Three-qubit quantum error correction with superconducting circuits

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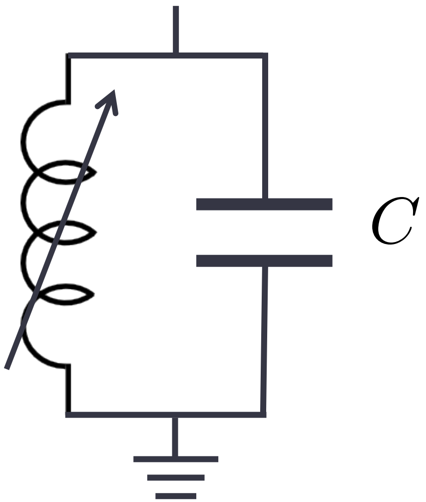
Outline

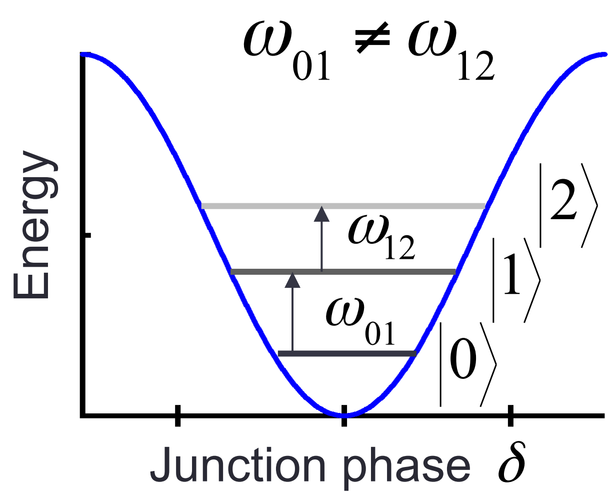
- Introduction to superconducting qubits
- Adiabatic and sudden two-qubit phase gates
- GHZ states
- Efficient Toffoli gate using third-excited state
- Bit- and phase-flip error correction
- Outlook

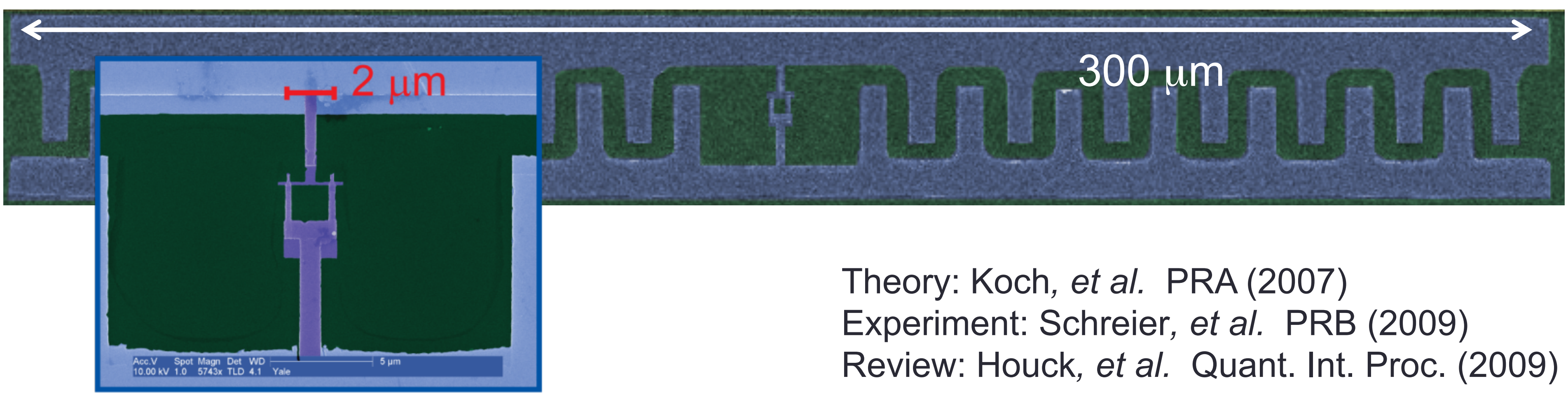
Superconducting transmon qubits

$$L = \frac{L_J}{\cos\delta}$$



Nonlinear
LC Oscillator

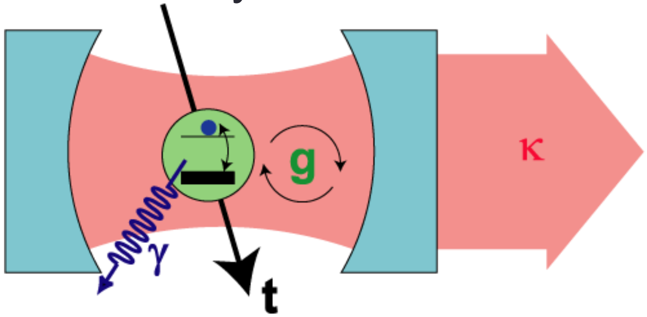




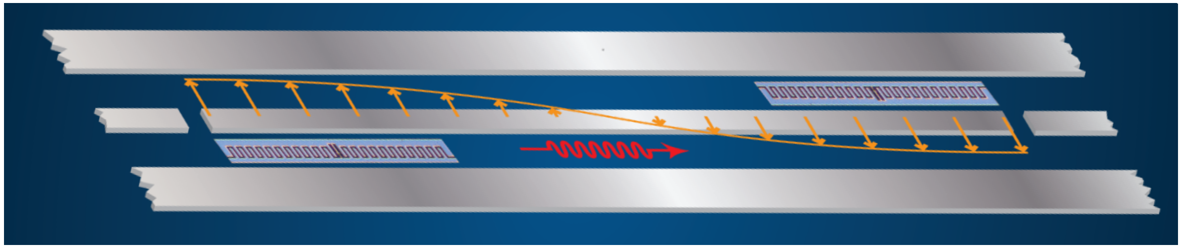
Theory: Koch, *et al.* PRA (2007)
Experiment: Schreier, *et al.* PRB (2009)
Review: Houck, *et al.* Quant. Int. Proc. (2009)

Circuit quantum electrodynamics

Cavity QED



Circuit QED



Couple transmon qubits to superconducting microwave resonator

- **Protection** from spontaneous emission
- Multiplexed qubit drives (**single-qubit gates**)
- Couple qubits together (**multi-qubit gates**)
- Qubit **readout**

Jaynes-Cummings Hamiltonian:

