Second International Conference on Quantum Error Correction
University of Southern California, **Dec. 5-9 2011**

Towards Optimal Constructions of Dynamically Corrected Quantum Gates

Lorenza Viola
Dept. Physics & Astronomy
Dartmouth College

The premise: DQEC

Dynamical Quantum Error Correction = Non-dissipative QEC:
Open-loop Hamiltonian engineering based on [a fixed set of] unitary control operations.

Simplest setting: Multipulse decoherence control for quantum memory ⇒ DD

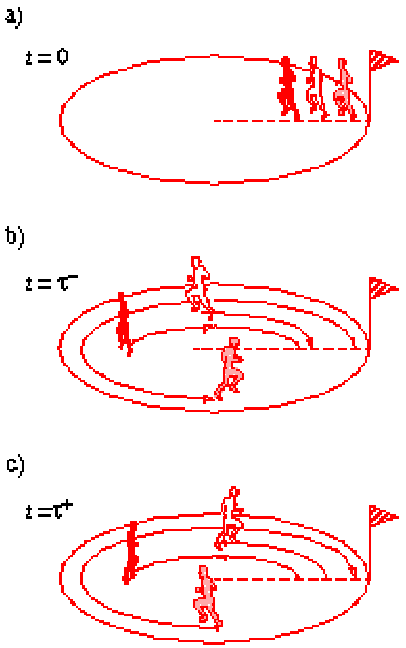
Key principle: Time-scale separation ⇒ Coherent averaging of interactions

Paradigmatic example: Spin echo ⟷ Effective time-reversal

Hahn 1950.

Theory: Average Hamiltonian formalism

Haeberlen & Waugh 1968; Waugh 1982...



Key feature: Open quantum system dynamics

✔ Error component includes coupling to a quantum environment/bath...
✔ Nature of environment need not be specified, except qualitatively...

$$\frac{\tau_{control}}{\tau_c}$$ Small parameter

➞ Perturbative error cancellation enforced

Theory challenges for practical DQEC-I

Goal: Better address timing and sequencing constraints...

Khodjasteh's talk

Even when BB assumption is accurate, min control time scale/pulse repetition rates are finite...

- Q1: What is the best possible DD performance for specified timing resources?
  - ↳ Bandwitdth-Adapted DD (BADD)

Hodgson, D'Amico & LV 2010; Uhrig & Lidar 2010;
Khodjasteh, Erdelyi & LV 2011.

- Q2: Can such performance be achieved for arbitrarily long storage time and how?
  - ↳ Bandwitdth-Adapted Long-time DD (BALDD)

Khodjasteh, Biercuk & LV, forthcoming.

Scalable lab implementations will face sequencing limitations from digital electronics...

- Q3: How can we ensure hardware compatibility and minimize DD sequencing complexity?
  - ↳ Walsh DD (WDD)

Hayes, Khodjasteh, LV & Biercuk, arXiv:1109.6002
PRA, in press.

Theory challenges for practical DQEC-2

Goal: Better address system and control limitations/non-idealities...

Even with otherwise perfect control, realistic control amplitudes are finite...

➞ Open-loop Hamiltonian engineering with bounded control inputs substantially harder:

$H_{\text{err}} \approx 0$ during each BB pulse, whereas EPG $= \mathrm{O}(\tau \, \|H_{\text{err}}\|)$ for 'fat' pulses...

- Q1: How [and how well] can we decouple with realistic pulses?

  ↳ Eulerian DD (EDD), RUDD...

  LV & Knill 2003; Pasini et al 2008; Uhrig & Pasini 2009...

- Q2: Can we suppress decoherence while effecting a non-trivial quantum gate?

  ↳ Strongly modulating pulses, Dynamically Corrected Gates (DCGs)...

  Fortunato et al 2002; Pryadko & Quiroz 2008...
  Khodjasteh & LV 2009...

- Q3: To what extent can DQEC compensate for decoherence and control errors together?

Outline:

I. DCGs, first-order and beyond – What they are and how to make them...

II. How to enhance DCG efficiency and flexibility – Toward combining 'DCG + OCT'...

I. Analytical DCG Framework

Khodjasteh & LV, PRL 102, 080501(2009); PRA 80, 032314 (2009);
Khodjasteh, Lidar & LV, PRL 104, 090501 (2010); [Longer paper coming soon...]

Control-theoretic setting

- Target system S couples to quantum bath B via interaction Hamiltonian H_{SB}:

$$H = \overbrace{[H_{S,\mathrm{g}} + H_{S,\mathrm{err}}]}^{\textbf{Pure-system}} \otimes \boldsymbol{I}_B + \boldsymbol{I}_B \otimes \overbrace{H_B}^{\textbf{Pure-bath}} + H_{SB} \equiv \sum_a S_a \otimes B_a$$

➞ System operators $\{S_a\}$ form Hermitian basis, with $S_0 = \boldsymbol{I}_S$ and $S_{a\neq 0}$ traceless.

➞ Bath operators $\{B_a\}$ are bounded but otherwise arbitrary [possibly unknown].

- Environment B is uncontrollable: Controller acts on system only,

$$H_{\mathrm{tot}}(t) \equiv H + H_{\mathrm{ctrl}}(t), \qquad H_{\mathrm{ctrl}}(t) = \sum_m \left(H_m \otimes \boldsymbol{I}_B\right) h_m(t) \quad \longleftarrow \textbf{Control inputs}$$

$$U_{\mathrm{ctrl}}(t) = T\exp\left\{-i \int_0^t ds\, H_{\mathrm{ctrl}}(s)\right\} \qquad \longleftarrow \textbf{Control propagator}$$

➞ Universal control on S may or may not require a non-zero [drift] system Hamiltonian.

