denotes the i th encoded qubit. These operators have the same commutation relations as the three components of spin angular momentum [i.e., they generate $su(2)$]. Under our assumption of a controllable Zeeman splitting, we can switch \bar{S}^z on or off, and hence can perform arbitrary rotations about the z axis of the encoded qubit. While we do not have direct access to \bar{S}^x , the three-step quantum circuit depicted in Fig. 2(a) yields this operation.

The ability to perform arbitrary rotations about the z and x axes suffices for performing arbitrary single-qubit operations, through a standard Euler angle construction [1]. To complete the universal gate set we also need a logic gate coupling different encoded qubits in a nontrivial manner, such as a CP gate. We have previously shown that the interaction $\bar{S}_1^z \bar{S}_2^z$ between logical qubits 1 and 2, and which generates a CP gate between these qubits, is equivalent to the interaction $S_2^z S_3^z$ between physical qubits 2 and 3 [15]. This can be implemented by the four-step quantum circuit depicted in Fig. 2(b). We note that this circuit also provides a way to cancel the anisotropic interaction by controlling the Zeeman splitting (see also Ref. [13]). In addition, the encoded qubit is a DFS against collective dephasing errors [2], so an automatic layer of error protection is built into these circuits.

IV. CASE 2: MAGNETIC FIELD PERPENDICULAR TO THE PLANE OF DOTS

We now analyze the more common case where the magnetic field is perpendicular to the x - y plane the quantum dots are on. First we consider $\vec{\beta} = \beta\vec{e}_x$ (or $\beta\vec{e}_y$), which can be along the direction \vec{R}_{12} from qubit 1 to qubit 2 [Fig. 1(b)]. As shown in the previous case [of Fig. 1(a)], the isotropic

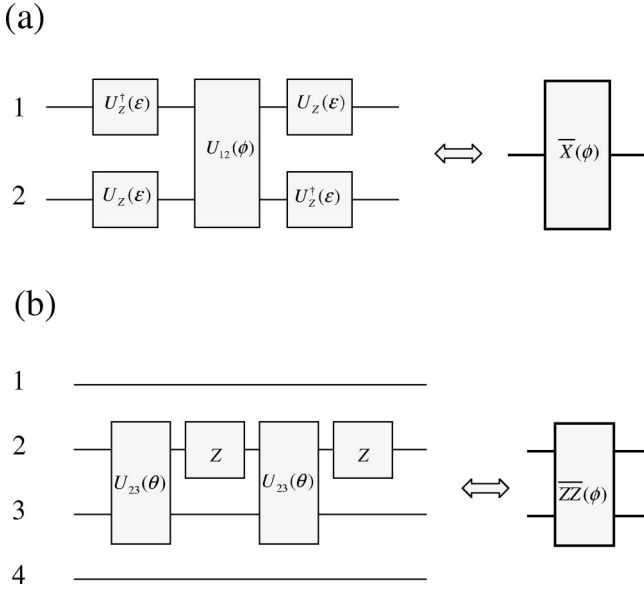


FIG. 2. Diagrams of circuits implementing logical operations in the $\vec{\beta} \parallel \vec{B}$ case. Lines denote physical qubits; time flows from left to right. The three-step circuit (a) implements the transformation $\bar{X}(\phi) \equiv \exp(-i\phi S_1^x) = \exp(-i\epsilon S_1^z) U_{12}(\phi/\sqrt{1+\beta^2}) \exp(i\epsilon S_1^z)$. Here ϕ is an arbitrary angle and $\epsilon = \arctan \beta$ is a time-independent constant [16]. The four-step circuit (b) implements the transformation $\bar{ZZ}(\phi) \equiv \exp(i\phi S_1^z S_2^z) = U_{23}(\theta) Z_2 U_{23}(\theta) Z_2$, where $\theta = \frac{1}{2} \phi / (1 + \gamma\beta^2)$. The notation used in the diagrams is $U_Z(\epsilon) \equiv \exp(-i\epsilon S^z)$, $Z \equiv i \exp(-i\pi S^z)$.

Heisenberg interaction and Zeeman splitting become universal for QC by using an encoding. In contrast, as we now show, H_{ij} together with the Zeeman splitting are universal *without encoding*. Since S^z is by our assumptions controllable, the problem for single-qubit rotations is to show how to generate S^x . We will explicitly be using the anisotropic perturbation to this end, so the speed of the S^x gate will be on the order of a few percent of the S^z gate. This is still reasonable since it is similar to, or even better than, the relative strength of the two-spin interaction and the external radiofrequency magnetic fields in NMR [1].