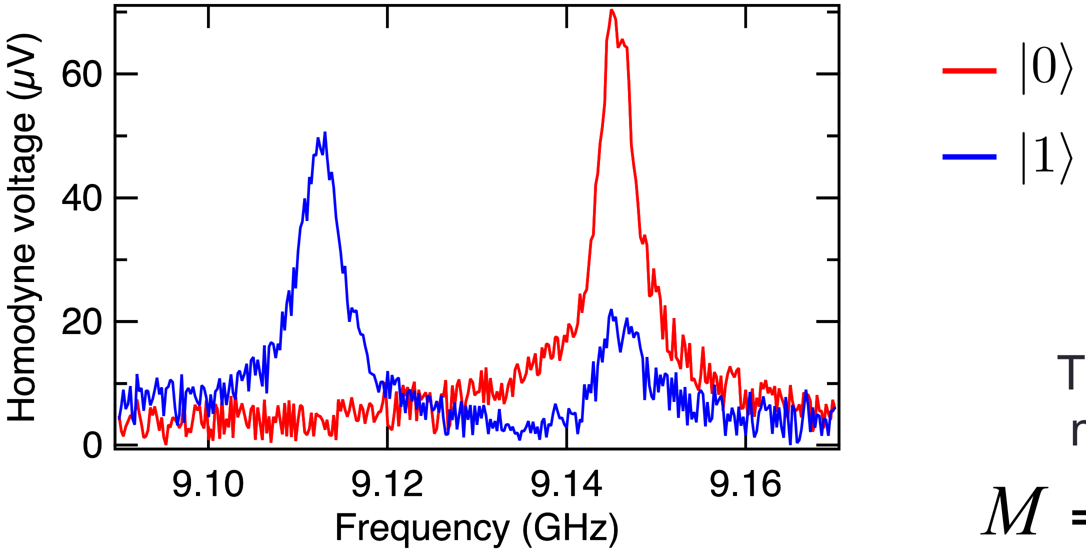
$$\mathrm{H} = \hbar\omega_{\mathrm{r}}(a^{\dagger}a + 1/2) + \hbar\omega_{\mathrm{a}}\sigma_{\mathrm{z}} + \hbar g(a^{\dagger}\sigma^{-} + a\sigma^{+})$$

Qubit readout

$$H = \hbar\omega_r(a^\dagger a + 1/2) + \hbar\omega_a\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Dispersive limit: $g \ll \Delta = \omega_a - \omega_r$

$$H \approx \hbar\left(\omega_r + \underline{\frac{g^2}{\Delta}\sigma_z}\right)(a^\dagger a + 1/2) + \hbar\omega_a\,\sigma_z/2$$



Three qubits: multiplexed

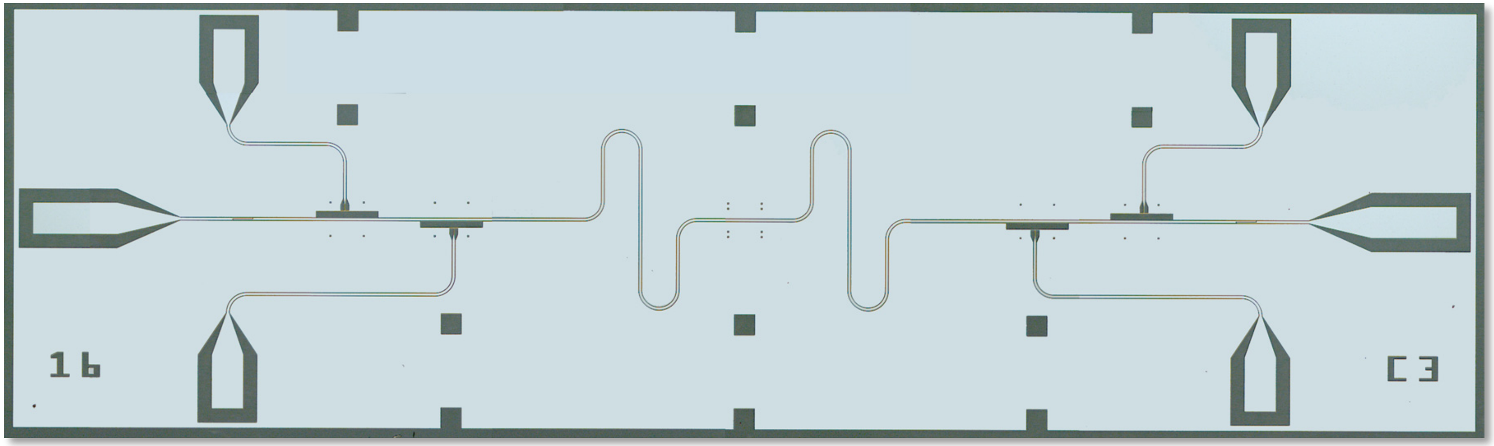
$$M = |000\rangle\langle 000|$$

State tomography

Reed, *et al.* *Phys. Rev. Lett.* **105,** 173601 (2010)

Four qubit cQED device

- Four transmon qubits coupled to single microwave resonator



- Three qubits biased at **6**, **7**, and **~8** GHz (and one above)
- Each has a **flux bias line** to control frequency in nanoseconds
  - **Two qubit gates**

DiCarlo, *et al. Nature* **467** 574 (2010)