➞ Semi-classical limit: Random modification of system Hamiltonian, $H_S \to H_S(t)$.

Error model assumptions

- Error model includes any deviation between actual controlled evolution and intended one:
  - ➞ Ideal gate propagator over duration T:

$$U^0(T) = Q \otimes \boldsymbol{I}_B = T\exp\left\{-i\int_0^T ds\,\left[H_{ctrl}(s) + H_{S,g}\right] \otimes \boldsymbol{I}_B\right\}$$

  - ➞ Actual gate propagator over duration T:

$$U(T) = T\exp\left\{-i\int_0^T ds\,\left[H_{ctrl}(s) + H_{S,g} + H_{err}\right]\right\} \equiv Q\exp(-iE_Q) \longleftarrow$$

Error action operator

Simplifying assumptions:

(A1) Perfect control – No errors are introduced by the controller;
(A2) Driftless system – Can effectively assume that $H_{S,\mathrm{g}} = 0,\ H_S \equiv H_{S,\mathrm{err}} \Rightarrow$

$$U_{ctrl}(T) \equiv Q = T\exp\left\{-i\int_0^T ds\ H_{ctrl}(s)\right\}, \quad U(T) = Q\,T\exp\left\{-i\int_0^T ds\ U^\dagger_{ctrl}(s) H_{err} U_{ctrl}(s)\right\}$$

- Focus on arbitrary linear [non-Markovian] decoherence on qubits:

$$H_{SB} = \sum_{i=1}^{n} \sum_{a=x,y,z} \sigma_a^{(i)} \otimes B_a^{(i)}, \qquad H_{\mathrm{err}} = I_S \otimes H_B + H_{SB}$$

  - ➞ Error operators we wish to suppress:

$$\Omega_{\mathrm{err}} = \mathrm{Span}\left\{\sigma_\alpha^{(i)} \otimes B_\alpha^{(i)} \mid B_\alpha^{(i)} \text{ nonzero in } H_{\mathrm{err}}\right\}$$

Control assumptions

- Error action operator leads to a natural measure to quantify EPG:

$$\text{EPG} = \left\|\underbrace{mod_B\left(E_Q\right)}_{\textbf{Non-pure-bath component}}\right\|_{op} \quad \Rightarrow \quad \|\underset{\substack{\uparrow \\ \textbf{Actual}}}{\rho_S(\tau)} - \underset{\substack{\uparrow \\ \textbf{Ideal}}}{\rho_S^0(\tau)}\|_1 \le \left\|mod_B\left(E_{Q[\tau]}\right)\right\|_{op}$$

- Control resources: Universal set of tunable 'primitive' Hamiltonians

e.g. $\left\{ h_x(t)\sigma_x^{(i)},\, h_y(t)\sigma_y^{(i)},\, h_{zz}(t)\sigma_z^{(i)}\otimes\sigma_z^{(j)} \right\}, \quad i, j = 1, \dots, n$

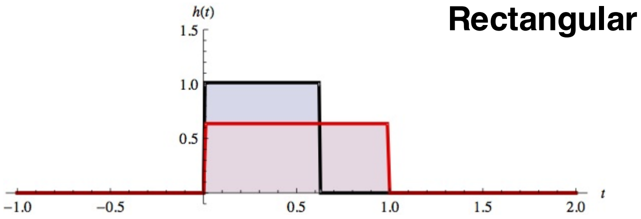
Assumptions:

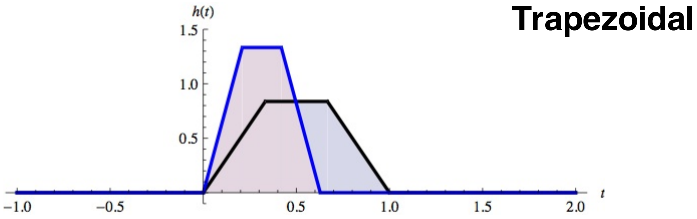
(C1) Finite-power and bandwidth constraint:

Bounded control amplitude, $h_a(t) \le h_{\text{max}}$, and minimum gate duration, $\tau_{\text{min}} > 0$;

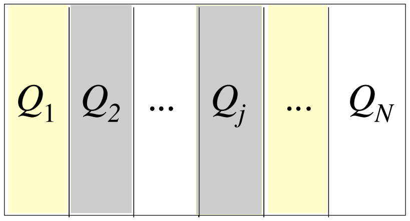
(C2) Strechable and scalable pulse profiles:

➡ Same primitive gate Q can be implemented with different pulse shapes and speed





Control objective



<u>DCG block structure</u>:

➞ Cascade N primitive gates.

➞ If each individual EPG is sufficiently small,

$$E_{(Q_1[\tau_1]\dots Q_N[\tau_N])} = E_{Q_1[\tau_1]} + P_1^\dagger E_{Q_2[\tau_2]} P_1 + \dots + P_{N-1}^\dagger E_{Q_N[\tau_N]} P_{N-1} + C^{[2+]},$$

$$P_{N-1} = Q_{N-1} \dots Q_1$$

as long as the [discrete-time] Magnus expansion converges, $\sum_i \left\| mod_B\left(E_{Q_i[\tau_i]}\right) \right\| < \pi$.

