Dressed Qubits

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(Received 19 March 2003; published 28 August 2003)

Inherent gate errors can arise in quantum computation when the actual system Hamiltonian or Hilbert space deviates from the desired one. Two important examples we address are spin-coupled quantum dots in the presence of spin-orbit perturbations to the Heisenberg exchange interaction, and off-resonant transitions of a qubit embedded in a multilevel Hilbert space. We propose a "dressed qubit" transformation for dealing with such inherent errors. Unlike quantum error correction, the dressed qubit method does not require additional operations or encoding redundancy, is insensitive to error magnitude, and imposes no new experimental constraints.

DOI: 10.1103/PhysRevLett.91.097904 PACS numbers: 03.67.Pp, 03.67.Lx

In theoretical models of physical systems implementing quantum computers it is common to make simplifying assumptions about the form of the underlying system Hamiltonian or the system's interaction with external controls. Such assumptions lead to an idealized system description that is convenient for the purpose of proving desirable properties, such as the ability to perform universal quantum computation (QC) [1]. Important examples of simplifying assumptions include the neglect of certain interactions (e.g., of spin-orbit coupling in models of quantum-dot quantum computers [2,3]) and the reduction of a multilevel Hilbert space to a two-dimensional one (thus neglecting off-resonant effects, e.g., in superconducting qubits [4–6]). Approaches for dealing with the actual system, S_A, as opposed to the idealized system, S_I, typically (though not always [7]) treat $\Delta \equiv S_A - S_I$ as a *problem* that needs to be overcome: Δ is considered an *inherent error.* For example, shaped pulses have been proposed to correct for the inevitable appearance of spin-orbit corrections in quantum dots [8], quantum error correcting codes have been shown to correct certain systematic qubit-qubit interaction errors [9], a problem also approached using NMR-inspired composite pulse sequences [10], and off-resonant transitions have been shown to be cancellable via a sequence of resonant pulses [6] or optimal control fields [11]. The motivation for removing Δ typically comes from the fact that there already exists a theory enabling convenient universal QC using S_I, but the same theory does not apply using S_A. For example, in interacting spin systems, such as quantum dots, it is known how to perform universal QC by manipulating only isotropic Heisenberg exchange interactions [12,13], an attractive prospect that eliminates the need for performing difficult single-qubit operations. In this case S_I is the purely isotropic Heisenberg Hamiltonian and Δ is the spin-orbit correction. The presence of Δ spoils the just mentioned universality result, and hence methods to cancel Δ have been proposed [8,14]. Is it always necessary to cancel Δ? Here we introduce a new method, termed "dressed qubits," that enables universal QC using S_A, provided it is known how to perform universal QC using S_I. This includes preparation and measurement of quantum information, in the dressed qubit basis. Thus, the ability to, e.g., perform universal QC using only Heisenberg interactions, translates directly into the ability to perform universal QC under Heisenberg interactions including spin-orbit coupling Δ, *without any extra overhead, and irrespective of the magnitude of* Δ. These are general features of the dressed qubits method that distinguish it from all schemes trying to correct for Δ, as opposed to working directly with S_A. The underlying idea is to find a unitary "dressing" transformation between the "ideal qubit basis" (the one for which universality results can be proven relatively easily) and the "dressed qubit basis" (corresponding to computation using S_A). We first give a general description of the dressed qubit method. We then illustrate the general results with examples of relevance to promising QC proposals.