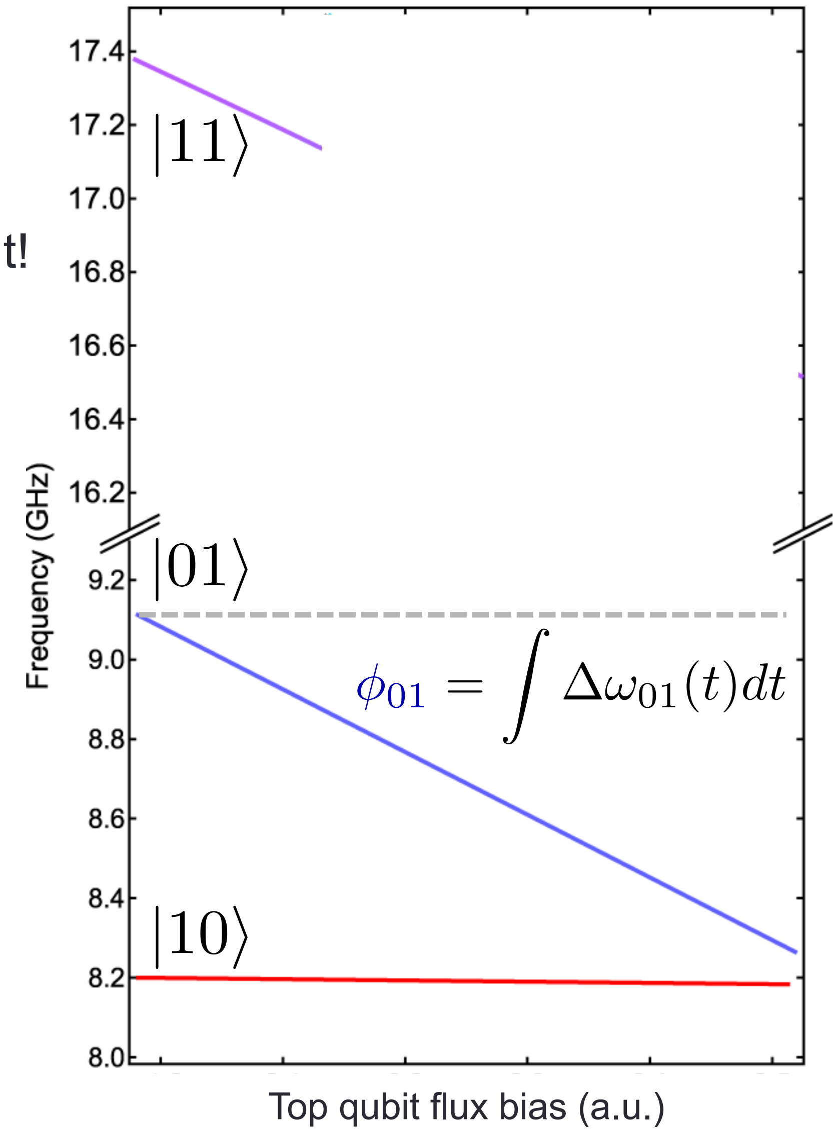
Adiabatic multiqubit phase gates

A **two qubit** phase gate can be written:

$$|00\rangle \longrightarrow |00\rangle$$
$$|01\rangle \rightarrow e^{i\phi_{01}}|01\rangle$$

Entanglement!

Interactions on **two excitation manifold** give entangling two-qubit conditional phases



Adiabatic multiqubit phase gates

A **two qubit** phase gate can be written:

$$|00\rangle \longrightarrow |00\rangle$$
$$|01\rangle \rightarrow e^{i\phi_{01}}|01\rangle$$
$$|10\rangle \rightarrow e^{i\phi_{10}}|10\rangle$$
$$|11\rangle \rightarrow e^{i(\phi_{01}+\phi_{10}+\phi_{11})}|11\rangle$$

Entanglement!

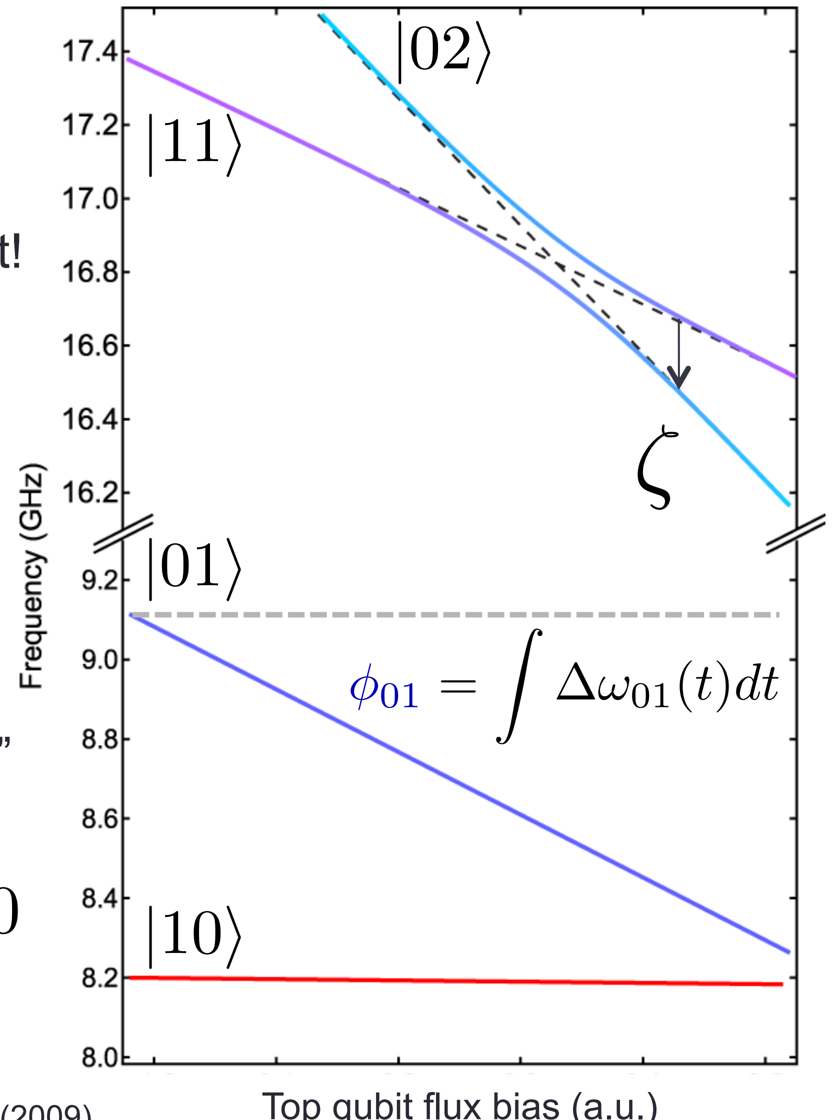
Interactions on **two excitation manifold** give entangling two-qubit conditional phases

$$\phi_{11} = -2\pi \int \zeta(t)dt$$

Can give a **universal** "Conditional Phase Gate"

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |10\rangle$$
$$|11\rangle \rightarrow -|11\rangle$$