- EPGs do not simply add: Individual errors are 'modulated' by the applied control path. Because control path is known, errors have a systematic dependence upon gate duration...

- Seek a control modulation s.t. the effect of H_{err} is perturbatively [coherently] averaged out:

1st-order DCGs:
Remove leading error

$$\text{EPG}_{\text{uncorrected}} = \left\| mod_B\left(E_{Q^{[0]}[\tau]}\right) \right\| \propto \tau + \text{O}(\tau^2)$$

$$\big\downarrow$$

$$\text{EPG}_{\text{corrected}} = \left\| mod_B\left(E_{Q^{[1]}[\tau^{[1]}]}\right) \right\| \propto \tau^2 + \text{O}(\tau^3)$$

Dynamically correcting NOOP

- If target gate $Q = I$, a solution is provided by EDD:

LV & Knill, PRL 90 (2003).

  - ➞ Primitive gates implement the generators $\{\gamma_l\} \in \Gamma$, $l=1,\ldots,L$, of a DD group $G = \{g_i\}$, $i=1,\ldots,G$.
  - ➞ Generators are applied by following an Eulerian cycle on the Cayley graph of G_{DD}.

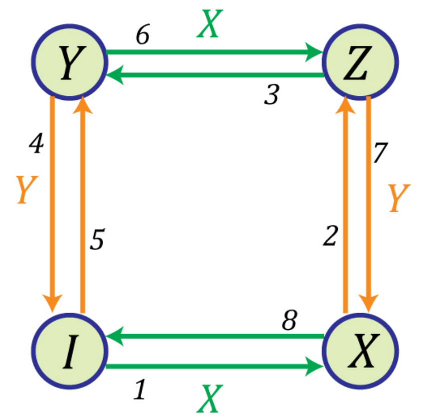
$$E_{EDD} = \sum_{l=1}^{L} \sum_{i=1}^{G} U_{g_i}^{\dagger} E_{\gamma_l} U_{g_i} + E_{EDD}^{[2+]}, \quad \| mod_B(E_{EDD}) \| = \| mod_B(E_{EDD}^{[2+]}) \| = O(\tau^2)$$

Linear decoherence on qubits:

$$G = \mathbb{Z}_2 \times \mathbb{Z}_2 \ \rightarrow \ \{I^{(\mathrm{all})}, X^{(\mathrm{all})}, Y^{(\mathrm{all})}, Z^{(\mathrm{all})}\}, \ \Gamma \equiv \{X, Y\}$$

- ➞ Collective generators can be implemented by collective primitive Hamiltonians:

$$X^{(all)} = X_1 \otimes \ldots \otimes X_n = \exp\left[-i \int_0^{\tau} h_x(s)(\sigma_x^{(1)} + \ldots + \sigma_x^{(n)})\, ds \right]$$



Euler cycle: X Y X Y Y X Y X

Cycle length: $\tau_{EDD} = (L x G)\tau$

- In EDD, no information about how primitive EPGs depend on control implementation is used/required!

  ➞ No-Go thm for 'black-box' DQEC: Only gates that commute with Ω_{err} can be achieved with 'control-oblivious' design...

DCGs beyond NOOP

- Simple[st?] way to evade No-Go: Identify two combinations of primitive gates that share same first-order error ⇒ [First-order] 'balance pair' for target gate Q:

$$Q_* = Q\exp(-iE_Q), \quad I_Q = \exp(-iE_Q)$$

- Modified Eulerian construction: Implement control path which begins at I and ends at Q on 'augmented' graph ⇒

(i) To non-identity vertex, attach edge labeled by I_Q

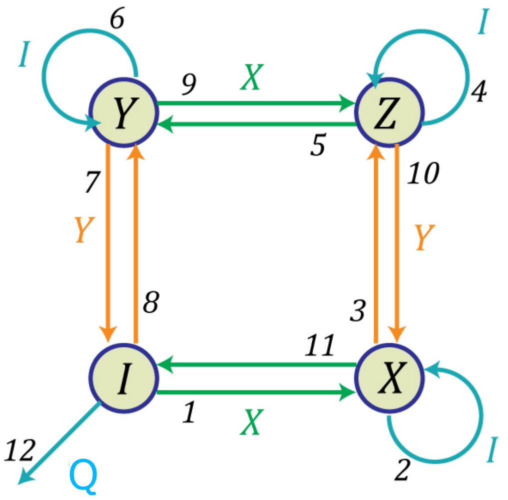
(ii) To identity vertex, attach edge labeled by Q_*

$$E_{DCG} = E_{EDD} + \sum_{i=1}^{G} U_{g_i}^{\dagger} E_Q U_{g_i} + E_{DCG}^{[2+]}$$

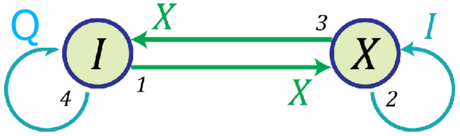
Total 1st-order error vanishes as long as the primitive errors E_{γ_l} and E_Q obey DD condition ⇒

$$\| mod_B(E_{DCG}) \| = \| mod_B(E_{DCG}^{[2+]}) \| = O(\tau^2)$$

Significantly smaller error wrto 'direct switching'.