Generalities.—Suppose that a system S_A of N (physical or encoded) qubits possesses a set of experimentally controllable Hamiltonians (or corresponding evolution operators) $\mathbf{H} = \{H_\alpha\}$ [or $\mathbf{U}(\boldsymbol{\theta}) = \{U_\alpha = e^{-i\theta_\alpha H_\alpha}\}$], which may be accompanied by inherent errors Δ. Correspondingly, there is an idealized set of Hamiltonians (or corresponding evolution operators) $\mathbf{H}^{\rm id} = \{H_\alpha^{\rm id}\}$ [or $\mathbf{U}^{\rm id}(\boldsymbol{\theta}^{\rm id}) = \{U^{\rm id} = e^{-i\theta_\alpha^{\rm id} H_\alpha^{\rm id}}\}$], which is universal: the set $\mathbf{H}^{\rm id}$ can be used to generate a transformation between an arbitrary N-qubit state $|\Psi^{\rm id}\rangle$ and any other such state. Assume that there exists a *fixed* unitary transformation V such that $H_\alpha = V^\dagger H_\alpha^{\rm id} V \; \forall \; \alpha$. Then we define V as the "dressing transformation" between $\mathbf{H}^{\rm id}$ and $\mathbf{H}$, and the states $|\Psi\rangle = V^\dagger |\Psi^{\rm id}\rangle$ (where $|\Psi^{\rm id}\rangle$ is an arbitrary N-qubit state of the idealized, or "bare" system S_I) and $|\Psi\rangle$ are the "dressed states." It follows that for all $|\Phi^{\rm (id)}\rangle, |\Psi^{\rm (id)}\rangle$

$$\langle \Phi | H_\alpha | \Psi \rangle = \langle \Phi^{\rm id} | H_\alpha^{\rm id} | \Psi^{\rm id} \rangle, \tag{1}$$

i.e., matrix elements in the dressed basis are identical to those in the idealized basis. Hence QC in the idealized and dressed (actual) system is equivalent. *The dressing transformation V need not be implementable experimentally.* We do, however, require that states can be prepared and measured in the dressed basis, so that this basis can be used for input and output. V may be separable: $V = \bigotimes_{j=1}^{N} V_j$. In this case, one can specifically define a dressed qubit represented by states $|0\rangle = V^\dagger |0^{\rm id}\rangle$ and $|1\rangle = V^\dagger |1^{\rm id}\rangle$, where $|0^{\rm id}\rangle$ and $|1^{\rm id}\rangle$ are the bare or idealized computational basis states. Such a separable dressing transformation retains essentially the features of $\mathbf{H}$ when it is transferred into $\mathbf{H}^{\rm id}$, meaning that a one- (two-)qubit operation in $\mathbf{H}$ is transferred into the corresponding one- (two-)qubit operation in $\mathbf{H}^{\rm id}$. Below we discuss both separable and nonseparable dressing transformations.

While the notion of dressed qubits is simple, it is usually not straightforward to find a valid V for a particular physical system, since one has to consider both single- *and* two-qubit operations in order for the general relation $H_\alpha = V^\dagger H_\alpha^{\rm id} V$ to hold. Surprisingly, we report here that some promising QC proposals with inherent errors are dressable.

Below we make repeated use of the following identity, valid for any set of operators $\{J_x, J_y, J_z\}$ satisfying the two su(2) [or so(3)] commutation relations $[J_z, J_x] = iJ_y$, $[J_y, J_z] = iJ_x$ (the third relation $[J_x, J_y] = iJ_z$ is not required):

$$\sqrt{1+\delta^2}\, e^{-i\varphi J_z} J_x e^{i\varphi J_z} = J_x + \delta J_y, \qquad \delta = \tan\varphi. \quad (2)$$