$$\phi_{01} = \phi_{10} = 0$$
$$\phi_{11} = \pi$$

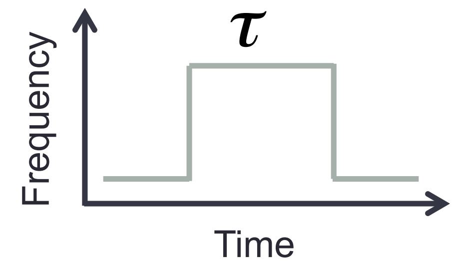


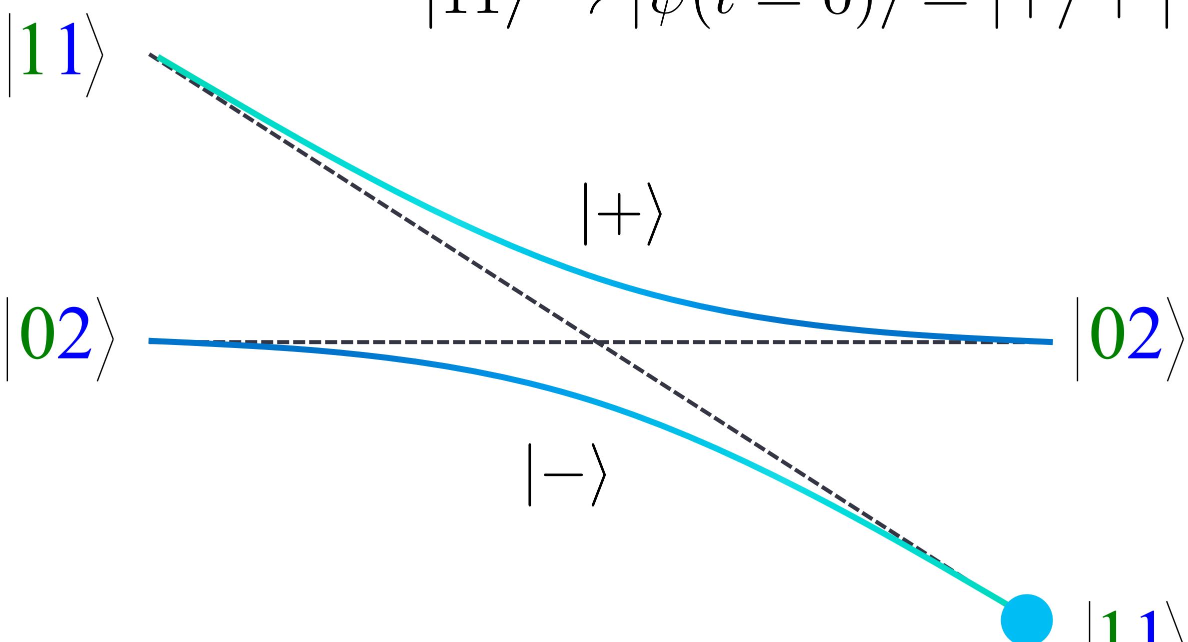
DiCarlo, *et al.* Nature **460**, 240 (2009)

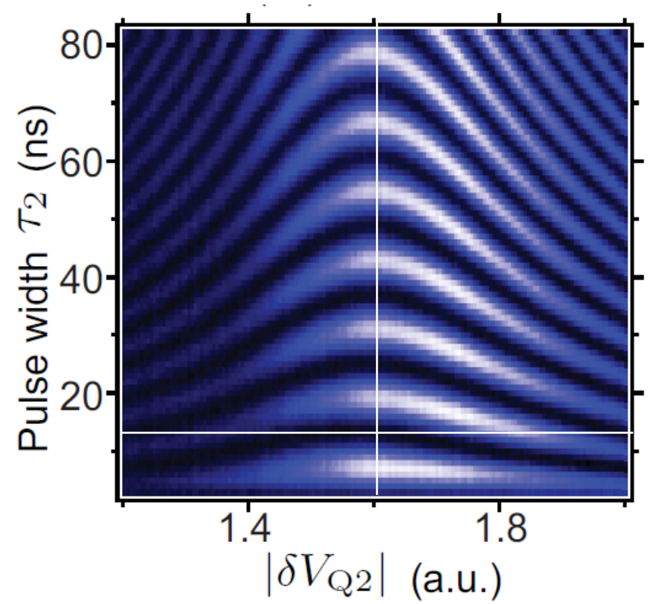
Sudden multiqubit phase gates

Suddenly move $|11\rangle$ into resonance with $|02\rangle$

$$|11\rangle \rightarrow |\psi(t=0)\rangle = |+\rangle + |-\rangle$$







$$|\psi(t)\rangle = e^{i\Delta t/2}|+\rangle + e^{-i\Delta t/2}|-\rangle$$

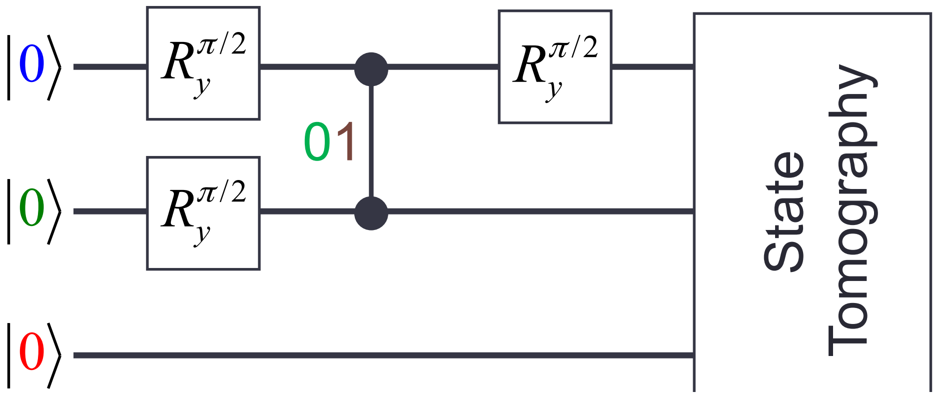
$$|\psi(t = 2\pi/\Delta)\rangle = -(|+\rangle + |-\rangle) \rightarrow -|11\rangle$$

$\tau = 12$ ns

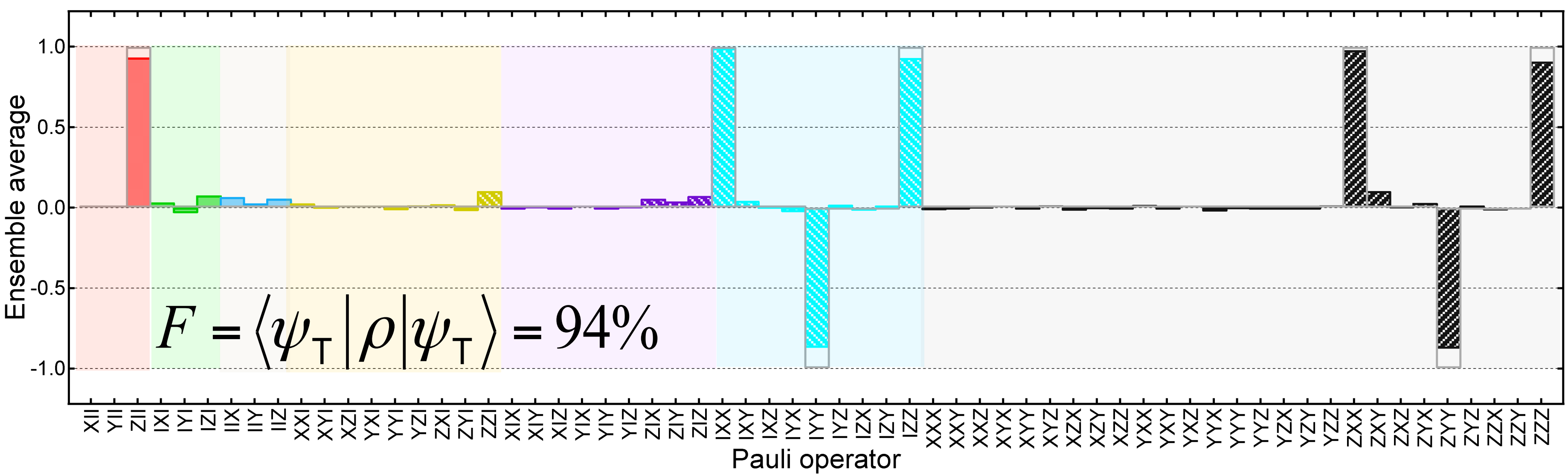
Or, transfer to $|02\rangle$ in 6 ns!