Euler path: $X\ I\ Y\ I\ X\ I\ Y\ Y\ X\ Y\ X$ Q



Euler path: $X\ I\ X$ Q

Finding gates with same error: Balance pairs

Key insight: Map out and exploit relationship between primitive error and control profile…

- Naïve balance pairs: Assume access to a stretchable and (sign-)reversible gating profile,

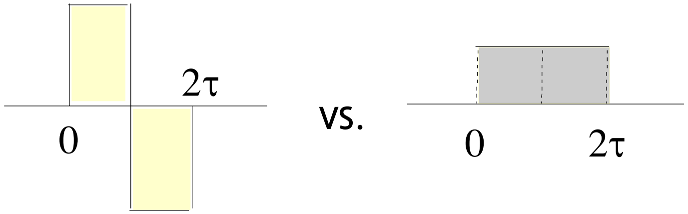
$$Q[\tau] = Texp\left\{-i\int_0^{\tau} h_Q(t) H_Q\, dt\right\} \qquad Q[s\tau] = Texp\left\{-\frac{i}{s}\int_0^{s\tau} h_Q\left(\frac{t}{s}\right) H_Q\, dt\right\}$$

$$E_Q[\tau] \longrightarrow E'_Q[s\tau] = s\, E_Q[\tau]$$

➡ Example of primitive gate combinations sharing the same [leading] error:

$$I_Q \equiv Q'[\tau]Q[\tau], \quad Q_* \equiv Q[2\tau],$$

$$mod_B(E_{I_Q}) = mod_B(E_{Q_*}) + O(\tau^2)$$



- Enhanced balance pairs: Assume access to [just] stretchable gating profiles,

➡ Portable primitive gate combinations sharing the same [leading] error:

$$I_Q \equiv I_Q^{[0]} = Q^{-1}[\tau]Q[2\tau], \quad Q_* \equiv Q_*^{[0]} = Q[\tau]Q^{-1}[\tau]Q[\tau], \quad mod_B(E_{I_Q}) = mod_B(E_{Q_*}) + O(\tau^2)$$

➡ DCG resource overheads:

8(2) + 3 x 4(2) ⇒ 20(8) primitives per DCG for linear decoherence(pure dephasing)

Beyond first-order DCGs...

Strategy: Increase order of cancellation by using recursive design ⇒ Concatenated DCGs

$\{Q^{[0]}\} \equiv$ Primitive gates; $\{Q^{[1]}\} \equiv$ *1*st-order DCGs; $\ldots$ $\{Q^{[m]}\} \equiv$ *m*th-order DCGs

- Balance pairs of order m may be given in terms of mth-order implementation of Q and Q^{-1}:

$$I_Q^{[m]} = Q^{-1,[m]}[\tau]\,Q^{[m]}[2^{1/(m+1)}\tau], \quad Q_*^{[m]} = Q^{[m]}[\tau]\,Q^{-1,[m]}[\tau]\,Q^{[m]}[\tau], \quad m \geqslant 0$$

$$mod_B(E_{I_Q}) = mod_B(E_{Q_*}) + O(\tau^{m+2})$$

- CDCG algorithm ⇒ Embed lower-order DCGs as components for EDDs and balance pairs:

① Set $m = 0$.

② Start with strechable mth-order primitive gates $Q = Q^{[m]}$ and their error model $\Omega_{\text{err}}^{[m]}$.

- ✔ $\Omega_{\text{err}}^{[m]}$ includes all errors uncorrected at level m ⇒ Identify smallest group $G^{[m]}$, of size G_m, that decouples all errors in $\Omega_{\text{err}}^{[m]}$.
- ✔ Represent the L_m generators of $G^{[m]}$ as primitive gates or combinations thereof.

③ Use generators for $G^{[m]}$ and the construction for mth-order balance pair to generate $Q^{[m+1)}$.

④ Repeat recursively by substituting the newly constructed gates for the old ones: $m=m+1$. Go to step 2.

CDCGs: Performance analysis

- Key step in establishing error bound ⇒ show that the Magnus expansion of error $E_{Q[m]}$ contains only terms that start at $O(\tau^{m+1})$ (modulo pure-bath terms)...
  - ➞ Starting at $m = 0$ with primitive gates of duration τ, duration at order m obeys

$$\tau_{m+1} = \Big[\underbrace{G_m L_m}_{\textbf{Euler path}} + 3 + (G_m - 1)(1+2^{1/(m+1)})\Big]\tau \quad \Rightarrow \quad \tau_{m+1} \leq [G_m(L_m+3)]^m\, \tau \equiv (\chi_m)^m\, \tau$$

Balance pairs

  - ➞ [Worst-case] error upper bound, $c = O(1)$:

$$\left\| mod_B(E_{DCG}^{[m]}) \right\| < c\,4^m (\chi_m)^{m^2+m} \left(\left\|H_{err}\right\|\tau\right)^{m+1}$$

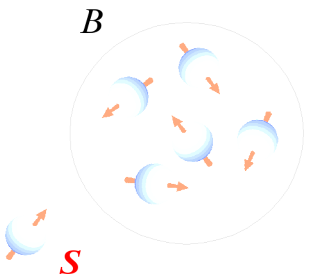
- For fixed minimum switching time τ, error bound implies optimal concatenation level,

$$m_{opt} = \left\lfloor -\frac{1}{2}\left[\log_\chi(4\,\tau\left\|H_{err}\right\|) + 1\right]\right\rfloor, \qquad \chi \equiv \chi_{opt}$$

below which CDCGs are guaranteed to perform better than first-order DCGs.

  - ➞ Smaller τ ⇒ Finer temporal resolution of CDCG 'digitized pulse profile'...