Eliminating off-resonant effects.—In many QC proposals a two-dimensional qubit subspace is embedded in a larger N-level Hilbert space. In such cases quantum logic operations typically mix the qubit subspace with the other states. This is known as "leakage," and is the result of unwanted off-resonant transitions [6,11]. Since it follows from time-independent perturbation theory that such transitions are stronger for levels closer to those supporting the qubit, we consider for simplicity first a three-level model with states $\{|k^{\rm id}\rangle\}_{k=0}^{2}$. The first two states represent the qubit. This example is highly relevant to superconducting QC proposals (e.g., the current-biased Josephson junction [4], and the persistent-current qubit [5]), where the qubit levels couple to a third level supported by the potential. Ideally $\{H_1^{\rm id} = f\sqrt{1+\delta^2}(c_0^\dagger c_1 + c_1^\dagger c_0), H_2^{\rm id} = \epsilon_1 n_1 \equiv \epsilon_1 c_1^\dagger c_1\}$, where $c_k^\dagger$ is a fermionic or bosonic creation operator for level k, or, in the case of a single-particle Fock space, a projection operator such that $c_k^\dagger c_l = |k\rangle\langle l|$. The representations of $H_1^{\rm id}$ and $H_2^{\rm id}$ in the two-dimensional qubit subspace are proportional to the Pauli matrices σ_x and σ_z, respectively, and generate an SU(2) group for all single-qubit operations, provided one can tune f, ϵ_1. Experimentally, instead, one typically obtains the actual Hamiltonian $H_1 = f[(c_0^\dagger c_1 + c_1^\dagger c_0) + \delta(c_1^\dagger c_2 + c_2^\dagger c_1)]$, where the last term is the undesirable off-resonant transition. Additionally, we now have $H_2 = \sum_{i=1}^{2} \epsilon_i n_i$, where we assume that $\epsilon_1 \neq \epsilon_2$. Effective, but costly schemes have been proposed to eliminate the systematic error due to δ [6,11]. As an alternative to eliminating δ, using a dressed qubit instead of $|0^{\rm id}\rangle$ and $|1^{\rm id}\rangle$ solves the problem at no extra cost: First, note that the set $\{X \equiv c_0^\dagger c_1 + c_1^\dagger c_0, Y \equiv c_1^\dagger c_2 + c_2^\dagger c_1, Z \equiv i(c_2^\dagger c_0 - c_0^\dagger c_2)\}$ satisfies su(2) or so(3) commutation relations. Hence a dressing transformation for the kth qubit is $V_k = \exp(i\varphi_k Z_k)$ where $\varphi_k = \tan^{-1}\delta_k$. Using Eqs. (1) and (2), it follows that for a dressed qubit $|\Phi\rangle_k = V_k^\dagger |\Phi^{\rm id}\rangle_k$ (a superposition of the states $|0^{\rm id}\rangle_k$ and $|2^{\rm id}\rangle_k$), $H_1 = f(X + \delta Y)$ acts as σ_x:

$$\langle\Psi|H_1|\Phi\rangle = \langle\Psi^{\rm id}|f\sqrt{1+\delta^2}(c_0^\dagger c_1 + c_1^\dagger c_0)\,|\Phi^{\rm id}\rangle.$$

Note that H_2 no longer acts as σ_z in the presence of n_2, since $[H_2(k), V_k] \neq 0$. However, it is always possible to effectively cancel n_2 by evolving it for time $2\pi/\epsilon_2$. This is clearly possible if ϵ_2 is tunable separately from ϵ_1; if not, then we set $\epsilon_1 = \epsilon_2/4$ and evolve for a time $2\pi/\epsilon_2$. This generates the phase gate diag$(1, i)$ in the dressed basis, which together with the Hamiltonian σ_x generates SU(2) [1]. To obtain a universal set of gates we need, in addition, an entangling two-qubit transformation. To this end, the dressing transformation is compatible, e.g., with the two-qubit Ising-like interaction $n_1(i)n_1(j)$, since then $n_1(k)n_1(l) = V_k^\dagger V_l^\dagger n_1(k)n_1(l)V_l V_k$. In general, if the single-qubit operations undergo the transformation V_k, the two-qubit interaction undergoes the transformation $V_k V_l$. This transformation must ensure that the actual two-qubit interaction becomes the idealized two-qubit interaction. In our discussion of exchange interactions below we give a nontrivial example.

The construction above is easily generalized to systems with $N > 3$ levels with an interaction of the form $H_1^{(N)} = f[c_0^\dagger c_1 + \sum_{j=2}^{N-1} \delta_j c_1^\dagger c_j + \text{H.c.}]$, describing leakage from state $|1^{\rm id}\rangle$ to all states $\{|j^{\rm id}\rangle\}_{j=2}^{N-1}$. Let $|\kappa|^2 = \sum_{j=2}^{N-1} |\delta_j|^2$; the triple $\{X^{(N)} \equiv c_0^\dagger c_1 + c_1^\dagger c_0, Y^{(N)} \equiv \frac{1}{|\kappa|}\sum_{j=2}^{N-1}(\delta_j c_1^\dagger c_j + \text{H.c.}), Z^{(N)} \equiv \frac{i}{|\kappa|}\sum_{j=2}^{N-1}(\delta_j c_j^\dagger c_0 - \text{H.c.})\}$ is an su(2) or so(3) algebra. Therefore $H_1^{(N)} = f(X^{(N)} + |\kappa| Y^{(N)}) = f\sqrt{1+|\kappa|^2}\, e^{-i\varphi Z^{(N)}} X^{(N)} e^{i\varphi Z^{(N)}}$, where $\varphi = \tan^{-1}|\kappa|$. The general-$N$ dressing transformation is thus $V = \exp[i\varphi Z^{(N)}]$ which creates a dressed qubit that is a superposition of states $|0^{\rm id}\rangle$ and $\{|j^{\rm id}\rangle\}_{j=2}^{N-1}$, and is again compatible with the Ising interaction, so that universality is preserved.

