Overcoming quantum noise in optical fibers

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Noise in optical telecommunication fibers is an important limitation on optical quantum data transmission. Unfortunately, the classically successful amplifiers cannot be used in quantum communication because of the no-cloning theorem. We propose a simple method to reduce quantum noise: the insertion of phase shifters and/or beam splitters at regular distance intervals into a fiber. We analyze in detail the case of qubits encoded into polarization states of low-intensity light, which is of central importance to various quantum information tasks, such as quantum cryptography and communication. We discuss the experimental feasibility of our scheme and propose a simple experiment to test our method.

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INTRODUCTION

Quantum communication (QC) has recently emerged as a subject of much interest, due to its applications in distributed quantum computation and quantum cryptography [1]. In QC, nonorthogonal quantum signals are typically transmitted through telecommunication fibers. Reducing noise in telecommunication fibers is crucial for QC applications, because the very weak signals carried by polarization states are usually employed. Ideally, a single photon in a superposition of two pulses separated in time with a controlled phase difference (i.e., $|\text{pulse } A\rangle + e^{i\theta}|\text{pulse } B\rangle$) may be used. However, on-demand single-photon sources remain an important technological challenge. Currently, weak coherent states are often employed as approximate single-photon signals. It is well known in quantum cryptography [2,3])—a branch of QC—that weak coherent states may open up loopholes in security because of the probabilistic existence of multiphoton signals [4]. Indeed, a weak coherent state, when phase randomized via decoherence, gives a Poisson distribution in photon numbers. An eavesdropper, Eve, may, in principle, measure the photon number in such a signal by a nondemolition measurement. Afterward, she can stop all single-photon signals from reaching the receiver, Bob. For each multiphoton signal, she can steal one photon and keep it in her quantum memory and send the rest of the signals to Bob by using, for example, an ideal channel instead. Since Eve now has an exact copy of the quantum state transmitted to Bob, this creates a significant challenge in the security of quantum cryptography [4]. Thus, attenuation losses and decoherence in QC are a major issue and methods for reducing such quantum noise are therefore important. Unfortunately, the classically successful amplifiers [such as erbium-doped fiber amplifier (EDFA) [5]] cannot be used in QC because of the no-cloning theorem [6], and new methods must be explored.

Here, we propose a method to reduce noise in the transmission of quantum optical signals in a telecommunication fiber. Our method is inspired by the theory of quantum dynamical "bang-bang" (BB) decoupling [7]. However, a key novelty of our work is the following: we propose to implement BB control in space, rather than time, through the insertion at regular intervals of a sequence of simple linear optical elements (phase shifters and/or beam splitters) in sections of a telecommunication fiber. We also discuss the experimental feasibility of our scheme and propose a few simple experimental tests. We do not expect our method to improve the fidelity of classical light transmission compared to, e.g., EDFA amplifiers, since our method turns out to be quite sensitive to reflection from optical elements and deviations from average fiber homogeneity, which is not the case for classical amplifiers.

QUANTUM NOISE IN OPTICAL FIBERS

An optical fiber provides boundary conditions that guide light along a (locally) straight trajectory. An ideal fiber allows modes of traveling photons to propagate through unchanged. A real fiber induces noise (dispersion, loss, decoherence) compared to the ideal case. The method we introduce in this paper is designed to cancel in principle all quantum noise. The dominant classical light loss mechanisms in an optical fiber are UV absorption, Rayleigh backscattering, OH absorption, and Raman scattering and lead to typical attenuation rates, for state-of-the-art commercial silica telecommunication fiber, of about 0.25 db/km [5]. These mechanisms are active also in the quantum regime [8,9]. All noise processes affecting quantum light in optical fibers are derivable from a microscopic Hamiltonian describing (i) the direct interaction between photons and the optical (dielectric) material of a fiber and (ii) the indirect interaction between photons and quasiparticle excitations of the fiber material, such as polaritons and photon-phonon coupling. These indirect interactions are, of course, in turn derivable from a microscopic Hamiltonian that takes into account matter-matter interactions in the fiber and couples them to photons. The derivation of the resulting effective interactions (e.g., a nonlinear Schrödinger equation) from such microscopic Hamiltonians has been covered in detail, e.g., in [8–10].

The starting point of our analysis is the observation that all interactions involving photons can be written in terms of polynomials in the bosonic raising and lowering operators $b_j^\dagger, b_j$ (where j is the mode of the traveling photons [8,9]). A polynomial of order N describes an N-photon process, and typically the cross section of interactions decreases with increasing N. In the case of the nonrelativistic quantum electrodynamics of charged particles one can decompose the photon-matter interaction Hamiltonian into linear and quadratic terms with respect to the photon field, $H_I=H_I^l+H_I^q$, where the linear part is

$$H_I^l = \sum_j (b_j\hat{B}_j^\dagger + b_j^\dagger\hat{B}_j), \quad (1)$$

where the "bath" operators $\hat{B}_j$ depend only on the variables of charged particles and/or quasiparticles and the quadratic part H_I^q is a function of the bilinear operators $b_i^\dagger b_j$, $b_i^\dagger b_j^\dagger$, and b_ib_j. Higher-order interactions may originate from relativistic effects. In general H_I^q, which makes no contribution to one-photon processes, is much smaller than H_I^l [11]. Therefore, the quadratic term can usually be neglected.