CDCGs: Illustrative results



$$|\psi_S\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\rho_B = \boldsymbol{I}_B\, 2^{-N}$$

➞ Case study: Electron spin qubit undergoing hyperfine decoherence in QD

$$H_{error} = \boldsymbol{I}_S \otimes \sum_{k=1}^{N} D_{kl}\, \vec{I}_k \cdot \vec{I}_l + \vec{\sigma} \otimes \sum_{k=1}^{N} A_k \vec{I}_k, \quad N=5$$

$$\Omega^{[m]}_{\text{err}} \equiv \Omega_{\text{err}} = \text{Span}\{\sigma_a \otimes B_a, a = x, y, z\}, \text{ independent upon } m$$

$$G^{[m]} \equiv G = \mathbb{Z}_2 \times \mathbb{Z}_2 \;\to\; \{I, X, Y, Z\}, \Gamma \equiv \{X, Y\} \Rightarrow \chi_m \equiv \chi = 4 \times 5 = 20$$

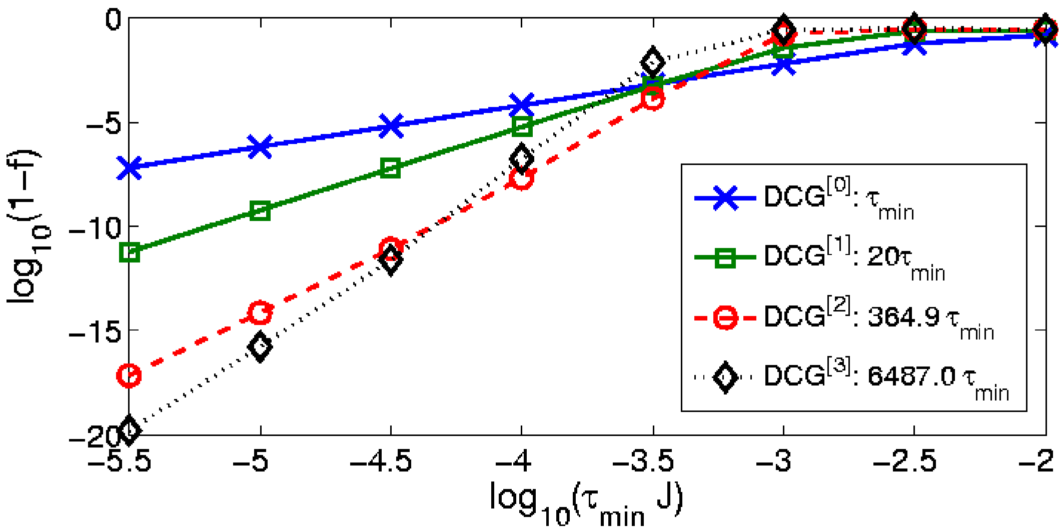
Gate sequence [right to left!] for mth-order DCG:

$$Q^{[m+1]} = Q_*^{[m]} X^{[m]} Y^{[m]} X^{[m]} Y^{[m]} Y^{[m]} I_Q X^{[m]} I_Q Y^{[m]} I_Q X^{[m]}$$

➞ Target gate: $Q = \exp(-i\frac{2\pi}{3}\sigma_x)$

➞ Steeper error-corrected 'slopes' achieved as concatenation level grows, if switching time is small.

➞ Fidelity improvement by as many as 13 orders of magnitude...



CDCGs in the lab?...

Hayes et al, arXiv:1104.1347;
Hayes, Khodjasteh, LV & Biercuk, arXiv:1109.6002.

➞ Recently implemented Mølmer-Sørensen 'composite' gate sequences can be interpreted as CDCGs under a simple error model...

$$U_Q(t) = \exp[S_N(\alpha(t)a^\dagger - \alpha(t)^* a)]\, Q\,, \quad Q = \exp[-i\Phi(t)\, S_N^2]$$ Spin-dependent gate

$$\alpha(t) = \frac{\Omega}{2}\int_0^t \exp[-i(\delta+\Delta)s]\,ds$$ Δ = Detuning error $\ll \delta$

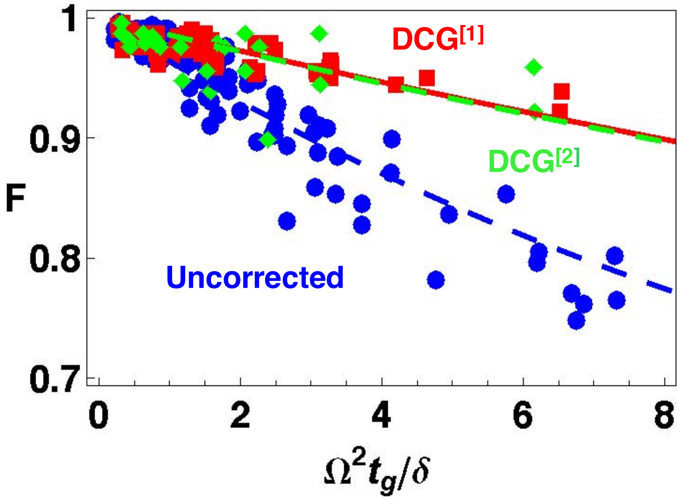
Gate relies on 'disentangling' spin and motional degrees of freedom at $t_g = j2\pi/\delta \quad \Rightarrow$
Residual spin-motional entanglement results in error action $-iE_Q(t) = S_N(\alpha(t)a^\dagger - \alpha(t)^* a)$

➞ Key simplifications:

(1) Ideal spin-flips (X gates) can be effected;
(2) Target gate commutes with X;
(3) Error action anti-commutes with X gate $\Rightarrow$

$$X\,Q\exp(-iE_Q)\,X\,Q\exp(-iE_Q) =$$
$$Q^2\,X\exp(-iE_Q)\,X\exp(-iE_Q) = Q^2$$

➞ 1st-order implementation of gate Q^2 ⇒ can iterate to achieve higher-order suppression...