Now note that $|1\rangle = |1^{\rm id}\rangle$. Therefore *preparation* amounts to initializing all qubits in the state $|1\rangle$, and *measurement in the dressed basis* amounts to observing just the $|1\rangle$ state. This can be done similarly to the technique of cycling transitions in trapped ions, by coupling the $|1\rangle$ state to an auxiliary level and observing fluorescence [15].

The dressed qubit is the natural computational basis given the actual "leaky" interaction H_1, and there is no need to eliminate the leakage term contained in H_1: this term represents leakage only with respect to the "unnatural" computational basis $|0^{\mathrm{id}}\rangle, |1^{\mathrm{id}}\rangle$. The dressed qubit is "natural" in the sense that there is no need to physically implement the dressing transformation: it is inherent in the actual Hamiltonian.

Encoded QC using Heisenberg interaction with anisotropy.—The Heisenberg exchange interaction $J\mathbf{S}_k \cdot \mathbf{S}_l$ between spins $\mathbf{S}_k$ and $\mathbf{S}_l$ is central to a number of the most promising solid-state QC proposals, including electrons in quantum dots [2] and donor atoms in Si arrays [16]. It has been shown to be universal for QC, without (more difficult to implement) single-qubit gates, provided one encodes a logical qubit into the state of several spins [12,13]. In reality, the idealized Heisenberg Hamiltonian is, however, perturbed by an anisotropic term arising due to spin-orbit interactions: the actual Hamiltonian is

$$H_{kl} = J\{\mathbf{S}_k \cdot \mathbf{S}_l + \mathbf{D} \cdot \mathbf{S}_k \times \mathbf{S}_l + \gamma(\mathbf{S}_k \cdot \mathbf{D})(\mathbf{S}_l \cdot \mathbf{D})\}, \quad (3)$$

where $\mathbf{D} \in \mathbb{R}^3$ is known as the Dzyaloshinski-Moriya vector in solid-state physics, and $\gamma = (\sqrt{1 + |\mathbf{D}|^2} - 1)/|\mathbf{D}|^2$. Kavokin has estimated that $|\mathbf{D}|$ is in the range 0.01–0.8 in coupled quantum dots in GaAs [3]. This is at least 2 orders of magnitude beyond the current fault-tolerance threshold estimates of quantum error correction theory [17]. For this reason the anisotropic perturbation has been considered a problem and strategies have been designed to cancel it. For example, it can be removed to first order by shaped pulses [8], canceled in the absence of an external magnetic field and in the presence of single-qubit operations [14], or *used* in order to generate a universal gate set that, however, incurs some timing overhead [7]. These approaches to dealing with the spin-orbit term are motivated by universal QC with either the usual (bare) choice of S_z eigenstates as qubits [7,14], or with encoded qubits [8]. Here we show that *dressed qubits, defined with respect to the actual Hamiltonian H_{kl}, offer a solution that is fully compatible with the encoded qubits approach, at no extra overhead and without any approximations, other than the assumption that $\mathbf{D}$ is time independent.* The residual time dependence of $\mathbf{D}$ (that arises via the spin-orbit constant from switching of J during the execution of quantum gates [3]) is small enough that it can be corrected using quantum error correcting codes [14].