Let us substantiate these arguments by briefly reviewing the corresponding nonrelativistic electrodynamics. Consider particles α with charge q_α and mass m_α, which constitute the optical material of a fiber. Let $\mathbf{r}_\alpha$ and $\mathbf{p}_\alpha$ be the position and momentum of particle α and $\mathbf{A}(\mathbf{r})$ be the vector potential of the photon field. The system-bath Hamiltonian that describes the dynamics can be written, in the Coulomb gauge, as

$$H = H_0 + H_I,$$

$$H_0 = H_M + H_P. \quad (2)$$

Here H_M depends only on the variables of the charged particles. $H_P=\Sigma\hbar\omega_j(b_j^\dagger b_j+1/2)$ is the free photon Hamiltonian, where b_j and $b_j^\dagger$ are the photon annihilation and creation operators in the normal vibrational mode j of the field identified by the wave vector $\mathbf{k}_j$, the polarization ε_j, and the frequency $\omega_j=ck_j$, where c is the speed of light in vacuum. Then the linear part with respect to the photon field [11] is

$$H_I^l = \sum_\alpha \left(\frac{q_\alpha}{m_\alpha}\mathbf{p}_\alpha\cdot\mathbf{A}(\mathbf{r}_\alpha) + \frac{g_\alpha q_\alpha}{2m_\alpha}\mathbf{S}_\alpha\cdot\mathbf{B}(\mathbf{r}_\alpha)\right)$$
$$= \sum_j (\hat{B}_j^\dagger b_j + \hat{B}_j b_j^\dagger), \quad (3)$$

where for a cubic box with dimension L the operator $\hat{B}_j$ can be expressed as

$$\hat{B}_j^\dagger = -\sum_\alpha \frac{q_\alpha}{m_\alpha}\sqrt{\frac{\hbar\omega_j}{2\varepsilon_0 L^3}}e^{i\mathbf{k}_j\cdot\mathbf{r}_\alpha}\left(\mathbf{p}_\alpha\cdot\varepsilon_j + \frac{ig_\alpha}{2c}\mathbf{S}_\alpha\cdot\mathbf{k}_j\times\varepsilon_j\right),$$

which only depends on the variables of charged particles. Here g_α is the g factor, ε_0 is the permittivity of free space, and $\mathbf{S}_\alpha$ is the spin of particle α. Note that the interaction is linear in the operators b_j and $b_j^\dagger$.

The quadratic part of the interaction Hamiltonian is found to be

$$H_I^q = -\sum_\alpha \frac{q_\alpha^2}{2m_\alpha}\mathbf{A}^2(\mathbf{r}_\alpha) \quad (4)$$

and is a function of the bilinear operators $b_i^\dagger b_j$, $b_i^\dagger b_j^\dagger$, and b_ib_j.

Under the long-wavelength approximation, where the spatial variations of the electromagnetic field over the size of the particles is negligible, first-order perturbation theory of H_I^l results in the widely applied dipole interaction (e.g., [10] and references therein). Some effective interactions, such as atom-mediated photon-photon interactions and nonlinear photon-photon interactions (Kerr effect), have been derived without consideration of H_I^q [8–10]. We provide more details on these effective interactions in a later section. For simplicity of presentation we will first design an "*anti-linear*-decoherence fiber" by considering H_I^l only. Later on we show how to treat higher-order interaction terms. It is important to stress that in essence our method hardly depends on the details of the interaction, but *depends on the statistics of photons as bosons.* For this reason our method is very general and is in principle applicable to the entire phenomenology of quantum noise processes affecting photons in fibers, though its practical applicability is a matter of being able to satisfy certain constraints that will be discussed in detail below.

ANTI-LINEAR-DECOHERENCE FIBER

We first consider quantum data transmission through a telecommunication fiber with noise induced by H_I^l. Since H_I^l describes the absorption and creation of photons, it generates photon loss, among other processes. To simplify, we suppose that a *polarization* photon is transmitted from end A to end B. One can define a logical qubit supported by $|0\rangle_L = b_1^\dagger|\mathrm{vac}\rangle$ and $|1\rangle_L = b_2^\dagger|\mathrm{vac}\rangle$ where the mode indices refer to the two polarization states. The initial state at end A is $|\Psi_A\rangle=(a|0\rangle_L+b|1\rangle_L)|M\rangle$, where $|M\rangle$ is the state of the bath (dielectric material and quasiparticle excitations in the fiber). At the time $T=X/v$ (where X is the distance between A and B, and v is the average speed of light in the fiber) the wave function is $|\Psi(T)\rangle=U(T,0)|\Psi_A\rangle$, where the evolution operator is (in units where $\hbar=1$) $U(T,0) \approx e^{-iH(N\Delta)\tau}\cdots e^{-iH(2\Delta)\tau}e^{-iH(\Delta)\tau}$, where $H(k\Delta)\equiv\frac{1}{\Delta}\int_{(k-1)\Delta}^{k\Delta}[H_I(x)+H_0(x)]dx$ is the average Hamiltonian over the kth segment, where H_0 is a sum of the matter (and/or excitations) and photon self-Hamiltonians, $\tau=\Delta/v$, and we have assumed that $N=X/\Delta$ is large in order to expand the normal-ordered exact propagator $U(T,0)=:\exp\{-i\int_A^B[H_I(x)+H_0(x)]dx\}:$. I.e., we have neglected deviations from average fiber homogeneity, $\delta_k=\langle\{H(k\Delta)-[H_I(k\Delta)+H_0(k\Delta)]\}^2\rangle$ [$U(T,0)$ can easily be expressed including such second- and higher-order moments using a Magnus expansion, and it is known how to generalize BB decoupling to treat such higher moments, at the expense of more BB pulses [12]] The interaction H_I entangles the output wave function at end B with the material or excitations in the fiber. By standard arguments it follows that, therefore, the quantum information encoded into the photon state will decohere [1].