To generate the S^x gate we first introduce a simple three-step quantum circuit, that will serve as a building block for other gates,

$$V = U_{12}(\pi/\sqrt{1+\beta^2}) Z_1 Z_2 U_{12}(-\pi/\sqrt{1+\beta^2})$$

(note that $Z_1 Z_2$ can be implemented in one parallel step). Contrary to its appearance, this gate is actually separable for qubits 1 and 2. This allows us to use it for creating single-qubit gates, e.g., the following eight-step circuit:

$$\exp(-i4\epsilon S_1^x) = Z_1 V Z_1 V.$$

Recall that $\epsilon = \arctan \beta$ and β is fixed (given) so that this circuit is *discrete* (only whole multiples of the angle 4ϵ can be generated). Alternatively, the following 17-step circuit yields a continuous S^x gate [17]:

$$\begin{aligned} \exp(i\phi S_1^x) &= \exp(i\delta S_1^z) V \exp(i\eta S_1^z) V Z_1 Z_2 V \\ &\quad \times \exp(-i\eta S_1^z) V Z_1 Z_2 \exp(-i\delta S_1^z), \end{aligned}$$

with $\delta = \pi/2 - \arctan(\tan \eta/2 \cos 2\epsilon)$, and continuous angle $\phi = 2 \arccos(1 - 2 \sin^2 \eta/2 \sin^2 2\epsilon)$, controlled in terms of η . However, $|\phi|$ is bounded because of ϵ . For example, its maximum is approximately $\pi/12$ if $\epsilon = 0.03$. Therefore, in order to perform exactly a single-qubit S^x gate with larger angle, one can first use V to approximate the needed gate and then converge using the 17-step continuous gate.

To complete the discussion of universality we again need to generate a logic gate coupling qubits. Such a two-qubit operation can be obtained in terms of the following (not necessarily optimized) 55-step quantum circuit,

$$\begin{aligned} \exp(-i\phi S_1^z S_2^z) &= Z_2 \exp\left(-i\frac{\epsilon}{2}(S_1^x - S_2^x)\right) U_{12}(\varphi) Z_2 \\ &\quad \times \exp(i\epsilon S_2^z) U_{12}(\varphi) \exp\left(i\frac{\epsilon}{2}(S_1^x - S_2^x)\right), \end{aligned}$$

where the arbitrary angle $\phi = 2\varphi\sqrt{1+\beta^2}$ is controlled in terms of the angle φ in H_{ij} . This gate is therefore no longer slow. Note that $e^{i\phi(S_1^x - S_2^x)}$ can be implemented as above by a 17-step quantum circuit. Further note that since *any* entangling gate is universal (together with single-qubit gates) [18], in practice one may be able to reduce our 55-step circuit, e.g., using geometric time-optimal control methods [19].

V. CASE 3: GENERAL TIME-INDEPENDENT CASE

The general $\vec{\beta} \perp \vec{B}$ case is where $\vec{\beta} = \beta_x \vec{e}_x + \beta_y \vec{e}_y$, i.e., time independent and somewhere in the x - y plane [Fig. 1(c)]. However, this case is equivalent up to a unitary rotation to the $\vec{\beta} = \beta \vec{e}_x$ case. Specifically, the transformation $e^{i\omega(S_1^z + S_2^z)} U_{12}(\varphi) e^{-i\omega(S_1^z + S_2^z)}$ [where $\omega = \arctan(\beta_y/\beta_x)$], rotates $\vec{\beta}$ so that it becomes parallel to \vec{e}_x . The treatment above then applies provided we everywhere replace β by $\sqrt{\beta_x^2 + \beta_y^2}$.

It is worth noting that in the present case of time-independent $\vec{\beta}$, similarly to Ref. [5] where efficient gate sequences for the isotropic Heisenberg interaction were obtained, we did not employ the short-time approximation, i.e., made use only of finite-time steps. In contrast to the numerically derived circuits of Ref. [5], our circuits are based on analytical results, and can be understood using elementary angular momentum theory [16,17].

