Universal Fault-Tolerant Quantum Computation in the Presence of Spontaneous Emission and Collective Dephasing

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A universal and fault-tolerant scheme for quantum computation is proposed which utilizes a class of error correcting codes that is based on the detection of spontaneous emission (of, e.g., photons, phonons, and ripplons). The scheme is compatible with a number of promising solid-state and quantum-optical proposals for quantum computer implementations, such as quantum dots in cavities, electrons on helium, and trapped ions.

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The most severe obstacle in the path towards the dramatic speedup offered by future quantum information processing (QIP) devices is decoherence: the process whereby a quantum system becomes irreversibly entangled with an uncontrollable environment (''bath''). This causes information loss and may degrade the operation of a quantum computer to the point where it can be efficiently simulated classically [1]. One can formally model decoherence processes in QIP as being due to operators $\{S_i\}$ acting on the system qubits $\{i\}$, that are coupled to bath operators B_i in a system-bath interaction Hamiltonian $H_{SB} = \sum_i S_i \otimes B_i$. Two of the main proposals to combat decoherence in QIP are *quantum error correction codes* (QECCs) and *decoherence free subspaces* (DFSs). In QECCs multiqubit states define quantum ''code words,'' with the special property that they are distinguishable (orthogonal) after the occurrence of errors, i.e., decoherence. Appropriate nondestructive measurements yield an ''error syndrome,'' which can be used for recovery from the errors [2,3]. The DFS approach similarly invokes code words, but it does not require active measurement and recovery, since the encoded states are chosen so as to be immune from decoherence: a state $|\psi_n\rangle$ is decoherence-free if $S_i|\psi_n\rangle = c_i|\psi_n\rangle$, where c_i is a scalar that *does not depend on* $|\psi_n\rangle$ [4,5]. This condition, which we refer to as *the DFS condition* below, assumes that there is a *symmetry* in the system-bath interaction, such as ''collective decoherence,'' wherein H_{SB} is qubit-permutation-invariant [6,7]. A number of studies have pointed out the advantages of combining the QECC and DFS approaches [5,8,9]. Of particular relevance is the recent work by Alber *et al.* [9], who introduced a new class of hybrid DFS-QECC codes, known as ''detected-jump correcting (DJC) quantum codes.'' These codes, which we review below, are particularly useful in the case of spontaneous emission errors: $S_i = |0\rangle_i\langle 1|$, where $|1\rangle_i$ ($|0\rangle_i$) is the excited (ground) state of, e.g., an atom i. The DJC codes improve upon earlier work on QECC in the presence of spontaneous emission [10] in that they take advantage of knowing where the emission event occurred (which qubit). This assumes that the mean distance between qubits exceeds the wavelength of the emission. The work by Alber *et al.* [9] left open the question of computation with these codes [11].

We show here how to perform universal, fault-tolerant quantum computation (QC) on a class of the DJC hybrid codes, in the presence of spontaneous emission and collective dephasing errors. The latter are errors that arise when the system-bath interaction can be written as $H_{SB} = S_z \otimes B_z$, where $S_z = \sum_i \sigma_i^z$ (σ_i^z is the Pauli σ^z matrix acting on the ith qubit), and have been extensively discussed before, both theoretically [5,7,12,13] and experimentally [14,15]. We show below that in order to accomplish this *we need only control the coupling constants* J_{ij}^z *and/or* J_{ij} *appearing in an anisotropic, exchange-type system Hamiltonian*: $H_S = \sum J_{ij}(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y) + J_{ij}^z\sigma_i^z\sigma_j^z$. The case $J_{ij}^z \neq 0$ ($J_{ij}^z = 0$) is known as the XXZ (XY) model. These types of Hamiltonians naturally appear in a number of promising proposals for implementing quantum computers, in which spontaneous emission, as well as collective dephasing errors, are important sources of decoherence. For example, the quantum Hall [16], quantum dots [17], dimer atoms in a solid host [18], and atoms in cavities [19] proposals are all of the XY type and suffer from photon and phonon emission, while the electrons on helium proposal [20] is of the XXZ type and suffers in addition from ripplon emission. The phonon-mediated ion-ion interaction in the Sørensen-Mølmer scheme for trapped-ion QC [21] is equivalent to an XY model, and this proposal too suffers from spontaneous emission of photons and phonons, as well as from collective dephasing [15]. Other sources of decoherence can also appear in all proposals, but as shown in [13], using appropriate pulse

sequences generated by the XY Hamiltonian, they can be reduced to the collective dephasing type. The idea of universal QC using the XY or XXZ interaction has been considered before, starting with [17], where the XY interaction had to be supplemented with arbitrary single-qubit operations. In [22] it was shown how to perform universal QC using the XY interaction supplemented with static single-qubit energy terms (e.g., a Zeeman splitting) and an encoding into two qubits; in [23] universal gate sequences were given for the XY interaction alone, using an encoding into three qubits; and in [22,24] universal gate sequences using the XXZ interaction were found for encodings into two or more qubits. Here we use an encoding into four or more qubits that has the additional, significant advantage of offering protection (using a QECC) against spontaneous emission errors.