We derive a dressing transformation by constructing a set of su(2) operators for H_{kl} of Eq. (3). The operators $\{X_{kl} \equiv \mathbf{S}_k \cdot \mathbf{S}_l - (\mathbf{S}_k \cdot \mathbf{n})(\mathbf{S}_l \cdot \mathbf{n}), Y_{kl} \equiv \mathbf{n} \cdot (\mathbf{S}_k \times \mathbf{S}_l), Z_{kl} \equiv \frac{1}{2}\mathbf{n} \cdot (\mathbf{S}_l - \mathbf{S}_k)\}$, where $\mathbf{n}$ is a unit vector, form such a set. Further note that $[Z_{kl}, (\mathbf{S}_k \cdot \mathbf{n})(\mathbf{S}_l \cdot \mathbf{n})] = 0$. It therefore follows by direct substitution from Eq. (2) that

$$W_{kl} = e^{-i(1/2)\epsilon \mathbf{n} \cdot (\mathbf{S}_k - \mathbf{S}_l)} \quad (4)$$

is a transformation such that $H_{kl} = W_{kl}^\dagger H_{kl}^{\mathrm{id}} W_{kl}$, where $H_{kl}^{\mathrm{id}} = \sqrt{1 + |\mathbf{D}|^2} J \mathbf{S}_k \cdot \mathbf{S}_l$ is the isotropic Heisenberg interaction, $\epsilon = \tan^{-1}|\mathbf{D}|$, and $\mathbf{n} = \mathbf{D}/|\mathbf{D}|$. Alternatively, the set $\{X_{kl}, Y_{kl}, Z_l \equiv \mathbf{n} \cdot \mathbf{S}_l)\}$ satisfies the pair of su(2) commutation relations $[Z_l, X_{kl}] = iY_{kl}$, $[Y_{kl}, Z_l] = iX_{kl}$ (but $[X_{kl}, Y_{kl}] \neq iZ_l$). It again follows from Eq. (2) that $T_l = e^{i\epsilon \mathbf{n} \cdot \mathbf{S}_l}$ is a transformation such that

$$H_{kl} = T_l^\dagger H_{kl}^{\mathrm{id}} T_l = T_k H_{kl}^{\mathrm{id}} T_k^\dagger. \quad (5)$$

Let us now recall the encoding under which the Heisenberg interaction becomes universal for QC. The most economical encoding uses the two total spin $S = 1/2$ representations of three spin-1/2 particles to encode a qubit [18]. A convenient choice of encoded-qubit basis states are the two states, $|0_L^{\mathrm{id}}\rangle_z = |s\rangle_{12}|\uparrow\rangle_3$ and $|1_L^{\mathrm{id}}\rangle_z = \sqrt{2/3}|\uparrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3 - \sqrt{1/3}|t\rangle_{12}|\uparrow\rangle_3$, where $|s\rangle_{12} = (|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)/\sqrt{2}$ and $|t\rangle_{12} = (|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2)/\sqrt{2}$ are the singlet and triplet states of spins 1, 2, respectively. The z subscript indicates that these two states have total spin projection $S_z = +1/2$. Because H_{kl}^{id} is a scalar of total spin, a qubit can also be represented by states with quantization axis along an arbitrary direction $\mathbf{n}$; in this case we use the (obvious) notation $|0_L^{\mathrm{id}}\rangle_{\mathbf{n}}, |1_L^{\mathrm{id}}\rangle_{\mathbf{n}}$, and write an arbitrary encoded-qubit state as $|\Phi^{\mathrm{id}}\rangle_l = a|0_L^{\mathrm{id}}\rangle_{\mathbf{n}l} + b|0_L^{\mathrm{id}}\rangle_{\mathbf{n}l}$ $(|a|^2 + |b|^2 = 1)$. In [13] a convenient set of universal gates was found for the $|0_L^{\mathrm{id}}\rangle_z, |1_L^{\mathrm{id}}\rangle_z$ encoding: sequences chosen from $\mathbf{U}^{\mathrm{id}} = \{U_{12}^{\mathrm{id}}(\theta), U_{23}^{\mathrm{id}}(\theta), U_{45}^{\mathrm{id}}(\theta), U_{56}^{\mathrm{id}}(\theta), U_{34}^{\mathrm{id}}(\theta)\}$ are universal for two qubits encoded into the states of spins 1–3 and 4–6, respectively, where $U_{kl}^{\mathrm{id}}(\theta) = \exp(-i\theta \mathbf{S}_k \cdot \mathbf{S}_l)$. The first four gates serve as logical single-qubit operations for the two encoded qubits; the last operation, $U_{34}^{\mathrm{id}}(\theta)$, serves to entangle the two encoded qubits via a controlled-phase (CZ) gate [1]. Let us now show how to construct logic gates directly in terms of the actual interaction H_{kl}.