In order to solve this problem of decoherence, we draw inspiration from the idea of BB decoupling via time-dependent pulses [7] (we note that a method for finding such pulses directly from empirical data was proposed in [13]). We first recall the action of a phase shifter. It is simple to show [using the Baker-Campbell-Hausdorff (BCH) formula [14]] for a boson that

$$e^{i\phi\hat{n}}b^{\dagger}e^{-i\phi\hat{n}} = e^{i\phi}b^{\dagger}, \quad e^{i\phi\hat{n}}be^{-i\phi\hat{n}} = e^{-i\phi}b, \tag{5}$$

where $\hat{n}=b^{\dagger}b$ is a boson number operator. Physically, the operation $e^{i\pi\hat{n}}$ is a π phase shifter (it puts a phase of π between the number states $|0\rangle$ and $|1\rangle$, not to be confused with our logical qubit states). Defining the π-phase-shifter operator

$$\Pi = \Pi^{\dagger} = e^{i\pi(\hat{n}_1+\hat{n}_2)}, \tag{6}$$

we therefore have

$$\Pi H \Pi = H_0 - H_I^l, \tag{7}$$

because the photons term of H_0 is $\Sigma\hbar\omega_j(n_j+1/2)$, so that $[H_0,\, n_1+n_2]=0$. The crucial point is that *the sign of the linear term of the interaction Hamiltonian has been negated by the action of two phase shifters—i.e., effectively time reversed.* Now, if we install thin phase shifters inside the fiber at positions $x=0,\Delta,2\Delta,\,\ldots,$ from A to B, the evolution will be modified to

$$U'(T,0) \approx e^{-iH(N\Delta)\tau}\cdots\Pi e^{-iH(2\Delta)\tau}\Pi e^{-iH(\Delta)\tau}\Pi$$
$$\equiv [N,\ldots,\Pi,2,\Pi,1,\Pi],$$

where in the second line we have introduced a self-explanatory notation that will be used repeatedly below. Note that in writing this expression we have neglected the variation of H inside the phase shifter; this will hold provided that the phase-shifter width is much smaller than the distance over which deviations δ_k from average fiber homogeneity become significant. Further note that we are applying the "parity-kick" version of BB decoupling [7,15], but are implementing it in space, rather than time. Now assume that the *average* Hamiltonians over two successive segments are equal:

$$H_I^l((k+1)\Delta) = H_I^l(k\Delta),$$

$$H_0((k+1)\Delta) = H_0(k\Delta). \tag{8}$$

The better this approximation, the better our method will perform; we address deviations in the Appendix. In this case, to first order in τ and using Eq. (7), we have an exact cancellation of H_I^l between successive segments:

$$e^{-iH((k+1)\Delta)\tau}\Pi e^{-iH(k\Delta)\tau}\Pi = e^{-iH((k+1)\Delta)\tau}e^{-i\Pi H(k\Delta)\Pi\tau} = e^{-2iH_0(k\Delta)\tau}. \tag{9}$$

This yields the overall evolution operator

$$U'(T,0) = e^{H_0(N\Delta)\tau} = e^{H_0(X)\tau};$$

i.e., the evolution is completely decoherence free, in analogy to the ideal BB limit of infinitely fast and strong pulses [7].

ROUGH ESTIMATE OF REQUIRED INTER-PHASE-SHIFTER DISTANCE

Because of the in-principle equivalence between the BB method and the quantum Zeno effect [16], the proposed method can only work if the phase shifters are inserted at small intervals Δ over which coherence loss is quadratic ("Zeno like"), rather than exponential ("Markovian"). A reliable estimate of Δ requires a first-principles calculation which is beyond the scope of the present work; we present a phenomenological model for a detailed estimate of Δ in the Appendix. Here we give a rough *upper bound* estimate of this distance. We assume that the linear term of the interaction Hamiltonian gives rise to the 0.25 dB/km (5×10^{-2}) *classical* loss figure in a telecommunication fiber. Our main approximation now consists in further assuming that the insertion of phase shifters into the fiber causes a reduction of loss from first to second order, and we use this to estimate the Δ required in the *quantum* case. Thus, imagine a distributed quantum computing scenario where small-scale quantum computers are connected by optical fibers of length about 1 km. Our goal is to have reliable quantum computation within the fault-tolerance threshold value of a 10^{-4} error rate for each elementary quantum logical operation. (We remark that for reliable quantum *communication of entangled photon pairs*, the current error rate of about 5×10^{-2} is already acceptable provided one allows the application of entanglement purification [17]; our scheme is significantly simpler.) Therefore, we need to cut down the loss figure from 5×10^{-2} to, say, 10^{-4}. Suppose we need to insert N phase shifters within 1 km of a telecommunication fiber. Denote the attenuation between a pair of phase shifters by l. Then, without the N phase shifters, we have $(1-l)^N=0.95$. For a sufficiently large N, we can expand the expression binomially and obtain the approximation $lN=0.05$. Now, with the insertion of phase shifters, we simply assume that the attenuation between two phase shifters is due to a second-order contribution of the form l^2. We further assume that those contributions sum up in the usual addition. Therefore, we have $l^2N=10^{-4}$. This yields $l=2\times10^{-3}$ and $N=25$. Recalling that two phase shifters are needed per cancellation step, we see that about 50 phase shifters have to be inserted in a distance of 1 km which translates to one phase shifter every 20 m. This figure is merely a rough upper-bound estimate on the distance Δ between two phase shifters for our scheme to be useful; one can also determine Δ via the experiment we propose below. Also note that we have assumed here that the fiber is straight as is typically done in theoretical models. In order to regain the straight fiber approximation, in the case of a curved fiber Δ is upper bounded by the local radius of curvature.

While in spirit our method is similar to BB decoupling [7], a major advantage here is that we do not need to apply any time-dependent pulses, which may result in significant uncertainties such as gate errors and off-resonance transitions. Instead, the phase shifters may be incorporated into the fiber directly during the manufacturing process. Alternatively, time-independent (say, electronic or pressure) controls may be applied at various points of a telecommunication fiber to achieve the action of pulse shifters.