Detected jump-corrected codes.—In the DJC codes method, the Markovian quantum trajectories approach [25] is used to describe decoherence. This approach is equivalent to the Lindblad semigroup master equation [26]. The evolution is decomposed into two parts: a conditional *non-Hermitian* Hamiltonian H_C, interrupted at random times by occurrence of random errors. For errors such as spontaneous emission, where the jump can be detected by observation of the emission, the quantum trajectories approach also provides a way to combine QECCs and DFSs [9]. The DFS takes care of the conditional evolution, whereas the QECC deals with the random jumps that couple DFS states with states outside of the DFS. Formally, the conditional Hamiltonian is given by [25]: $H_C = H_S - \frac{i}{2}\sum_i \kappa_i S_i^\dagger S_i$, where κ_i are (in our case) the spontaneous emission rates. The DFS in the quantum jump approach is given by the eigenspace of the *collective* operator $C \equiv \sum_i \kappa_i S_i^\dagger S_i$. The symmetry that leads to the DFS condition being satisfied is $\kappa_i \equiv \kappa$. For n qubits and spontaneous emission errors, we then have $C = \kappa \sum_{i=1}^n |1\rangle_i\langle 1|$, and the DFS with maximal dimension is comprised of (''computational'') basis states with $\frac{n}{2}$ 1's and $\frac{n}{2}$ 0's. It has dimension $\binom{n}{n/2}$ and eigenvalue $n/2$ under C. From here on, we work exclusively with this DFS.

Consider such a DFS encoding into $n = 4$ qubits ($n = 2$ qubits already yields a logical qubit, but $n = 4$ is the smallest such *generalizable* example, in the sense of the multi-encoded-qubit scheme discussed below). It protects against the conditional evolution, so what remains is to protect against the jumps. As shown in [9], if we assume knowledge of the position of errors by observing the emission, then one can use states in this DFS in order to construct a QECC that encodes one logical qubit:

$$|0\rangle_L = \frac{|1010\rangle + |0101\rangle}{\sqrt{2}}, \qquad |1\rangle_L = \frac{\pm(|0110\rangle + |1001\rangle)}{\sqrt{2}}, \tag{1}$$

where the choice of sign is + (−) if $J_{12} < 0$ (> 0), as will be clarified below, and for simplicity we assume from here on that $J_{ij}^z \geq 0$. The general QECC condition [2] that keeps the errors from scrambling the code words $|\psi_n\rangle$ takes the following form, provided we know which of the errors indexed by i has occurred:

$$\langle\psi_m|S_i^\dagger S_i|\psi_n\rangle = \Lambda_i \delta_{mn}, \tag{2}$$

where Λ_i is a number independent of the code words [9]. This is easily verified for the code in Eq. (1). Therefore, this code offers complete protection against the detected-jump spontaneous emission process. Note that in addition to the states in Eq. (1) the state $|2_L\rangle = (|0011\rangle + |1100\rangle)/\sqrt{2}$ also satisfies the QECC condition (2) and is inside the DFS.

Alber *et al.* [9] gave a combinatorial design-theory method for generalizing the code of Eq. (1). We now describe a class of these codes that come with natural encoded qubit operations, that allow for *universal, scalable, and fault-tolerant QC*. Our *protocol* is as follows: Computation is performed during the conditional evolution periods, while the system is in a DFS. If a jump is detected, it must first be corrected (as in QECC) before computation can resume. We note that the performance of DJC codes in the presence of imperfections such as detection inefficiencies, unequal decay rates κ_i, and time delay between error detection and recovery operations has been analyzed in [9(c)], with favorable conclusions.