The lth logical qubit is encoded by physical qubits $3l - 2, 3l - 1, 3l$. We define an arbitrary lth dressed qubit by

$$|\Phi\rangle_l = V_{3l-2,3l}^\dagger |\Phi^{\mathrm{id}}\rangle_l, \quad (6)$$

where $V_{3l-2,3l}$ is the dressing transformation with $V_{kl} = (W_{kl})^2 = T_k T_l^\dagger$ as given in Eq. (4). Consider how *single-qubit operations* act on this dressed qubit. Let $U_{kl}(\theta) = \exp(-i\theta H_{kl})$. It follows from Eq. (5) that $U_{12}(\theta)|\Phi\rangle_1 = V_{13}^\dagger U_{12}^{\mathrm{id}}(\theta)|\Phi^{\mathrm{id}}\rangle_1$, and similarly $U_{23}(\theta)|\Phi\rangle_1 = V_{13}^\dagger U_{23}^{\mathrm{id}}(\theta)|\Phi^{\mathrm{id}}\rangle_1$. Therefore matrix elements of $U_{12}(\theta)$ and $U_{23}(\theta)$ in the dressed basis are identical to those in the idealized basis. Thus all single encoded-qubit operations can be performed using H_{kl}, provided the dressed basis is used.

Now consider *two-qubit operations.* First, by using a sequence of swaps, $U_{kl}^{\mathrm{id}\dagger}(\frac{\pi}{4})U_{lm}^{\mathrm{id}}(\theta)U_{kl}^{\mathrm{id}}(\frac{\pi}{4}) = U_{km}^{\mathrm{id}}(\theta)$, we can replace the entangling gate $U_{34}^{\mathrm{id}}(\theta)$ by $U_{15}^{\mathrm{id}}(\theta)$ or $U_{26}^{\mathrm{id}}(\theta)$. If we arrange the physical qubits as shown in Fig. 1, $U_{15}^{\mathrm{id}}(\theta)$ is a nearest neighbor interaction. Next, using Eq. (6),

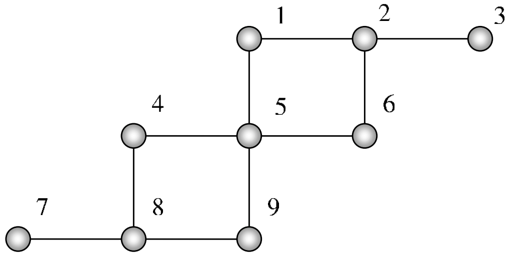

FIG. 1. Geometry for 3-spin encoding. Each row represents a single encoded qubit.

a two-encoded-qubit dressed state is $|\Phi\rangle_1|\Phi\rangle_2 = V_{13}^\dagger V_{46}^\dagger|\Phi^{\rm id}\rangle_1|\Phi^{\rm id}\rangle_2$. We then have $U_{15}(\theta)|\Phi\rangle_1|\Phi\rangle_2 = V_{13}^\dagger V_{46}^\dagger U_{15}^{\rm id}(\theta)|\Phi^{\rm id}\rangle_1|\Phi^{\rm id}\rangle_2$, meaning that $U_{15}(\theta)$ plays the same role in the dressed basis as does $U_{15}^{\rm id}(\theta)$ in the idealized basis. Therefore the set $\mathbf{U} = \{U_{12}(\theta), U_{23}(\theta), U_{45}(\theta), U_{56}(\theta), U_{15}(\theta)\}$ is universal for dressed qubits and has the same matrix representations as in the idealized basis. With the arrangement shown in Fig. 1, spins 15, 26, 48, 59, ... are nearest neighbors, and H_{kl} interactions between them can be used to generate a CZ gate between any pair of encoded qubits.