ANTI-BILINEAR-DECOHERENCE FIBER

We now consider higher-order processes. Although they are generally weak, the bilinear interactions appearing in H_I^q may still cause decoherence. A direct harmful consequence is to change the polarization direction, through a term such as $b_1^\dagger b_2$. In the classical case, the fiber structure can be designed so that a *known* polarization direction can be preserved [18]. In the quantum case the polarization direction is *not known* prior to the transmission and the classical method is not applicable. In this case one must in general consider a system-bath Hamiltonian that is a linear combination of all 10 possible independent bilinear terms: $\mathcal{A}=\{b_1^\dagger b_2, b_2^\dagger b_1, (b_1^\dagger)^2, (b_2^\dagger)^2, (b_1)^2, (b_2)^2\}$, $\mathcal{B}=\{b_1 b_2, b_1^\dagger b_2^\dagger\}$, $\mathcal{C}=\{b_1^\dagger b_1, b_2^\dagger b_2\}$ (the grouping will be clarified momentarily). It can be shown that all 10 of these terms can be eliminated by installing 18 linear optical devices that include beam splitters in addition to phase shifters—i.e., in 16 elementary steps (we combine beam splitting and phase shifting into one step). This result is based on Eq. (5) and the identities [which follow directly from Eq. (5)]

$$e^{i\phi\hat{n}}(b^\dagger)^2 e^{-i\phi\hat{n}} = e^{2i\phi}(b^\dagger)^2, \quad e^{i\phi\hat{n}}(b)^2 e^{-i\phi\hat{n}} = e^{-2i\phi}(b)^2. \tag{10}$$

The role of the beam splitter is to eliminate the set of operators $\mathcal{C}$; the beam splitter is inserted after the first eight steps. The 16-step result can be considerably simplified in a realistic situation wherein the two polarizations used to represent our qubit are degenerate. In this case $\mathcal{C}$ becomes $b_1^\dagger b_1 + b_2^\dagger b_2$, which generates an *overall* phase and hence will not cause decoherence. In this degenerate case, as we now show, we need only phase shifters to eliminate all contributions to decoherence. Let

$$\Pi_i = e^{i\pi\hat{n}_i}, \quad \Gamma = e^{i\pi(\hat{n}_1-\hat{n}_2)/2}, \tag{11}$$

i.e., a pair of phase shifters. It follows immediately from Eqs. (5) and (10) that

$$\Gamma^\dagger \mathcal{A} \Gamma = \mathcal{A},$$

while

$$\Gamma^\dagger \mathcal{B} \Gamma = \mathcal{B}, \quad \Pi^\dagger \mathcal{A} \Pi = \mathcal{A}, \quad \Pi^\dagger \mathcal{B} \Pi = \mathcal{B} \tag{12}$$

(where $\Pi = \Pi_1 \Pi_2$ was used in Eq. (7)). From these and the results for the "anti-linear-decoherence fiber," the sequence $\Omega_{12} \equiv [2, \Pi, 1, \Pi]$ does not contain any linear terms, but still contains all bilinear terms. Then, the sequence

$$\Omega_{1234} \equiv [\Omega_{34}, \Gamma^\dagger, \Omega_{12}, \Gamma] = [4, \Pi, 3, \Pi\Gamma^\dagger, 2, \Pi, 1, \Pi\Gamma] \tag{13}$$

has, in four elementary phase-shifter steps, eliminated H_I^l as well as $\mathcal{A}$ and, in particular, the polarization-direction-changing terms $b_1^\dagger b_2$ and $b_2^\dagger b_1$: at this point we have a *polarization-preserving fiber.* Note that the composite terms can be combined into a single phase shifter—i.e.,

$$\Pi\Gamma^\dagger = e^{i\pi(\hat{n}_1+3\hat{n}_2)/2},$$

$$\Pi\Gamma = e^{i\pi(3\hat{n}_1+\hat{n}_2)/2}. \tag{14}$$

The only remaining bilinear terms at this point are the counterrotating terms $\mathcal{B}=\{b_1 b_2, b_1^\dagger b_2^\dagger\}$, which are typically neglected in the rotating-wave approximation [11]. To eliminate them, nevertheless, we note that

$$\Pi_1 \mathcal{B} \Pi_1 = -\mathcal{B}.$$

Therefore the sequence that eliminates *all* linear and bilinear terms for degenerate qubit states is

$$[\Omega_{5678}, \Pi_1, \Omega_{1234}, \Pi_1] = [8, \Pi, 7, \Pi\Gamma^\dagger, 6, \Pi, 5, \Pi\Gamma\Pi_1, 4, \Pi, 3, \Pi\Gamma^\dagger, 2, \Pi, 1, \Pi\Gamma\Pi_1],$$

which involves eight elementary phase-shifter steps (note that $\Pi\Gamma\Pi_1 = e^{i\pi(5\hat{n}_1+\hat{n}_2)/2}$). At this point we have a fiber that is completely free of both linear and bilinear decoherence-causing terms for degenerate polarization qubits.

We can repeat the mixed-classical-quantum rough distance estimate above by simply assuming that now contributions to decoherence come only due to third order in l: $l^3 N = 10^{-4}$. This leads to $N = 5/\sqrt{20} \approx 1.2$, and recalling that eight phase shifters are needed per cancellation step, we arrive at an upper-bound estimate of about ten phase shifters per km or one phase shifter every 100 m. These phase shifters must be introduced in addition to the ones used above for cancellation of first-order effects. We have again assumed here that the fiber is straight; local curvature may impose a lower upper bound.