VI. CASE 4: GENERAL TIME-DEPENDENT CASE

Finally, we also consider the general case with $\vec{\beta}$ and γ both time dependent, $\vec{\beta}(t)$ in the x - y plane [Fig. 1(d)]. In contrast to the time-independent case, gates now have to be implemented using the short-time approximation: $e^{A\Delta t} e^{B\Delta t} = e^{(A+B)\Delta t + O(\Delta t^2)}$ for operators A and B that do not necessarily commute, and $\Delta t \ll 1$. While this is less accurate than the exact circuits given above, it is nevertheless a valuable

and common tool in discussions of universality [1,2,20]. The short-time evolution operator corresponding to H_{ij} has the same form as before: $U_{12}(\Delta\phi)$ with $\Delta\phi=J\Delta t$, except that now J is an average value of the coupling constant in the time interval from 0 to Δt . Assuming that all time-dependent parameters do not vary appreciably within the short time Δt , a two-qubit CP gate is given by the repeated four-step circuit

$$\exp(-i\phi S_1^z S_2^z) \approx [U_{12}(\phi/4n)Z_1Z_2U_{12}(\phi/4n)Z_1]^{2n}, \quad (2)$$

where $\phi=n\Delta\phi$. The approximation improves with increasing n . Since $\phi/4n \ll 1$ we only need to know the detailed properties of the evolution operator around time zero. Next we must generate the single-qubit S^x gate. To do so we combine a short-time and a finite-time circuit. First,

$$e^{-i\phi(S_1^z S_2^y - S_1^y S_2^z)} \approx (e^{i\omega(S_1^z + S_2^z)} U_{12}(-\Delta\phi) Z_1 Z_2 \\ \times U_{12}(\Delta\phi) Z_1 Z_2 e^{-i\omega(S_1^z + S_2^z)})^n,$$

where $\phi=n\Delta\phi\sqrt{\beta_x^2+\beta_y^2}$ and we have used the short-time approximation. Then the single-qubit S^x gate is given in terms of the following circuit:

$$e^{-i\phi S_1^x} = e^{i\pi S_1^z S_2^z} e^{-i\phi(S_1^z S_2^y - S_1^y S_2^z)} Z_2 \\ \times e^{-i\phi(S_1^z S_2^y - S_1^y S_2^z)} e^{-i\pi S_1^z S_2^z}.$$

This completes the generation of single-qubit gates, and thus proves universality of our available interactions in the time-dependent case.

VII. MANAGING DECOHERENCE

A discussion of universal quantum computation is incomplete without a consideration of decoherence, the process whereby quantum information is degraded through the interaction of qubits with their environment. In principle, three of the major methods for resisting decoherence, quantum error correcting codes [1,4,14], DFSs [2–4], and fast or strong ‘‘bang-bang’’ (BB) pulses [21] are compatible with our universality results. As mentioned above, in the case of $\vec{\beta}||\vec{B}$ we have used an encoding into a DFS that is automatically resistant to collective dephasing errors. We have recently shown how, starting from a general (linear) system-bath coupling, to actively create the conditions for collective decoherence by applying BB pulses generated by the isotropic Heisenberg interaction [22]. In this case an encoding into a three- or four-qubit DFS is possible, which resists the re-

maining collective errors. Leakage errors (which would arise due to corrections to the short-time approximation invoked in BB theory) can likewise be eliminated using only the isotropic Heisenberg interaction [23]. We conjecture that the same (creation of collective decoherence, leakage elimination) should be possible using the available interactions we considered here. Even without encoding, the use of BB pulses should serve to significantly enhance the robustness of our circuits under decoherence.

We further note that some of our circuits already have a form of decoherence-resistance built into them. For example, the form of Eq. (2) is that of a parity-kick operation [21], which implies that this circuit eliminates all Hamiltonians (including system bath) containing system operators which anticommute with Z_1Z_2 and Z_1 or Z_2 . In fact the same consideration shows that this circuit also eliminates the undesired anisotropic interaction in the more complicated case in which the strength of the anisotropic interaction is not proportional to that of the Heisenberg interaction.

VIII. CONCLUSIONS

We have introduced a new set of universal Hamiltonians: the Zeeman splitting and anisotropic Heisenberg interaction. This set is of direct relevance to quantum computing in solid-state systems that rely on spin-spin interactions [9,10]. Until recently, most studies of such systems assumed an isotropic Heisenberg interaction, which, however, is an approximation due to spin-orbit coupling and other perturbations [6,11]. Instead of trying to cancel the resulting anisotropy [12,13], we showed here how to advantageously use the anisotropy. We analytically derived circuits which implement universal quantum logic in a variety of geometries of interest, for both time-independent and time-dependent perturbations. In the former case, depending on geometry and type of gate implemented, these circuits come with an overhead of between 3 and at most 55 extra pulses. We hope that the methods presented here will enhance the prospects of quantum information processing in those promising quantum computing proposals, where the inherent anisotropy of the exchange interaction cannot be ignored.