Next, we need to show that dressed qubits can be prepared and measured. Both can be performed in a manner analogous to the procedure proposed in [13] for the idealized Heisenberg Hamiltonian. In that case the computational basis state $|0_L\rangle_{\mathbf{n}} = |s\rangle_{12}|\uparrow\rangle_{3\mathbf{n}}$ can be prepared by turning on a strong exchange interaction between spins 1, 2, and a moderately strong magnetic field $B\mathbf{n}$ (such that $k_bT \ll g\mu_B B < J$): the system then relaxes to the ground state $|s\rangle_{12}$ and spin 3 is oriented along $\mathbf{n}$. The dressed state $|0_L\rangle = V_{13}^\dagger|s\rangle_{12}|\uparrow\rangle_{3\mathbf{n}} \propto e^{i\epsilon\mathbf{n}\cdot\mathbf{S}_1}|s\rangle_{12}|\uparrow\rangle_{3\mathbf{n}}$ can be similarly prepared since it follows that $V_{13}^\dagger|s\rangle_{12}$ is the ground state of the actual Hamiltonian $H_{12} = V_{13}^\dagger H_{12}^{\rm id}V_{13}$. Computation can then begin, with gates applied from the set $\mathbf{U}$. The measurement scheme in [13] relies on distinguishing a singlet $|s\rangle_{12}$ from a triplet $|t\rangle_{12}$ (e.g., using Kane's ac capacitance scheme [16]), since this is a measurement of whether the encoded qubit is in the state $|0_L\rangle_z = |s\rangle_{12}|\uparrow\rangle_3$ or not (thus the state of spin 3 does not enter). In essence this is a measurement of the idealized observable $H_{12}^{\rm id}$; in reality this becomes a measurement of the actual observable H_{12}, which will serve to determine whether the encoded qubit is in the state $|0_L\rangle$. We have thus described a complete scheme for universal QC with the anisotropic Heisenberg Hamiltonian. Our conclusions remain valid for encodings into more than three qubits [12,19]. Finally, we note that a dressing transformation can also be found for the case of QC in the *anisotropic XXZ* model, $H_{kl} = J_{kl}(S_k^xS_l^x + S_k^yS_l^y + \delta S_k^zS_l^z + \Delta)$, $\Delta = S_k^xS_l^y - S_k^yS_l^x$, $\delta \neq 0$, in the presence of nonuniform Zeeman splittings [19].

Non-separable dressing transformation.—So far we have discussed only separable dressing transformations $V = \bigotimes_{j=1}^N V_j$. As illustrated by the following simple example, a nonseparable dressing transformation V may be used to deal with problems such as a one-qubit operation accompanied inherently by weak two-qubit coupling. Given N qubits, suppose one can turn on S_k^z and $S_k^zS_l^z$ perfectly, while turning on $f^yS_k^y$ induces a small inherent error $f^y\delta(S_k^xS_{k+1}^z + S_k^xS_{k-1}^z)$, with $\mathbf{S}_{N+1} = \mathbf{S}_1$. This error can be approximately eliminated by a nonlocal dressing transformation $V = \exp(i\delta\sum_{k=1}^N S_k^zS_{k+1}^z)$ if $\delta \ll 1$, since $S_k^y \approx V[S_k^y + \delta(S_k^xS_{k+1}^z + S_k^xS_{k-1}^z)]V^\dagger$.

In conclusion, we have introduced a general method, dressed qubits, that eliminates arbitrarily strong inherent errors in QC proposals, without introducing any encoding overhead. Such errors arise when the actual Hamiltonian driving the system differs from the desired one. Two important physical examples we have discussed in detail illustrate the power of the method: elimination of off-resonant transitions in multilevel systems in which a qubit is embedded, and elimination of the inherent spin-orbit induced anisotropy accompanying the Heisenberg interaction in spin-based QC proposals.

D. A. L. acknowledges support from D-Wave Systems, Inc., and under the DARPA-QuIST program (managed by AFOSR under Agreement No. F49620-01-1-0468). We thank the staff at D-Wave Systems, Inc., for valuable comments.

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