GENERAL DECOHERENCE ELIMINATION

So far we have considered linear and bilinear photon terms in the interaction Hamiltonian. The most general two-mode photon-related term in a Hamiltonian is $b_1^{\dagger r} b_1^s b_2^{\dagger k} b_2^l$. Provided $r \neq s$ and $k \neq l$ the identity

$$e^{i(\alpha n_1+\beta n_2)} b_1^{\dagger r} b_1^s b_2^{\dagger k} b_2^l e^{-i(\alpha n_1+\beta n_2)} = e^{i[(r-s)\alpha+(k-l)\beta} b_1^{\dagger r} b_1^s b_2^{\dagger k} b_2^l$$

shows that such a term can be eliminated using only phase shifters. For example, when $r+s+k+l$ is an odd number, our considerations in the linear case show that the term can be eliminated using the phase shifter Π, while $b_1^{\dagger 2} b_2^2$ can be eliminated using $e^{-i(\pi/2)n_1}$. High-order terms with $r,s,k,l > 1$ arise if one considers the relativistic contribution, and they appear also in most of the effective photon scattering theories. It should be clear that if such terms arise, they can be reduced using additional phase shifters or beam splitters in the case $r=s$ and/or $k=l$, which arise due to terms containing photon number operators.

CONNECTION TO KNOWN LEADING LOSS MECHANISMS IN OPTICAL FIBERS

As mentioned in a previous section the leading loss mechanisms in optical fibers are well characterized: UV absorption, Rayleigh backscattering, OH absorption, and infrared absorption. It is useful to quickly review how these processes arise and then are treated by our method. Consider, for example, the case of Rayleigh backscattering. We base our discussion on the standard reference [19] (for a general description of absorption see p. 168; the cross section of Raleigh scattering is given on pp. 371–373). The discussion starts [19] [Eq. (4.9.9)] from the *dipole approximation* to our general photon-matter interaction Hamiltonian, Eq. (3):

$$\hat{H}_{ED} = ie\sum_{\mathbf{k}}\sum_{\lambda}\sum_{i,j}(\hbar\omega_k/2\varepsilon_0 V)^{1/2}\mathbf{e}_{\mathbf{k}\lambda}\cdot\mathbf{D}_{ij}\{\hat{b}_{\mathbf{k}\lambda}\exp(i\mathbf{k}\cdot\mathbf{R}) - \hat{b}^{\dagger}_{\mathbf{k}\lambda}\exp(-i\mathbf{k}\cdot\mathbf{R})\}|i\rangle\langle j|, \tag{15}$$

where $|i\rangle$ is the interacting charged particle state or the eigenstate of H_M, $\mathbf{R}$ is the atom position, $V=L^3$ is the volume, $\mathbf{D}_{ij}=-e\langle i|\Sigma\mathbf{r}_\alpha|j\rangle$ are the matrix elements of the atomic dipole moment, and λ is the polarization. A general scattering transition rate τ is [19] [Eq. (7.7.2)]:

$$\frac{1}{\tau} = \sum_f\sum_{\mathbf{k}_{sc}}\left|\sum_l \frac{\langle n-1,1,f|\hat{H}_{ED}|l\rangle\langle l|\hat{H}_{ED}|n,0,1\rangle}{n\omega-\omega_l}\right|^2 \times\frac{2\pi}{\hbar^4}\delta(\omega_f+\omega_{sc}-\omega), \tag{16}$$

where $|1\rangle$ and $|f\rangle$ are the atomic ground state and final state. Initially, there are n photons with frequency ω and wave vector $\mathbf{k}$. At the end there are $n-1$ incident photons and a single scattered photon with frequency ω_{sc} and wave vector $\mathbf{k}_{sc}$. Then the cross section follows from the relation $\sigma(\omega) = V/cn\tau$, and the differential light-scattering cross section is $d\sigma(\omega)/d\Omega$. The differential cross section of Rayleigh scattering is the special case when the atom returns to its ground state, which is [19] [Eq. (8.8.1)]

$$\frac{d\sigma(\omega)}{d\Omega} = \frac{e^4\omega^4}{16\pi^2\varepsilon_0^2\hbar^2c^4}\left|\sum_l \frac{(\mathbf{e}_{sc}\cdot\mathbf{D}_{1l})(\mathbf{e}\cdot\mathbf{D}_{l1})}{\omega_l-\omega} + \frac{(\mathbf{e}\cdot\mathbf{D}_{1l})(\mathbf{e}_{sc}\cdot\mathbf{D}_{1l})}{\omega_l+\omega}\right|^2, \tag{17}$$

where the parameters are obtained from the matrix elements of $\hat{H}_{ED}$.

The important equation is Eq. (16) above: it shows that Rayleigh scattering originates from the interaction $\hat{H}_{ED}$. Clearly, the differential cross section of Rayleigh scattering vanishes when $\hat{H}_{ED}$ is zero. *Our spatial BB method does just that: it effectively eliminates the interaction* $\hat{H}_{ED}$. Of course, this is not unique to Rayleigh scattering, which is just one of the processes derived from considering various cases involving $\hat{H}_{ED}$. For example, photon absorption and emission are mainly related to transitions involving two atomic or molecular levels. The corresponding matrix element for absorption is [19] [Eq. (4.10.1)]

$$\langle n_{\mathbf{k}\lambda}-1,2|\hat{H}_{ED}|n_{\mathbf{k}\lambda},1\rangle = i\hbar g_{k\lambda}\exp[i(\omega_0-\omega_k)t+i\mathbf{k}\cdot\mathbf{R}]n_{\mathbf{k}\lambda}^{1/2}, \tag{18}$$

where $g_{k\lambda}=(e\omega_k/2\varepsilon_0\hbar V)^{1/2}\mathbf{e}_{\mathbf{k}\lambda}\cdot\mathbf{D}_{12}$. The radiative lifetime is

$$1/\tau_R = 2\pi\sum_{\mathbf{k}}\sum_{\lambda}g_{\mathbf{k}\lambda}^2\delta(\omega_k-\omega_0), \tag{19}$$

and, of course, it follows from Eq. (18) that this absorption is prevented when $\hat{H}_{ED}$ is zero.