II. Progress towards Optimized DCG Framework

Recap thus far...

The good... ☺

- CDCGs offer a proof-of-concept that arbitrarily accurate decoherence suppression solely based on open-loop control is possible in principle.
  - ➞ Highly portable – only need qualitative knowledge of environment and strechable controls...
  - ➞ Fully analytical – rigorous performance analysis and [often] physical insight...
  - ➞ Can concatenate with composite pulses for robustness under systematic control errors...

The bad... 😐

- Ignoring the system Hamiltonian [driftless assumption] can be a serious oversimplification.
  - ➞ What if H_S is required for universality and synthesizing primitive gates is non-trivial?...
  - ➞ How to construct balance pairs if the relationship between errors and control is not manifest?...

The ugly... ☹

- CDCG constructions can be very inefficient...
  - ➞ Single qubit, $n=1$: Sequence length grows exponentially with concatenation level...
  - ➞ Multiple qubits, n: Sequence length typically [also] grows exponentially with system size...

$$G^{[m]} \equiv G_{\text{adv}} \simeq (\mathbb{Z}_2 \times \mathbb{Z}_2)^{\times n} \quad \Rightarrow \quad G_{\text{adv}} = 4^n,\ L_{\text{adv}} = 2,\ \chi_{adv} = 4^n \times 5$$

The CDCG framework revisited

- For a given target gate, the actual control outcome is partitioned into ideal and error action:

  ➞ Actual gate propagator over duration T:

$$U(T) = T\exp\{-i\int_0^T ds\,[H_{ctrl}(s) + H_{S,g} + H_{err}]\} = Q\exp(-iE_Q)$$

CDCG framework provides a constructive recipe for finding a solution to

$$(1)\qquad Q\,e^{i\phi} - T\exp\{-i\int_0^T ds\,[H_{ctrl}(s) + H_S]\} = 0$$

Gate synthesis

$$(2)\qquad mod_B(E_Q) \propto \tau^m + O(\tau^{m+1})$$

Error cancellation

provided that (1) perfect [universal] gate synthesis can be achieved if $H_{\text{err}}=0$, and (2) a systematic relationship can be found between control and error for each segment.

- The more detail is available about error model and control specification, the lesser the need for portable DCG constructions ⇒ Optimize for specific control scenarios.

- Numerically optimized CDCGs: Rely on numerical search methods to solve one/both the above equations, by restricting solutions [control variables] within the admissible domain.

  ➞ Similar in spirit to strongly modulating pulses, OCT approaches...

Boosting efficiency via parametric optimization

Address the two problems separately [for now] – Problem 1: Retain driftless assumption…

- Strategy: Exploit freedom in describing control profiles to optimize parametrically EPG.

① Choose a desired pulse shape and parametrization – e.g., rectangular.

$$Q(\{h_l \tau_l\}) = T\exp\{-i\int_0^T ds\, H_{ctrl}(s)\} = \prod_{l=1}^n \exp[-i h_l H_l \tau_l], \quad T = \sum_{l=1}^n \tau_l$$

✔ Gate synthesis is automatically accommodated.

② Obtain symbolic expansion of error action E_Q in terms of perturbative error operators.

✔ For a given sequence, error can be evaluated parametrically order-by-order.

③ Search numerically for parameters that cancel prefactor for each algebraically independent term, while implementing desired gating action:

E.g., for 2nd-order DCG:

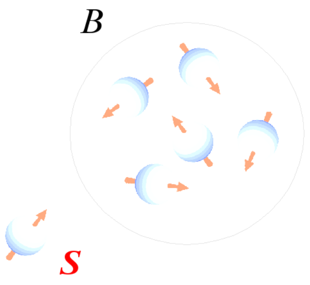
$$mod_B(E_Q^{[2]}) \propto \tau^3 + O(\tau^4) \;\Rightarrow$$

$$\begin{cases} z_1 \equiv mod_B(E_Q^{(1)}[\{h_l, \tau_l\}]) = 0 \\ z_2 \equiv mod_B(E_Q^{(2)}[\{h_l, \tau_l\}]) = 0 \end{cases}$$

The existence of arbitrary-order DCGs guarantees existence of a solution to search problem.

➡ Explicit expressions for z_1 and z_2 depend on problem specification…

Illustrative results



$$|\psi_S\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\rho_B = \boldsymbol{I}_B\, 2^{-N}$$

➞ Case study: Single qubit coupled to purely dephasing spin bath

$$H_{error} = \boldsymbol{I}_S \otimes \sum_{k=1}^{N} D_{kl}\, \vec{I}_k \cdot \vec{I}_l + \sigma_z \otimes \sum_{k=1}^{N} A_k I_{kz}, \quad N = 5$$

Objective: 2nd-order DCGs. Assume rectangular [reversible] profiles.