Example: universal encoded logic operations for the 4-qubit DJC code.—In order to perform universal QC, we first identify a set of generators of all encoded single-qubit transformations. As is well known, arbitrary single-qubit transformations can be generated from Hamiltonians via time evolution, using a standard Euler angle construction: $e^{-i\omega \mathbf{n}\cdot\boldsymbol{\sigma}} = e^{-i\beta\sigma^z}e^{-i\theta\sigma^x}e^{-i\alpha\sigma^z}$. This is a rotation by angle ω about the axis $\mathbf{n}$, given in terms of three successive rotations about the z and x axes. Let us now suppose that we have at our disposal a controllable XXZ Hamiltonian, as defined above. This gives us the ability to switch on/off, separately, the Hamiltonian terms $T_{ij} \equiv \frac{1}{2}(X_iX_j + Y_iY_j)$ and Z_iZ_j, where $X_i \equiv \sigma_i^x$, etc. These operators preserve the number of 0's and 1's [24,27]. Since this implies that they cannot take states outside of the DFS, it follows that they are *naturally fault-tolerant* [12]. Now suppose that we turn $|J_{12}|$ (J_{13}^z) on for a time t such that $|J_{12}|t/\hbar = \theta$ ($J_{13}^z t/\hbar = \theta$). Then:

$$e^{-i\theta T_{12}}|\epsilon\rangle_L = \cos\theta|\epsilon\rangle_L - i\sin\theta|\bar{\epsilon}\rangle_L, \qquad e^{-i\theta Z_1Z_3}|0\rangle_L = e^{-i\theta}|0\rangle_L, \qquad e^{i\theta Z_1Z_3}|1\rangle_L = e^{i\theta}|1\rangle_L, \tag{3}$$

where $\epsilon = 0$ or 1, and $\bar{\epsilon} = (\epsilon + 1)\mathrm{mod}2 = \mathrm{NOT}(\epsilon)$. These equations show that T_{12} and Z_1Z_3 have precisely the action of single-qubit σ^x and σ^z transformations, on the code states in Eq. (1), and that this code space is perfectly preserved under T_{12} and Z_1Z_3. We denote logical X (Z) operations on the ith encoded qubit by $\bar{X}_i$ ($\bar{Z}_i$). Thus, $\bar{X}_1 = T_{12}$ and $\bar{Z}_1 = Z_1Z_3$ and we have the ability to

generate arbitrary encoded single-qubit transformations in the XXZ model. This is particularly relevant for the electrons on helium proposal [20].

However, in many QC proposals of interest, it is either inconvenient to separately control J^z_{ij}, or such exchange interactions vanish [16–19,21]. We must then resort to controlling only the XY term. Now, as shown in [22], using the "encoded recoupling" method, it is possible to generate $Z_{2i-1}Z_{2j-1}$ operations with arbitrary i, j as long as one can control an XY Hamiltonian. Define $C^{\phi}_{A} \circ B \equiv \exp(-i\phi A)B\exp(i\phi A)$, then [22,24,28]:

$$2C^{\pi/2}_{(1/2)T_{2i,2j-1}} \circ (C^{\pi/2}_{T_{2i-1,2i}} \circ T_{2i-1,2j-1}) = Z_{2i-1}Z_{2j-1}. \quad (4)$$

The procedure given in Eq. (4) is a five-step implementation of the Ising interaction $Z_{2i-1}Z_{2j-1}$. For $i = 1, j = 2$ this yields $\bar{Z}_1$, and we have all we need for encoded single-qubit transformations in the XY model.

The one apparent disadvantage of the procedure in Eq. (4) is that in 1D it requires next-nearest neighbor interactions (this is inevitable with an XY interaction in 1D [24]), but note that these interactions are still nearest-neighbor on a 2D triangular qubit lattice. Let us also note that application of $T_{2i-1,2j-1}$ [as arises in Eq. (4); e.g., T_{13} is needed for the implementation of $\bar{Z}_1$], maps the code state $|1\rangle_L$ to a superposition of $|1\rangle_L$ and $|2\rangle_L$. While $|2\rangle_L$ is not part of our encoded qubit, it is part of the DJC code [it is in the DFS and satisfies the QECC condition (2)], so that the fault tolerance of our procedure is not violated.