Note how $\hat{H}_{ED}$, which is effectively eliminated by our method, involves the bosonic raising and lowering operators $\hat{b}_{\mathbf{k}\lambda}, \hat{b}^{\dagger}_{\mathbf{k}\lambda}$. The reason that our method is so general is that it acts directly on these operators and "time-reverses" $\hat{H}_{ED}$ by flipping their sign.

PROPOSAL FOR AN EXPERIMENT

As mentioned above, a crucial requirement for the success of our proposed method is to insert the optical elements at intervals over which the coherence loss is still quadratic, rather than exponential. An experiment to test for this regime is thus useful. This could be done by monitoring the coherence (in particular, loss) locally, by focusing onto the edge of the fiber and collecting light into a photon-counting device (since the absolute intensity would be very small). By moving the focus along the fiber, one should be able to track the decay as a function of distance from the fiber entry point and observe the required quadratic-to-exponential transition, yielding an estimate of Δ.

To actually test the method in the presence of phase shifters, one could repeat the above experiment with a single fiber and write some phase-shift segments into it (as in the manufacturing of fiber Bragg gratings), at intervals bounded above by those determined from the first experiment. We note that a point of some potential concern is the impedance mismatch between air and the phase-shifter material, which will lead to reflection. Let n_i $(i=1,2)$ denote the indices of refraction: the reflected amplitude is $(n_2-n_1)/(n_2+n_1)$, which leads, at normal incidence, to 4% loss per air-glass interface. However, a standard antireflection coating can solve the problem: a quarter-wave layer of material at $\sqrt{n_1n_2}$ between the two materials (two equal reflections out of phase cancel out). In fibers the index changes will be smaller and reflection is typically neglected. Moreover, by writing a smooth phase profile as in the experiment proposed above, the reflection problem essentially disappears.

Once Δ has been estimated, one can proceed to directly test our method, as follows. Take two fiber segments and write a π phase shifter (PS) into each. Attach them colinearly (i) in the order PS-fiber-PS-fiber, (ii) in the order fiber-PS-PS-fiber, and perform a photon counting measurement. Our method should reduce attenuation in (i) by comparison to (ii).

CONCLUSIONS

We have proposed a method to reduce quantum noise in optical fibers via the insertion of phase shifters at appropriately spaced intervals. We have shown that, in principle, this method can eliminate all quantum noise processes that do not involve photon number operators in the system-bath Hamiltonian; when such terms do arise, the phase shifters need to be supplemented with beam splitters, and our conclusions remain. Thus, with simple linear-optical devices, quantum noise in optical fibers can be drastically reduced. This conclusion has potentially important implications for quantum communication (and its variants, quantum cryptography and distributed quantum computing) via optical fibers. The practical feasibility of our method hinges on the required distance between phase shifters. We have given a rough upper-bound estimate of several meters based on known attenuation rates. The Appendix presents a more detailed calculation that predicts a range of distances, depending on the bath spectral density appropriate for a fiber. Ultimately we believe that the best way to test our proposal is to perform the relatively straightforward experiment that it implies.

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APPENDIX: DETAILED MODEL FOR ESTIMATING Δ

Recall that our main approximation was the assumption of average fiber homogeneity, Eq. (8). In this appendix we relax this assumption in order to estimate an upper bound on the distance Δ between phase shifters. We do this by considering corrections to order τ^2 and the nonideal case

$$H_0((k-1)\Delta) = H_0(k\Delta) + \varepsilon P_k,$$
$$H_I^l((k-1)\Delta) = H_I^l(k\Delta) + \varepsilon Q_k, \tag{A1}$$

where $\varepsilon \ll 1$ and we take P_k, Q_k to be independent, identically distributed (IID) Gaussian, local, and time-dependent operator-valued corrections. This phenomenological model of fiber inhomogeneity may be the result of material nonuniformity along the fiber (such as local defects), slow time-dependent fluctuations in fiber properties, or even the quadratic interaction (4). By virtue of the central limit theorem it will be accurate in the case of a *large number* of defects. We assume that the effective BB time interval τ is chosen to be on the order of the small parameter ε (though we make no attempt to estimate ε). In this case, using the BCH formula $e^A e^B = e^{A+B+[A,B]/2+\cdots}$ to second order (i.e., keeping only terms of order $\varepsilon, \tau, \varepsilon^2, \varepsilon\tau, \tau^2$), we find, instead of the ideal Eq. (9),

$$\begin{aligned} &e^{-iH((k-1)\Delta)\tau}\Pi e^{-iH(k\Delta)\tau}\Pi \\ &= e^{-iH((k-1)\Delta)\tau} e^{-i\Pi H(k\Delta)\Pi\tau} \\ &= e^{-i[H_0(k\Delta)+H_I^l(k\Delta)+\varepsilon(P_k+Q_k)]\tau} e^{-i[H_0(k\Delta)-H_I^l(k\Delta)]\tau} \\ &\approx \exp\{-i\tau[2H_0(k\Delta)+\varepsilon(P_k+Q_k)] - \tau^2[H_I^l(k\Delta),H_0(k\Delta)]\} \end{aligned} \tag{A2}$$

where in the second line the effect of the phase shifters was to flip the sign (and thus cancel) the $H_I^l(k\Delta)$ term. To the same order of accuracy the overall evolution operator becomes

$$U'(T,0) \approx e^{-iH_0(0)T} \exp\left\{-\tau^2 \sum_{k=1}^{N/2} [H_I^l(2k\Delta), H_0(2k\Delta)]\right\} \times \exp\left\{-i\varepsilon\tau \sum_{k=1}^{N/2} (P_{2k}+Q_{2k})\right\}. \tag{A3}$$