Recall:

$$\tau_{m+1} = \left[G_m L_m + 3 + (G_m - 1)(1 + 2^{1/(m+1)}) \right] \tau$$

➞ Analytical generic DCG:

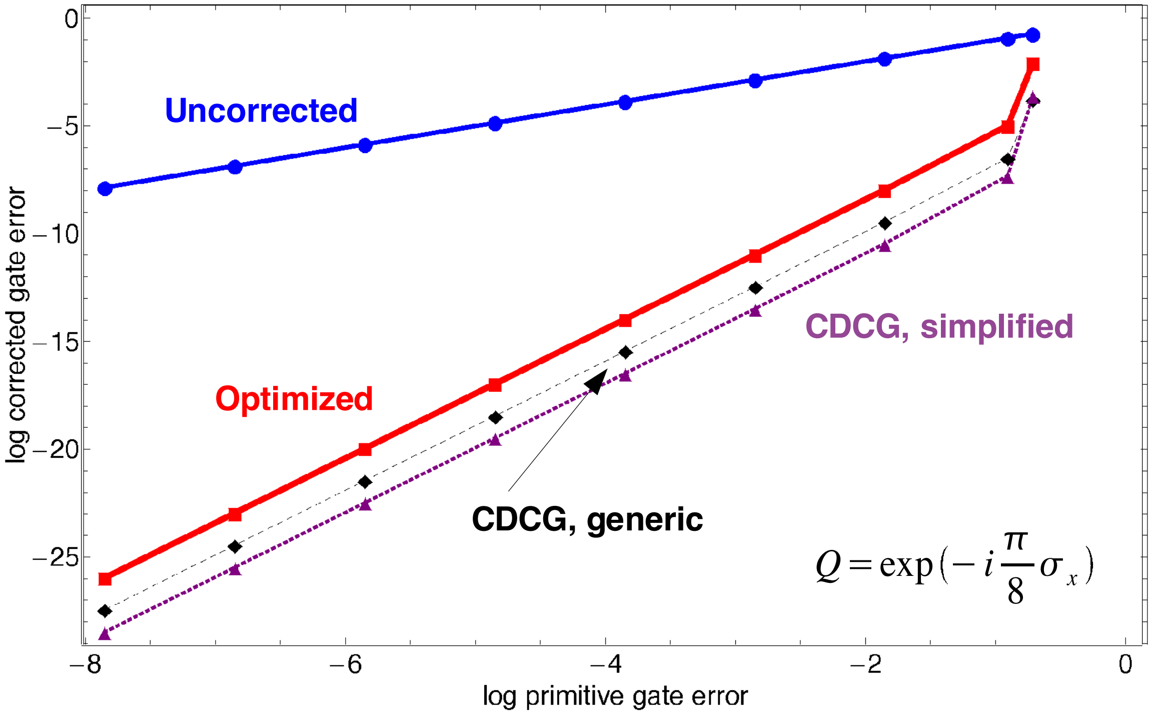
$$\tau_2 = (14 + 3\sqrt{2})\, 20\, \tau \approx 365\, \tau$$

➞ Analytical simplified DCG:

$$\tau_2 = (14 + 3\sqrt{2})\, 8\, \tau \approx 146\, \tau$$

➞ Numerically optimized DCG:

$$\tau_2 \approx 21\, \tau$$



Accommodating drift via robust optimization

Address the two problems separately [for now] – Problem 2: Focus on first-order DCGs...

- Strategy: Search for simultaneous solution to gate synthesis and error cancellation conditions.

  ➞ Simplest setting: Single qubit, [effectively] closed system, piecewise-const controls:

$$H_{tot} = H_S + H_{err} + H_{ctrl}(t) = \omega\sigma_z + \epsilon\sigma_z + h(t)\sigma_x$$

Relevant to singlet-triplet qubit in DQD: ω ➔ magnetic field gradient, $h(t)$ ➔ exchange splitting

Foletti et al, Nature Phys. 2009; Grace et al, arXiv:1105.2358.

➞ Drift is required for complete controllability but prevents a simple relationship between duration of each control segment and associated error action to be found...

✔ Cannot simply redefine $H'_{\text{err}} = H_{\text{err}} + \omega\sigma_z$ – need not be small... only x-direction controllable...

Objective: Determine control solution that cancels [minimizes] simultaneously

(1) Fidelity loss in the absence of error (gate synthesis) $\Rightarrow$ $z_1(\{h_l, \tau_l\}) \equiv \| U_{ctrl}(T) - Q \|$

(2) Effect of error Hamiltonian up to the 1st-order in ϵ $\Rightarrow$ $z_2(\{h_l, \tau_l\}) \equiv \| E_Q^{[1]} \|$