Generalization: DJC code encoding several qubits.—We now introduce an encoding that generalizes the code in Eq. (1) to arbitrary numbers of encoded qubits. Let

$$|\tilde{0}\rangle_i \equiv |0_{2i-1}1_{2i}\rangle, \qquad |\tilde{1}\rangle_i \equiv -\mathrm{sgn}(J_{2i-1,2i})|1_{2i-1}0_{2i}\rangle.$$

We then define a code as follows:

$$|\epsilon_L\rangle_1 \otimes \cdots \otimes |\epsilon_L\rangle_{n-1} = \frac{|\tilde{\epsilon}\rangle_1 \cdots |\tilde{\epsilon}\rangle_{n-1}|\tilde{0}\rangle_n + \text{conj.}}{\sqrt{2}}, \quad (5)$$

where $\epsilon = 0$ or 1, and "conj." denotes the bitwise NOT of the first ket. The rate (number of encoded per physical qubits) of this class of codes is $r = \frac{n-1}{2n} \to_{n\to\infty} \frac{1}{2}$. As in the case of a single encoded qubit, Eq. (3), the generators of encoded σ^x and σ^z transformations are

$$\bar{X}_i = \tfrac{1}{2}(X_{2i-1}X_{2i} + Y_{2i-1}Y_{2i}), \qquad \bar{Z}_i = Z_{2i-1}Z_{2n-1}, \quad (6)$$

as is easily verified by checking their action on $|\tilde{0}\rangle_i, |\tilde{1}\rangle_i$. Using the Euler angle formula, we may construct arbitrary encoded single-qubit operations from $\bar{X}_i$ and $\bar{Z}_i$, using operations from within the XY or XXZ models only. The fact that we can apply such single encoded qubit operations on the code in Eq. (5) shows that *this code is equipped with a (formal) tensor product structure, and allows for scalable QC.*

At this point we are ready to show how to implement a controlled-phase (CP) gate, $\mathrm{CP}|x, y\rangle = (-1)^{xy}|x, y\rangle$ (where x, y are 0 or 1), which together with arbitrary single-qubit operations is universal for QC [29]. As is well known, the CP gate is generated by an Ising interaction $Z \otimes Z$ [29]. Thus, to generate a CP gate between *encoded* qubits i, j, we must consider $\bar{Z}_i\bar{Z}_j = (Z_{2i-1}Z_{2n-1})(Z_{2j-1}Z_{2n-1}) = Z_{2i-1}Z_{2j-1}$. In the XXZ model, such a two-body Ising interaction is directly controllable. In the XY model, we can generate it using the five-step procedure of Eq. (4). Furthermore, since a CP gate can be used to construct a SWAP gate [29], we need only use at most next nearest-neighbor interactions (in 1D; nearest-neighbor in 2D) in order to couple arbitrary pairs of encoded qubits. Finally, we stress that the combination of Eqs. (3), (4), and (6), and the result above for $\bar{Z}_i\bar{Z}_j$, is an explicit prescription for constructing arbitrary quantum circuits in terms of the XY and/or XXZ interactions.

Fault tolerant measurement and recovery.—An inherent assumption in the DJC codes method is that it is possible to observe which of the physical qubits underwent spontaneous emission [9]. This is a manifestly fault-tolerant measurement [3], in the sense that observing an error on a specific qubit cannot cause errors to multiply. Now consider recovery from spontaneous emission errors. If the error affects qubit $2i - 1$ ($2i$), the effect is $|\tilde{0}\rangle_i \mapsto |\tilde{0}\rangle_i$, $|\tilde{1}\rangle_i = |0_{2i-1}0_{2i}\rangle$ ($|\tilde{0}\rangle_i \mapsto |0_{2i-1}0_{2i}\rangle$, $|\tilde{1}\rangle_i = |\tilde{1}\rangle_i$). The recovery operation must therefore correspondingly take $|0_{2i-1}0_{2i}\rangle$ to $|\tilde{1}\rangle_i$ ($|\tilde{0}\rangle_i$), while not affecting $|\tilde{0}\rangle_i$ ($|\tilde{1}\rangle_i$). Corresponding unitary operations with the desired effect are cousins of the standard controlled-NOT [29], defined on the subspace of qubits $2i - 1, 2i$:

$$CX_1 = \begin{pmatrix} & & 1 & \\ & 1 & & \\ 1 & & & \\ & & & 1 \end{pmatrix}, \qquad CX_2 = \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & 1 & \\ & & & 1 \end{pmatrix}.$$