Let us evaluate the first exponential. Using Eqs. (1) and (2),

$$\begin{aligned} &-i[H_I^l(2k\Delta), H_0(2k\Delta)] \\ &= -i\sum_{j,j'} [(\hat{B}_j^\dagger(2k\Delta)b_j + \hat{B}_j(2k\Delta)b_j^\dagger), \\ &\hbar\omega_{j'}(2k\Delta)(\hat{n}_{j'}+1/2) + H_M(2k\Delta)] \\ &= -i\sum_{j=0,1} \{\hbar\omega_j(2k\Delta)\hat{B}_j(2k\Delta) + [\hat{B}_j(2k\Delta),H_M(2k\Delta)]\}b_j^\dagger \\ &-\{\hbar\omega_j(2k\Delta)\hat{B}_j^\dagger(2k\Delta) - [\hat{B}_j^\dagger(2k\Delta),H_M(2k\Delta)]\}b_j \\ &\equiv H', \end{aligned} \tag{A4}$$

where H' is an effective Hamiltonian (it is Hermitian), which plays the role of a Lamb shift [20]. We thus have for the first exponential in Eq. (A3):

$$\exp\left\{-\tau^2 \sum_{k=1}^{N/2} [H_I^l(2k\Delta), H_0(2k\Delta)]\right\} = \exp(-i\tau^2 H'), \tag{A5}$$

whose effect is an energy renormalization (i.e., a phase shift) and does not contribute to decoherence.

Next, consider the second exponential in Eq. (A3). The operator $G(t)$ defined through $\sum_{k=1}^{N/2}(P_{2k}+Q_{2k}) \sim \int_0^T [P(t)+Q(t)]dt \equiv \int_0^T G(t)dt$ is Gaussian distributed by our assumption that P_{2k}, Q_{2k} are Gaussian, IID random variables. We would like to estimate the average deviation in $U'(T,0)$ that results from its presence. Since $G(t)$ is Gaussian distributed the average can be computed as follows [21]:

$$\begin{aligned} &\left\langle \exp\left\{-i\varepsilon\tau \sum_{k=1}^{N/2} (P_{2k}+Q_{2k})\right\}\right\rangle \\ &\sim \left\langle \exp\left[-i\varepsilon\tau \int_0^T G(t)dt\right]\right\rangle \\ &= \exp\left[-i\varepsilon\tau \int_0^T \int_0^T \langle G(t)G(t')\rangle dt\, dt'\right] \\ &\equiv \exp[-\varepsilon\tau\Gamma(T)]. \end{aligned} \tag{A6}$$

Expressed in terms of Fourier components G_ω of $G(t)$ we have, for the decoherence factor

$$\Gamma(T) = \frac{1}{2}\int_0^\infty d\omega \langle G_\omega^2\rangle Q(\omega,T), \tag{A7}$$

where

$$Q(\omega,T) = \int_0^T \int_0^T dt\, dt' \cos[\omega(t-t')] = \left(\frac{2\sin(\omega T/2)}{\omega}\right)^2. \tag{A8}$$

But in the Gaussian case we have (as in the spin-boson model [21])

$$\langle G_\omega^2\rangle = \frac{1}{2}I(\omega)\coth\frac{\beta\omega}{2}, \quad \text{(A9)}$$

where $I(\omega)$ is the spectral density (of matter in the fiber) and β is the inverse temperature. Hence our result is that the correction is

$$\exp[-\varepsilon\tau\Gamma(T)] = \exp\left[-\varepsilon\tau\int_0^\infty d\omega\, I(\omega)\coth\frac{\beta\omega}{2}\left(\frac{\sin(\omega T/2)}{\omega}\right)^2\right]. \quad \text{(A10)}$$

The attenuation is thus strongly dependent upon the form of $I(\omega)$, but also depends sensitively on temperature. In particular, the thermal time scale $\hbar\beta$ is important in separating thermal effects from effects due purely to vacuum fluctuations [22]. In order to formally separate the two it is convenient to write

$$\coth\frac{\beta\omega}{2} = 1 + \overline{n}(\omega,\beta), \quad \text{(A11)}$$

where

$$\overline{n}(\omega,\beta) = \exp(-\beta\omega/2)/\sinh(\beta\omega/2) \quad \text{(A12)}$$

is the average number of field excitations at inverse temperature β.

In the limit of very low temperatures ($\beta \gg 1$) we have

$$\overline{n}(\omega,\beta) \overset{\beta\gg 1}{\approx} 2\exp(-\beta\omega) \quad \text{(A13)}$$

and we can analytically evaluate the integral in Eq. (A10)—e.g., for the class of Ohmic-type spectral densities—i.e., for the case

$$I(\omega) = \alpha\omega^n e^{-\omega/\omega_c}, \quad \text{(A14)}$$

where α is the coupling strength and ω_c is the high-frequency cutoff (note that α is not dimensionless). The result in the zero-temperature case is

$$\lim_{\beta\to\infty}\int_0^\infty d\omega I(\omega)\coth\frac{\beta\omega}{2}\left(\frac{\sin(\omega T/2)}{\omega}\right)^2 = \begin{cases} \frac{\alpha}{4}\ln[1+(\omega_c T)^2], & n=1, \\ \frac{\alpha}{2}\omega_c^{n-1}\Gamma(n-1)(1-[1+(\omega_c T)^2]^{(n-1)/2}\cos[(n-1)\arctan(\omega_c T)]), & n\neq 1. \end{cases} \quad \text{(A15)}$$