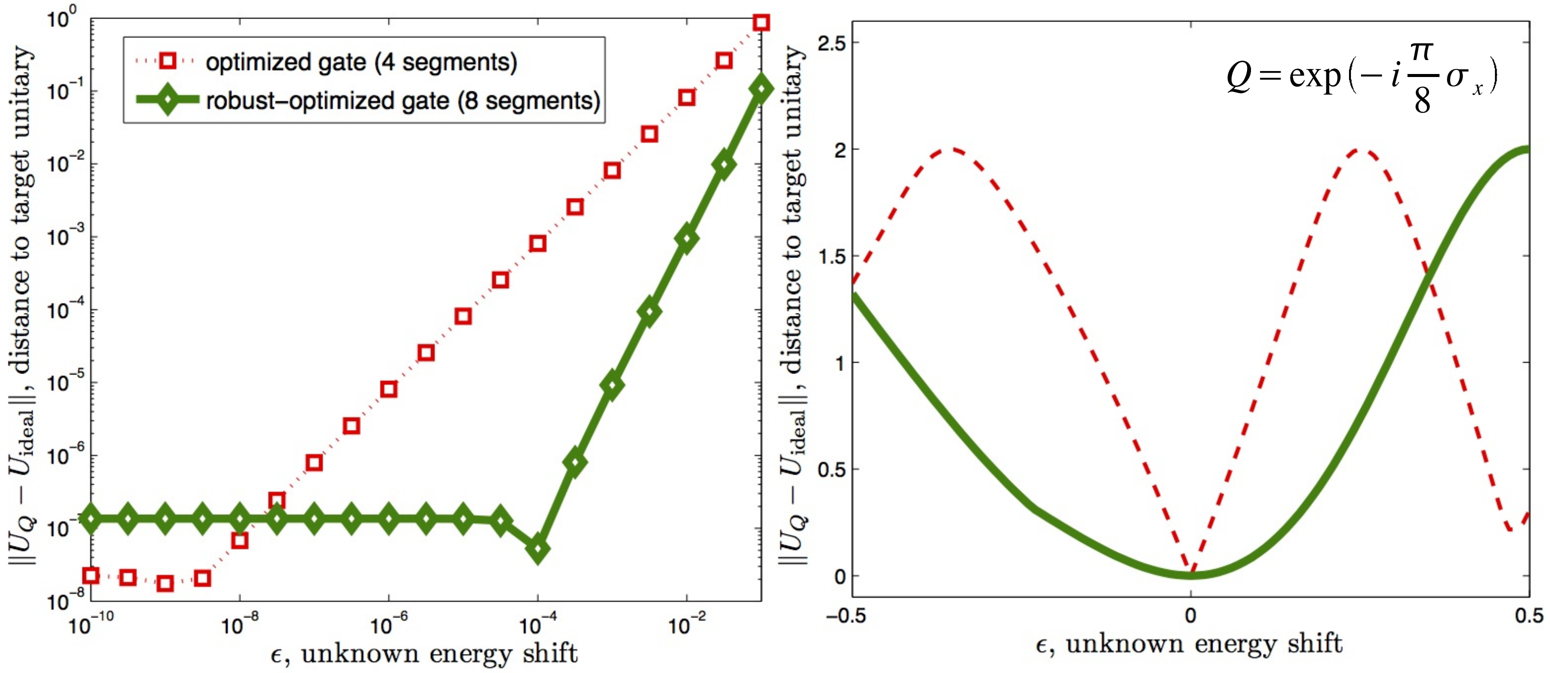
(Simplest choice) $z(\{h_l, \tau_l\}) = z_1(\{h_l, \tau_l\}) + z_2(\{h_l, \tau_l\})$

Illustrative results

➡ 'Twice easier' to synthesize gate vs. synthesizing a <u>robust</u> [1st-order DCG] gate:

Minimization of z_1 alone ⇒ 4 control segments suffice $[z_1^{\mathrm{min}} = 2.3 \cdot 10^{-8}]$

Minimization of $z_1 + z_2$ ⇒ At least 8 control segments required $[z^{\mathrm{min}} = 2.0 \cdot 10^{-7}]$



➡ 'Flatness' of DCG solution indicates its robustness compared to optimized gate

Optimized gate has higher fidelity in the limit $\epsilon \to 0$ $[\epsilon < 10^{-8}]$

DCG provides higher fidelity in a wide range $\epsilon > \epsilon_{min}$, $\epsilon_{min} T = O(z_1^{min} / z_2^{min})$ $[\epsilon_{min} \approx 10^{-4}]$

Conclusion

- DQEC – DD plus CDCGs – has the potential to reduce memory and gate errors below the level required by accuracy threshold for non-Markovian QEC.

See also Ng, Lidar & Preskill, PRA 2011.

  ➞ Make contact with filter-function formalism for classical noise settings...

  Green, Uys & Biercuk, arXiv:1110.6686.

  ➞ Explore DCGs with continuous driving fields...

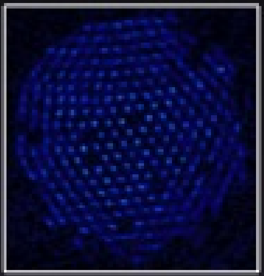
  Fanchini, Napolitano & Caldeira, arXiv:1005.1666; Chaudhry & Gong, arXiv:1110.4695.

- Plenty of room exists for improving the efficiency of CDCG constructions and for optimizing their performance under specific system/control assumptions.
  - ✓ Single-qubit setting:
    - ➞ Develop comprehensive numerically-optimized solution, make formal contact with OCT (analyze complexity, landscape and convergence properties) ...
  - ✓ Many-qubit setting:
    - ➞ Need to better exploit locality and sparsity of physical error models...

- Dedicated experimental realizations/benchmarking of DCGs needed...

Stay tuned...

Quantum Firmware Collaboration



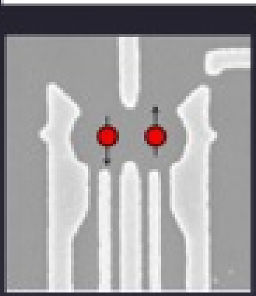
Michael J. Biercuk

www.physics.usyd.edu.au/~mbiercuk

Lorenza Viola

www.dartmouth.edu/~viola/

Postdoc position available!



Amir Yacoby

yacoby.physics.harvard.edu/

Support

NATIONAL SECURITY AGENCY
UNITED STATES OF AMERICA

SCIENCE TO SHAPE THE FUTURE OF THE ARMY
ARMY RESEARCH OFFICE
1951