Strauch *et al*., PRL (2003): proposed this approach

Entangled states on demand

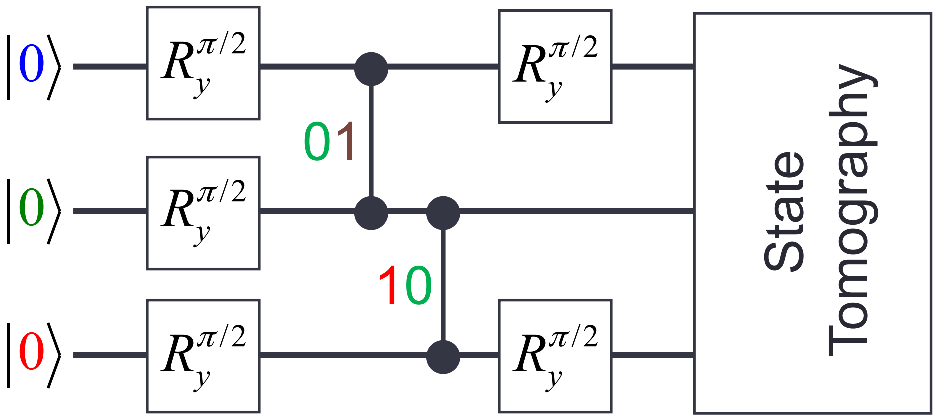


$$|\psi_T\rangle = |0\rangle \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

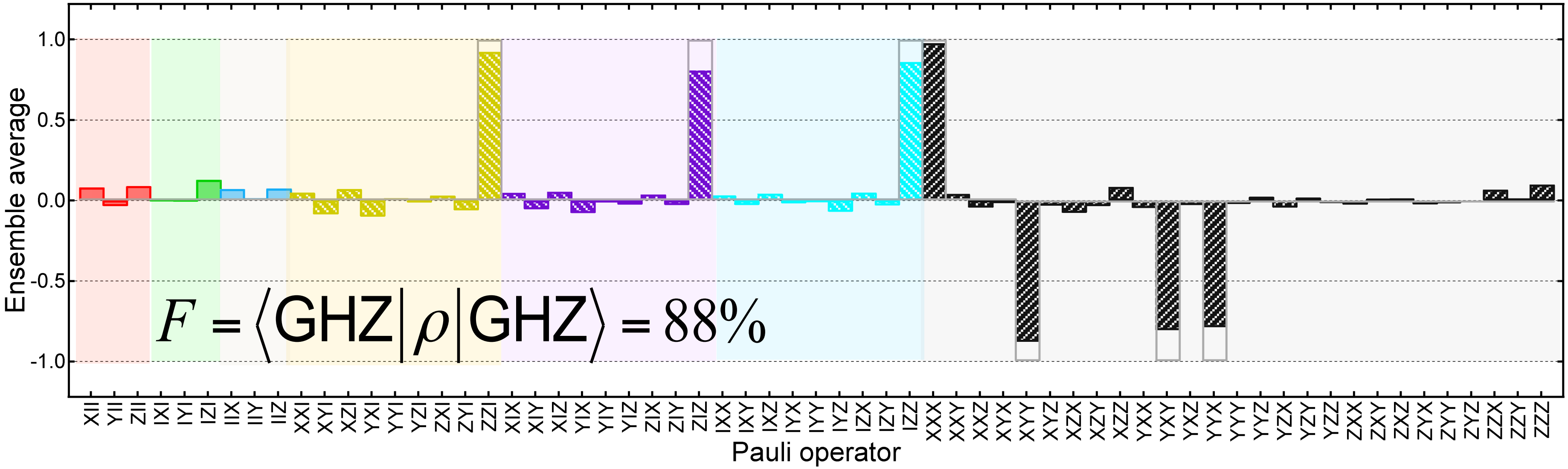


DiCarlo, *et al.* *Nature* **467** 574 (2010)

GHZ states



$$|\psi_{\text{target}}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$



DiCarlo, *et al.* *Nature* **467** 574 (2010)

Properties of GHZ-like states

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

All Z_iZ_j correlations are +1

$$|\psi_{QEC}\rangle = \alpha|000\rangle + \beta|111\rangle$$

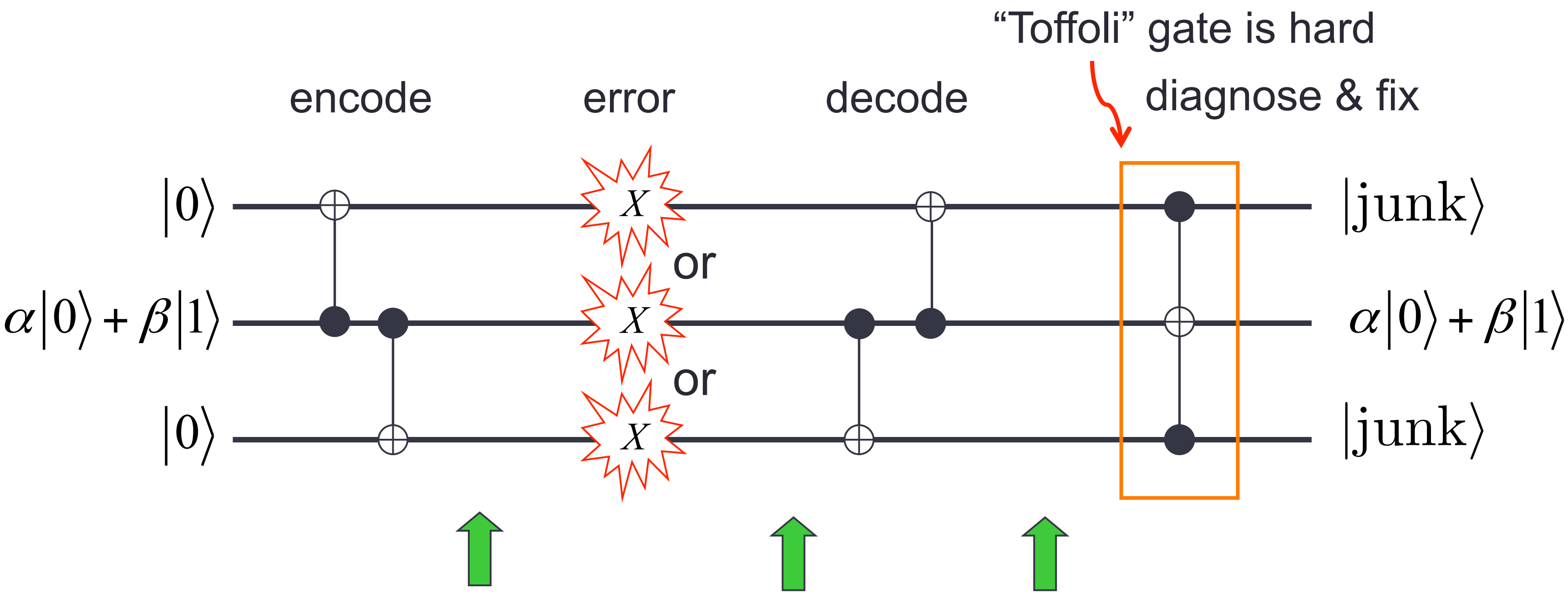
All Z_iZ_j correlations are still +1, *independent of* α *and* β