To obtain the nonzero-temperature correction in the approximation (A13) take these results, multiply by 2, replace ω_c by $\omega_c/(1+\beta\omega_c)$ everywhere, and add to the zero-temperature case. We tabulate a few cases of interest in the zero-temperature limit, letting $x \equiv \omega_c T$:

$$\lim_{\beta\to\infty}\exp[-\varepsilon\tau\Gamma(T)] = \begin{cases} (1+x^2)^{-\alpha\varepsilon\tau/4}, & n=1 \text{ (Ohmic)}, \\ \exp\left[-\frac{1}{2}\alpha\varepsilon\tau\omega_c\frac{x^2}{1+x^2}\right], & n=2 \text{ (super-Ohmic)}, \\ \exp\left[-\frac{1}{2}\alpha\varepsilon\tau\omega_c^2\frac{x^2(3+x^2)}{(1+x^2)^2}\right], & n=3 \text{ (Debye)}. \end{cases} \quad \text{(A16)}$$

Let $1-\delta(T)$ be the desired coherence value after time T (or distance X); then we need to solve for the phase-shifter spacing Δ from

$$\lim_{\beta\to\infty}\exp[-\varepsilon\tau\Gamma(T)] > 1-\delta(T). \quad \text{(A17)}$$

We find (assuming $\alpha > 0$)

$$\Delta^2 < -4v^2\ln[1-\delta(T)]/\ln[(1+x^2)], \quad n=1,$$

$$\Delta^2 < -\frac{2v^2}{\alpha\omega_c}\frac{1+x^2}{x^2}\ln[1-\delta(T)], \quad n=2,$$

$$\Delta^2 < -\frac{2v^2}{\alpha\omega_c^2}\frac{(1+x^2)^2}{x^2(3+x^2)}\ln[1-\delta(T)], \quad n=3. \quad \text{(A18)}$$

The present model is, unfortunately, too phenomenological to make a reliable estimate of Δ. Nevertheless, it is of some interest to see its prediction. E.g., we could wish to improve upon the current figure of merit of 0.25 db/km to the threshold value of $\delta(T)=10^{-4}$. Recall that $T=X/v$, $\tau=(\Delta/v)$ and we assumed $\tau\sim\varepsilon$. The coupling strength α is typically of order unity [7,23]; we shall set $\alpha=1$. We take $v=c/1.6$, the speed of light in a typical fiber, and $\delta(T)=10^{-4}$. The results in the three cases, with $x=1.6/3\times 10^{-5}\omega_c$, are displayed in Fig. 1, as a function of the high-frequency cutoff ω_c. As a rough reference, the Debye temperature of amorphous silica is $T_D=342$ K [24], yielding a Debye frequency estimate of $\omega_c=k_B T_D/\hbar=2\times 10^{13}$ Hz. The corresponding value of Δ is 6×10^5 m ($n=1$), 0.6 m ($n=2$), and 10^{-7} m ($n=3$).

This strong sensitivity to the decoherence model underscores the need for the proposed experiment in order to settle the question of the actual required distance between phase shifters. Nevertheless, one can make a heuristic argument

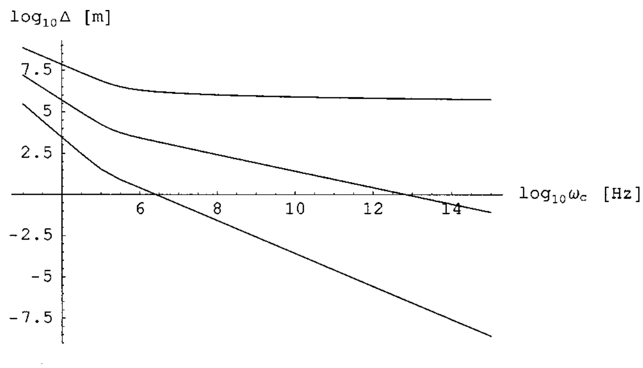

FIG. 1. Zero-temperature estimate of distance Δ between phase shifters (in meters), as a function of high-frequency cutoff ω_c (in Hz). Note the double-logarithmic scale. Upper, middle, and bottom curves correspond to $n=1$, 2, and 3, respectively, in Eq. (A18).

which favors the $n=1$ model. The argument is the following [25]: phenomenologically, decoherence effects in fibers are due to low-frequency fluctuations of the (optical-frequency) dielectric constants. The most obvious source of such fluctuations, though not the only one, is simply fluctuations of the total density at spatial Fourier component k equal to that of the light. Thus we are asking for the ω dependence of the imaginary part of the density autocorrelation function in the limit of small $k,\omega \ll ck$ (c=speed of sound). If one uses the standard "tunneling two-state system model" [21], this quantity should be linear in ω—i.e., $n=1$ in our language above. At first sight these considerations might seem not to settle the question, since for a perfectly uniform system fluctuations of the density should affect the two relevant components of the dielectric constant ε in exactly the same way and thus not affect their ratio, which is presumably what is important for decoherence of our qubit. However, because of the inhomogeneity at the microscopic scale, there should nevertheless be an effect which should be proportional to the rms fluctuations of the ratio of the quantities $d\varepsilon_V/d\rho$ and $d\varepsilon_H/d\rho$ over some characteristic length scale L (V =vertical, H=horizontal, ρ=density, and one may estimate $L=1/k$). The rms fluctuations are independent of ω, so one is still led to conclude that $n=1$. Considering the favorable scaling exhibited in the $n=1$ case as shown in Fig. 1, we believe that there is room for cautious optimism that our proposal can be made to work under conditions which are technologically feasible.

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