ACKNOWLEDGMENTS

This material is based on research sponsored by the Defense Advanced Research Projects Agency under the QuIST program and managed by the Air Force Research Laboratory (AFOSR), under agreement (Grant No. F49620-01-1-0468) (to D.A.L.).

-
- [1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
 [2] D. Bacon, J. Kempe, D. A. Lidar, and K. B. Whaley, Phys. Rev. Lett. **85**, 1758 (2000); J. Kempe, D. Bacon, D. A. Lidar, and K. B. Whaley, Phys. Rev. A **63**, 042307 (2001).
 [3] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997);

- L.-M. Duan and G.-C. Guo, Phys. Rev. A **57**, 737 (1998); D. A. Lidar, I. L. Chuang, and K. B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998).
 [4] E. Knill, R. Laflamme, and L. Viola, Phys. Rev. Lett. **84**, 2525 (2000).
 [5] D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, and K. B. Whaley, Nature (London) **408**, 339 (2000).

- [6] X. Hu, R. de Sousa, and S. Das Sarma, Phys. Rev. Lett. **86**, 918 (2001).
- [7] D. A. Lidar and L.-A. Wu, Phys. Rev. Lett. **88**, 017905 (2002); L.-A. Wu and D. A. Lidar, J. Math. Phys. **43**, 4506 (2002).
- [8] J. Levy, Phys. Rev. Lett. **89**, 147902 (2002); S. C. Benjamin, Phys. Rev. A **64**, 054303 (2001).
- [9] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); B. E. Kane, Nature (London) **393**, 133 (1998); J. Levy, Phys. Rev. A **64**, 052306 (2001); M. Friesen, P. Rugheimer, D.E. Savage, M. G. Lagally, D. W. van der Weide, R. Joynt, and M. A. Eriksson, e-print cond-mat/0204035.
- [10] R. Vrijen, E. Yablonovitch, K. Wang, H. W. Jiang, A. Balandin, V. Roychowdhury, T. Mor, and D. DiVincenzo, Phys. Rev. A **62**, 012306 (2000).
- [11] K. V. Kavokin, Phys. Rev. B **64**, 075305 (2001).
- [12] N. E. Bonesteel, D. Stepanenko, and D. P. DiVincenzo, Phys. Rev. Lett. **87**, 207901 (2001).
- [13] G. Burkard and D. Loss, Phys. Rev. Lett. **88**, 047903 (2002).
- [14] D. Aharonov and M. Ben-Or, e-print quant-ph/9906129; J. Preskill, Proc. R. Soc. London, Ser. A **454**, 385 (1998); A. M. Steane, e-print quant-ph/0207119.
- [15] L.-A. Wu and D. A. Lidar, Phys. Rev. A **65**, 042318 (2002).
- [16] In deriving this formula we neglected an overall phase factor $e^{i\phi(1+\gamma\beta^2)S_1^z S_2^z}$ and employed the identity $e^{-i\theta S^x} S^x e^{i\theta S^x} = S^x \cos \theta + S^y \sin \theta$, which we use repeatedly below.
- [17] Here we used the identity $e^{i\eta(S^z \cos 2\epsilon + S^y \sin 2\epsilon)} \times e^{-i\eta(S^z \cos 2\epsilon - S^y \sin 2\epsilon)} = e^{i\xi(S^x \cos \delta + S^y \sin \delta)}$, with angles as defined in the text.
- [18] J. L. Dodd, M. A. Nielsen, M. J. Bremner, and R. T. Thew, Phys. Rev. A **65**, 040301 (2002).
- [19] N. Khaneja, S. J. Glasser, and R. Brockett, Phys. Rev. A **65**, 032301 (2002).
- [20] S. Lloyd, Phys. Rev. Lett. **75**, 346 (1995).
- [21] L. Viola and S. Lloyd, Phys. Rev. A **58**, 2733 (1988); D. Vitali and P. Tombesi, *ibid.* **59**, 4178 (1999); P. Zanardi, Phys. Lett. A **258**, 77 (1999); M. S. Byrd and D. A. Lidar, Quant. Inf. Proc. **1**, 19 (2002).
- [22] L.-A. Wu and D. A. Lidar, Phys. Rev. Lett. **88**, 207902 (2002).
- [23] J. Kempe, D. Bacon, D. P. DiVincenzo, and K. B. Whaley, Quantum Inf. Comput. **1**, 33 (2001); M. S. Byrd and D. A. Lidar, Phys. Rev. Lett. **89**, 047901 (2002); L.-A. Wu, M. S. Byrd, and D. A. Lidar, *ibid.* **89**, 127901 (2002).