Flipped qubit	State	Z_1Z_2	Z_2Z_3
None	$\alpha\|000\rangle + \beta\|111\rangle$	+1	+1
Q_1	$\alpha\|100\rangle + \beta\|011\rangle$	-1	+1
Q_2	$\alpha\|010\rangle + \beta\|101\rangle$	-1	-1
Q_3	$\alpha\|001\rangle + \beta\|110\rangle$	+1	-1

Each error has a **different** observable!

Bit-flip error correction circuit

(measurement-free implementation)



Toffoli can be constructed with **five** two-qubit gates, but that's expensive

$\alpha|000\rangle + \beta|111\rangle$

$\alpha|010\rangle + \beta|101\rangle$

$\alpha|111\rangle + \beta|101\rangle$

$= (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$

GHZ state for $|\alpha| = |\beta|$

Can we do better?

P. Schindler et al. *Science* 332 1059 (2011)

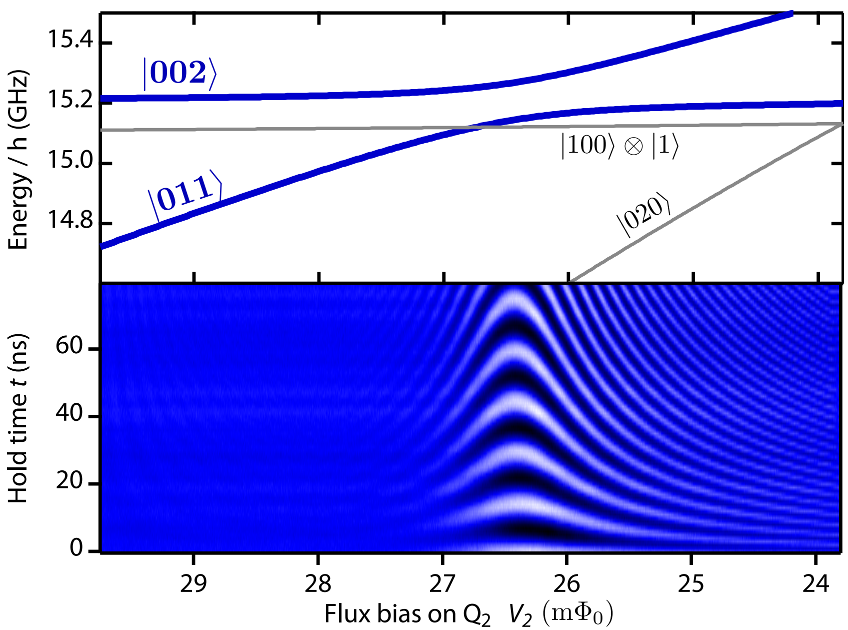
Nielsen & Chuang *Cambridge Univ. Press*

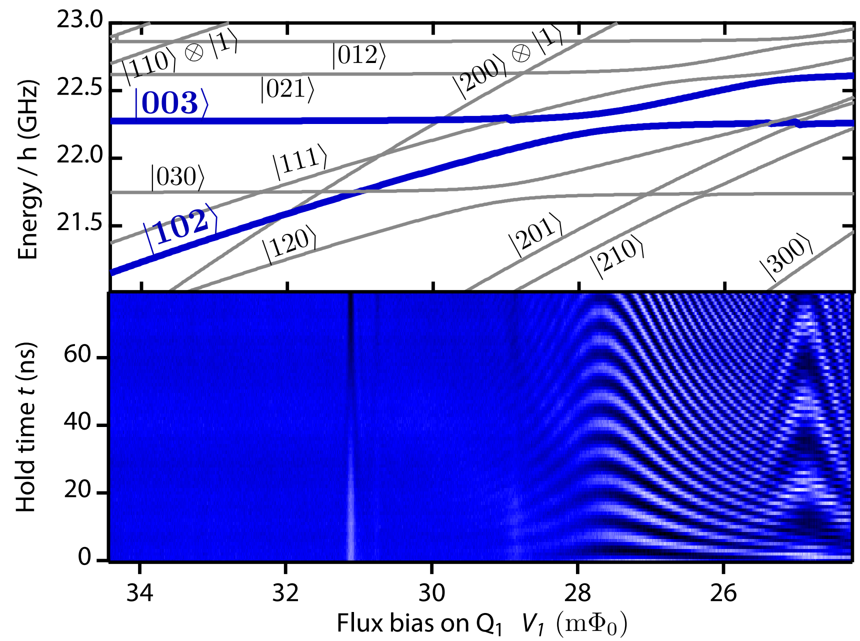
Toffoli gate with noncomputational states

Two-qubit gate requires two excitations $|11\rangle \circlearrowright |02\rangle$

Three-qubit interaction: third excited state $|111\rangle \circlearrowright |003\rangle$ The essence!