Now, in order to perform these recovery operations we must assume that in addition to an XY or XXZ Hamiltonian we have the ability to control single-qubit energies (i.e., control terms of the form $\omega_i Z_i$) and perform a Hadamard [$W = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right)$] gate, which is certainly reasonable in optics-based QC proposals [17–19,21] (where such single-qubit operations are executed through the application of laser pulses). This requirement is harder to satisfy in solid-state QC proposals that use gate voltages for single-qubit operations [16,20], but is not unreasonable. Note that the assumption that we can perform single-qubit operations is made only to enable recovery from spontaneous emission errors. It is needed since the XY and XXZ Hamiltonians preserve the number of 0's and 1's in each code word, while spontaneous emission lowers the number of 1's. Now, with the extra assumption we are able to construct CX_1 and CX_2 in seven (three) steps, assuming a controllable XY (XXZ) Hamiltonian. For example, $CX_1 = e^{-i(\pi/4)}(WP \otimes P^2)\exp(i\frac{\pi}{4}Z_1Z_2) \times (W \otimes P)$, where $P \equiv e^{-i(3\pi/4)Z} = e^{i(3\pi/4)}\mathrm{diag}(i, 1)$, and

we recall that five steps are needed to generate Z_1Z_2 from the XY Hamiltonian. To obtain CX_2, swap the order of the factors around the $\otimes$ symbols. Since we apply CX_1 and CX_2 only within a block encoding a single-qubit, the operations we perform can affect only that encoded qubit. Therefore, if the operations themselves are faulty the error cannot spread to other encoded qubits. This means that our recovery operations are fault tolerant [3].

State preparation and readout.—Finally, we must also show that our encoded states can be reliably prepared and read out. A general preparation technique is cooling to the ground state of a Hamiltonian. For this procedure to work, there should be an energy gap Δ between the code subspace and other states. Diagonalization of the XY Hamiltonian $J_{ij}T_{ij} = \frac{J_{ij}}{2}(X_iX_j + Y_iY_j)$ in the subspace of qubits i, j yields, depending on whether $J_{ij} > 0$ or <0, either the singlet state $|s\rangle_{ij} = \frac{1}{\sqrt{2}}(|0_i1_j\rangle - |1_i0_j\rangle)$ or the triplet state $|t\rangle_{ij} = \frac{1}{\sqrt{2}}(|0_i1_j\rangle + |1_i0_j\rangle)$, as the ground state, with energy $-|J_{ij}|$. Consider the case of a single encoded qubit and assume $J_{ij} > 0$: the ground state of the XY Hamiltonian $J_{12}T_{12} + J_{34}T_{34}$ is $|s_{12}\rangle \otimes |s_{34}\rangle$, which is exactly $\frac{1}{\sqrt{2}}(|0\rangle_L + |1\rangle_L)$, in terms of the code states of Eq. (1) with the choice of "$-$" for $|1\rangle_L$. In other words, cooling prepares a state that is in the code subspace, and application of the encoded logical operations derived above can rotate this initial state to any other desired encoded state. To prepare a state in the code subspace of $2n$ physical qubits, we turn on the pairwise XY Hamiltonian $\sum_{i=1}^{n} J_{2i-1,2i}T_{2i-1,2i}$, keep the temperature below Δ, and wait. The resulting ground state is $\otimes_{i=1}^{n}|s\rangle_{2i-1,2i}$, and a simple calculation shows that this state is in the code space:

$$\otimes_{i=1}^{n}|s\rangle_{2i-1,2i} = \otimes_{j=1}^{n-1}(|0_L\rangle_j + |1_L\rangle_j)/\sqrt{2}.$$

Identical conclusions hold when assuming $J_{ij} < 0$, with $|t\rangle_{2i-1,2i}$ replacing $|s\rangle_{2i-1,2i}$. Thus, cooling always prepares a state in the code subspace and can serve as an initialization procedure for our protocol. Measurement can be done analogously, i.e., by using the energy difference to distinguish a singlet from a triplet state on pairs of qubits encoding a logical qubit [27]. Thus, to distinguish $|0_L\rangle_j$ from $|1_L\rangle_j$, we first apply an encoded Hadamard gate to physical qubits $2j-1, 2j$, mapping $|0_L\rangle_j \to (|0_L\rangle_j + |1_L\rangle_j)/\sqrt{2}$ and $|1_L\rangle_j \to (|0_L\rangle_j - |1_L\rangle_j)/\sqrt{2}$, which by the preparation arguments above correspond to singlet and triplet states, depending on $\mathrm{sgn}(J_{2j-1,2j})$.

Conclusions.—We have studied a class of "detected-jump" codes that is capable of avoiding collective dephasing errors and correcting spontaneous emission errors on a single qubit. These codes are a hybrid of decoherence-free subspaces and active quantum error correction, and use $2n$ qubits to encode $n-1$. We have shown how to quantum compute universally and fault tolerantly on this class of codes, using Hamiltonians (XY- and XXZ-type) that are directly relevant to a number of promising solid-state and quantum-optical proposals for quantum computer implementations [16–21].

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