HYBRID DECOHERENCE-FREE ERROR-CORRECTING CODES VIA QUANTUM TRAJECTORIES

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Generalizing a proposal by Alber et al. (quant-ph/0103042), we develop a hybrid decoherence-free subspace (DFS) quantum error-correcting code (QECC) approach, utilizing the methodology of quantum trajectories. The DFS acts as a first layer of protection against errors due to the conditional evolution, while the QECC acts as a second layer and uses the DFS encoding to offer protection against random quantum jumps. We also study the effect of incorporation of quantum logic gates and hence the possibility of fault tolerant quantum computation. Finally we give an example of the method in cavity QED systems.

In this work we use a hybrid of decoherence free subspaces (DFS) and quantum error correction codes (QECC), along with the concept of quantum trajectories, to devise a coding scheme for qubits that protects them against spontaneous emissions and collective dephasing. This construction also allows for relatively easy means of universal computation with physically available Hamiltonians. It is also shown that this construction is fault tolerant in the sense that, for sufficiently low error rates, the errors do not propagate and quantum computation with this code does not add to the decoherence.

Suppose we have a physical system composed of n qubits (two-level quantum systems, with $|0\rangle$ corresponding to the ground state and $|1\rangle$ corresponding to the excited state). Spontaneous emission, induces a transition from $|1\rangle$ to $|0\rangle$, but leaves $|0\rangle$ unchanged. The operator generating this error on the ith qubit is $F_i = |0\rangle_1\langle i|$. In the quantum trajectories picture, the decoherence cycle has two ingredients: The unitary evolution is replaced by a conditional evolution dictated by a non-Hermitian *conditional Hamiltonian*, which is defined in terms of the system Hamiltonian H_s and the error generators F_i, as:

$$H_c = H_s - \frac{i}{2}\sum_i \lambda_i F_i^\dagger F_i, \tag{1}$$

where λ_i is some measure of the strength of errors acting on the ith qubit.[2] The second ingredient is random occurrence/application of the errors F_i, at random times on random qubits, for our case. In our system we assume all the qubits to be identical with respect to their interaction with the environment, which means, effectively, $\lambda_i = \lambda$.

Consider now the following scalable code structure for logical computational qubit states (as a subclass of a more general formalism introduced in Ref. 2), represented in binary format $|a_1a_2 \ldots a_m\rangle$: For each qubit, we assign 2 qubits in the code and add 2 extra fixed "base" qubits, that gives us $2m+2$ qubits:

$$|a_1a_2 \ldots a_m\rangle \rightarrow \frac{1}{\sqrt{2}}(|b_1b_2 \ldots b_m01\rangle + |c_1c_2 \ldots c_m10\rangle), \tag{2}$$

where

$$b_i = \begin{cases} 01 \text{ if } a_i = 0 \\ 10 \text{ if } a_i = 1 \end{cases}, c_i = \begin{cases} 10 \text{ if } a_i = 0 \\ 01 \text{ if } a_i = 1 \end{cases}. \tag{3}$$

The above code-words also span the DFS for the conditional Hamiltonian; the DFS condition in our case[4] will be given by,

$$F_i |\psi_j\rangle = |\psi_j\rangle\,, \tag{4}$$

in which $|\psi_j\rangle$ is a state in the subspace generated by the code words. One can easily check that this translates into using a fixed number of 1's (or 0's) in our code words and is satisfied in our case. This ensures that the code words are protected during the conditional evolution, but they also fulfill the QECC condition,[5] provided that we have the extra information about where (on which qubit) the error has occurred:[2]

$$\left\langle \psi_j \left| \mathbf{F}_i^{\dagger} \mathbf{F}_i \right| \psi_k \right\rangle = \delta_{jk}. \tag{5}$$

We then show that we can generate all the single qubit operations $(\mathrm{SU}(2)_i)$, using single-qubit "logical"[a] Hamiltonians: $X_i^L = X_{2i-1}X_{2i}$ and $Z_i^L = Z_{2i-1}Z_{2m+1}$. This only uses physical[b] Hamiltonians: $H_S = \sum J_{ij}(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y) + J_{ij}^z\sigma_i^z\sigma_j^z$, with controllable coefficients. One could also think of actual geometries that make the above interactions feasible even with the more common nearest-neighbor interactions.

To perform universal computation, we also need to generate entanglement between two arbitrary logical qubits. We use the fact that, with our construction, the logical Hamiltonian, Z_iZ_j, is just another two-body physical Hamiltonian term: $Z_i^LZ_j^L = Z_{2i-1}Z_{2m+1}Z_{2j-1}Z_{2m+1} = Z_{2i-1}Z_{2j-1}$. We further show how to use the Z_iZ_j logical Hamiltonian to generate entanglement. Alternatively, we prove that we can use the nearest neighbor XY interaction, $\sigma_x^i\sigma_x^{i+1} + \sigma_y^i\sigma_y^{i+1}$ and a $\sigma_z^i\sigma_z^{2m+1}$ interaction, to generate the CPHASE gate. This is done using a method called *encoded recoupling*.[6]

We note that, in two respects, this construction is fault tolerant. First, the operations we propose never take the states out of the code-space, hence never make them susceptible to spontaneous emissions. Second, the proposed operations satisfy the fault tolerance property in terms of the stabilizer formalism and its extension to continuous stabilizer. The stabilizer is the set of operators that keep the code words invariant. It is usually a discrete or continuous group generated by the operators $D(\vec{v})$, where $\vec{v}$ is an indexing vector parameter. The fault-tolerance condition is then given by[3]

$$\mathbf{H}\mathbf{D}(\vec{v})\mathbf{H} = \mathbf{D}(\vec{v'}), \tag{6}$$

where H is the desired unitary operation and the mapping from $\vec{v}$ to $\vec{v'}$ has to be one to one. It is shown that this condition is satisfied and also provides a method of fault-tolerant syndrome measurement for the QECC part of our code, which uses an ancillary qubit.

We finally provide a method for preparing the states in our code words initially and discuss realization possibilities using the available Hamiltonians in proposals such as quantum dots, or cavity QED with atoms.

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[a] encoded

[b] That means we do not need heterogeneous terms such as $\sigma_x^i\sigma_y^j$.