This interaction is small, so use intermediary $|111\rangle \rightarrow |102\rangle \circlearrowright |003\rangle$





Sudden transfer: $|011\rangle \rightarrow |002\rangle$

$|111\rangle \rightarrow |102\rangle$

Adiabatic interaction: $|102\rangle \circlearrowright |003\rangle$

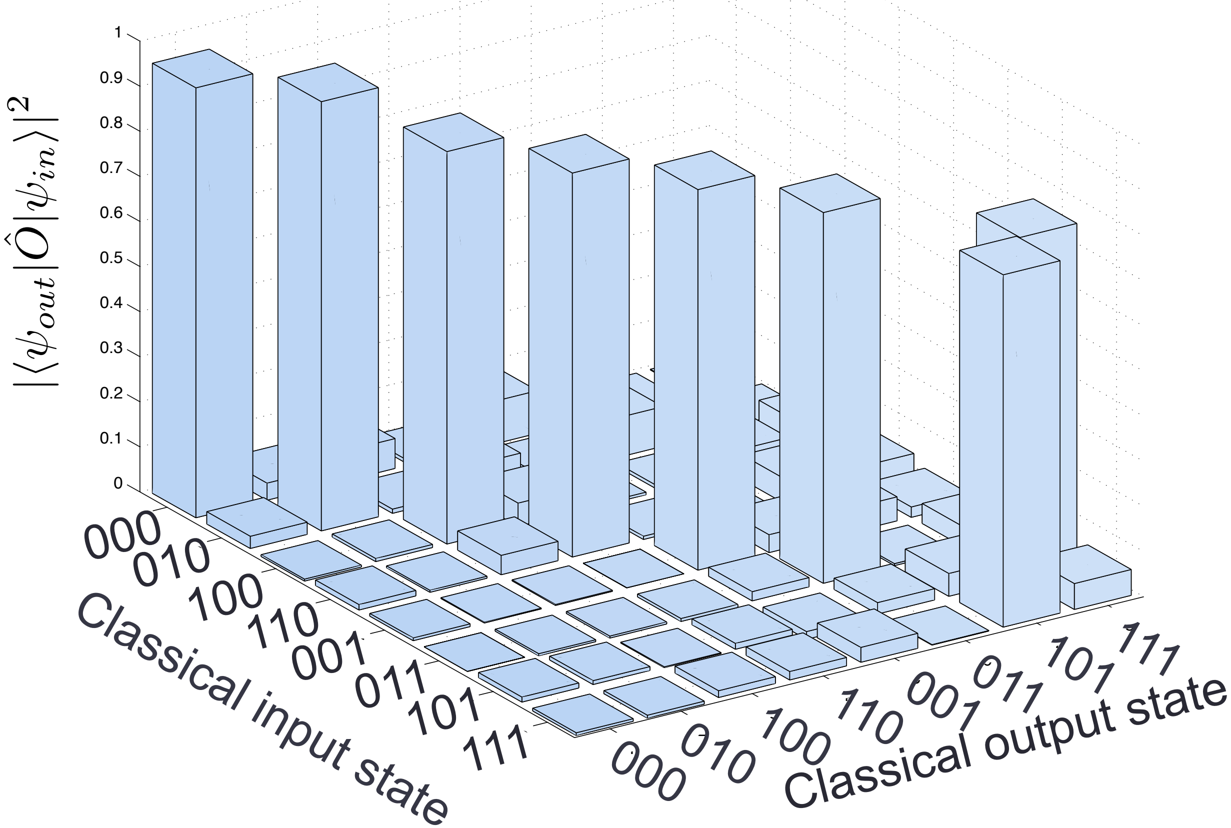
Three-qubit phase here!

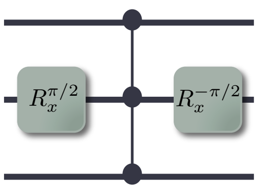
Classical truth table

How do we prove the gate works? First, **measure classical action**

Classically, a phase gate does nothing. So we dress it up to make it a **CCNOT**

$$|\text{input}\rangle \rightarrow \hat{O} \rightarrow |\text{output}\rangle$$





$\mathcal{F}$ = 86%

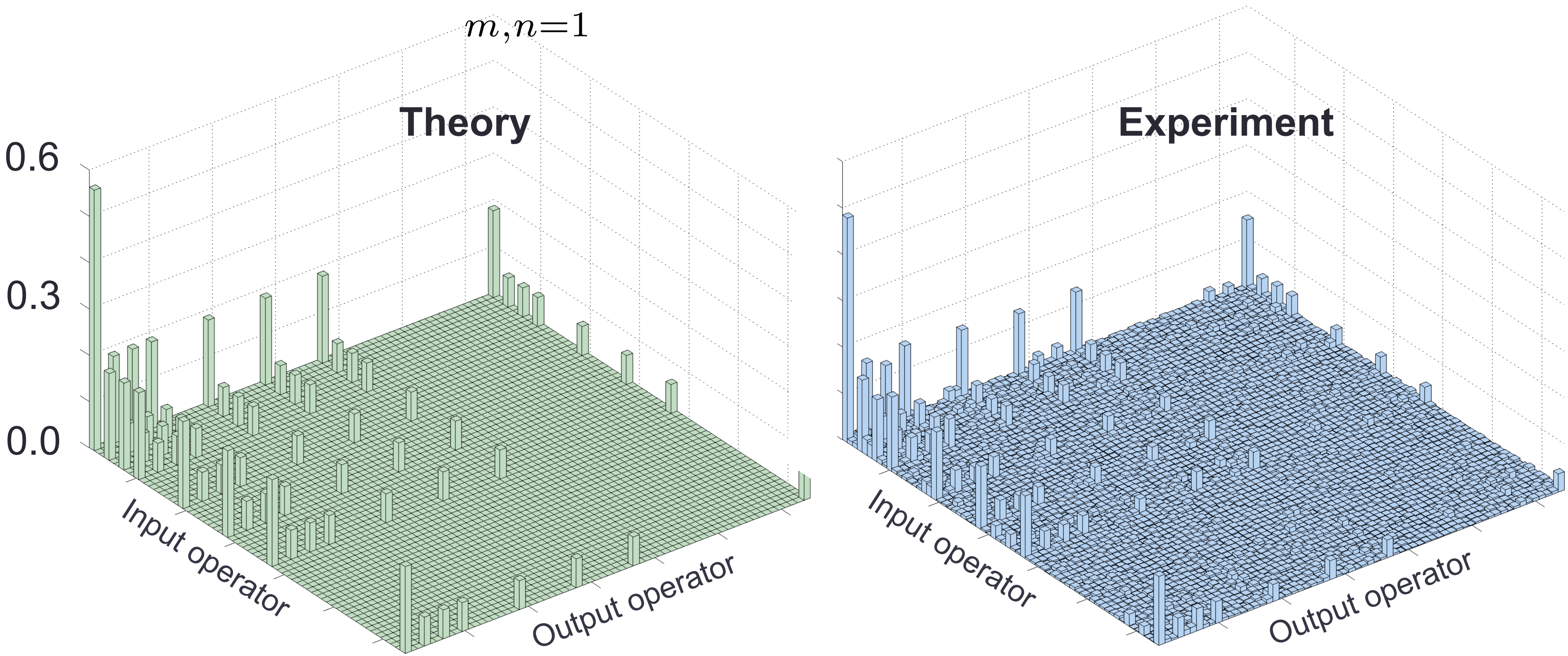
Quantum process tomography of CCPhase

Want to know the **action on superpositions:** $|\mathrm{input}\rangle \rightarrow \hat{O} \rightarrow |\mathrm{output}\rangle$

(but now with **64** basis states)

$$\rho_{\mathrm{out}} = P(\rho_{\mathrm{in}}) = \sum_{m,n=1}^{4^N} \chi_{m,n} A_m \rho_{\mathrm{in}} A_n^\dagger$$

Invert to find χ



4032 Pauli correlation measurements (90 minutes)

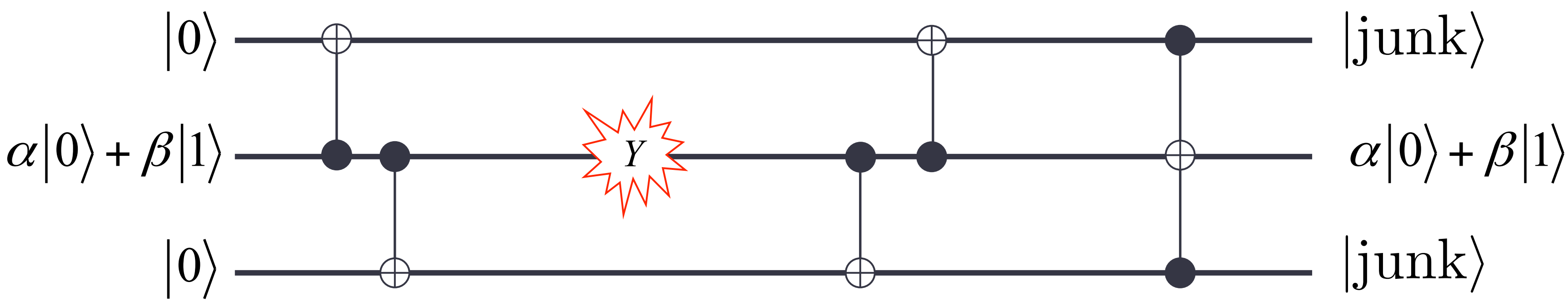
$\mathcal{F} = 77\%$

Bit-flip error correction with fast Toffoli

Prepare state $|{+X}\rangle$

"Error" rotation
by some angle

Correct



Encode in
three-qubit
state

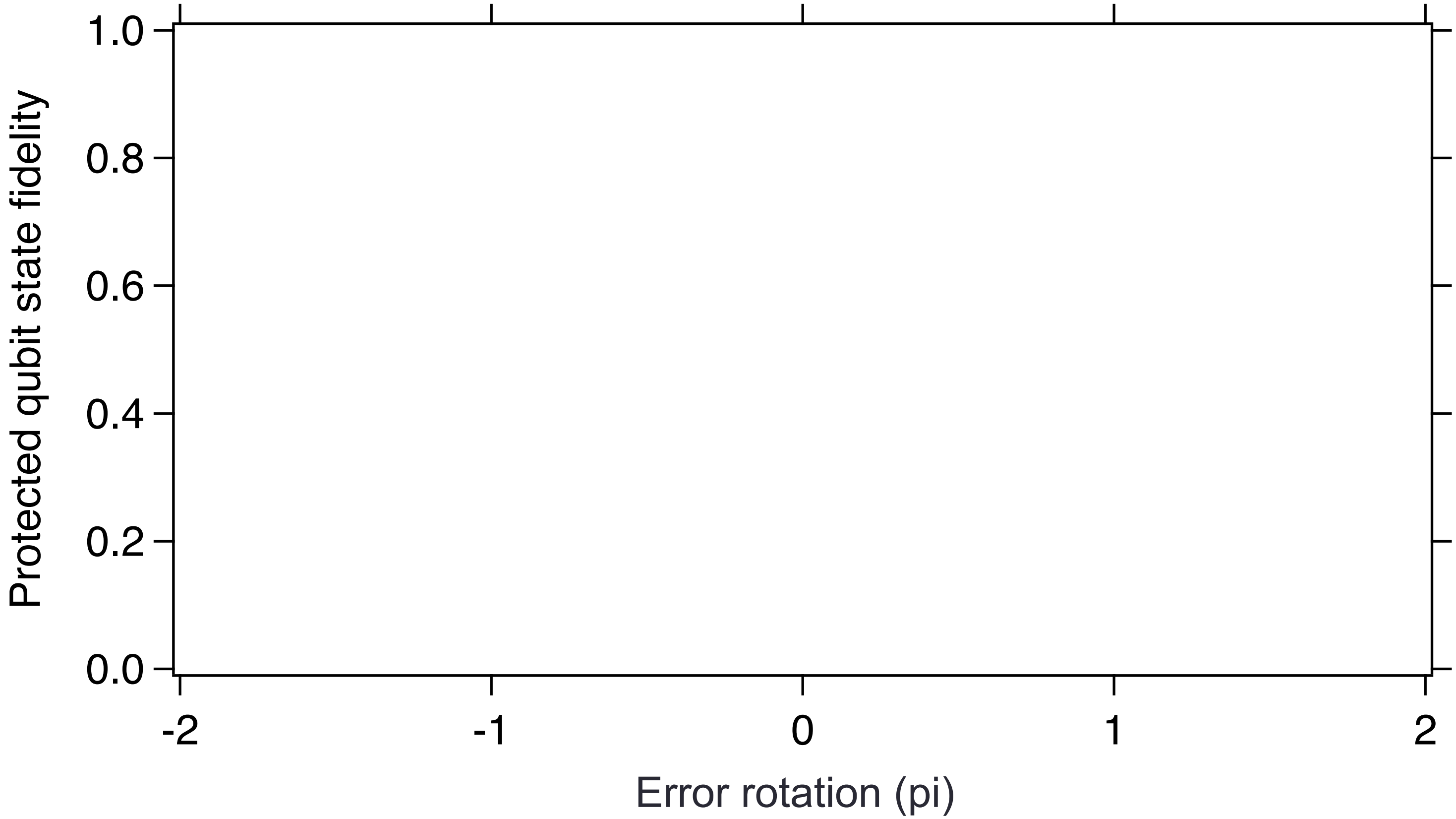
Decode
error
syndromes

Measure single-
qubit state
fidelity to $|{+X}\rangle$

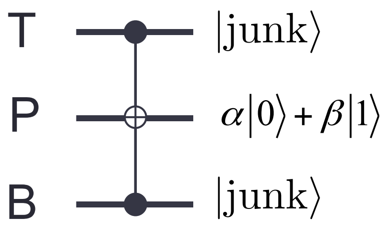
Ideally, there should be no dependence of fidelity on the error rotation angle

Correction fidelity vs. error rotation

Encode, single known error, decode, fix, and measure resulting state fidelity

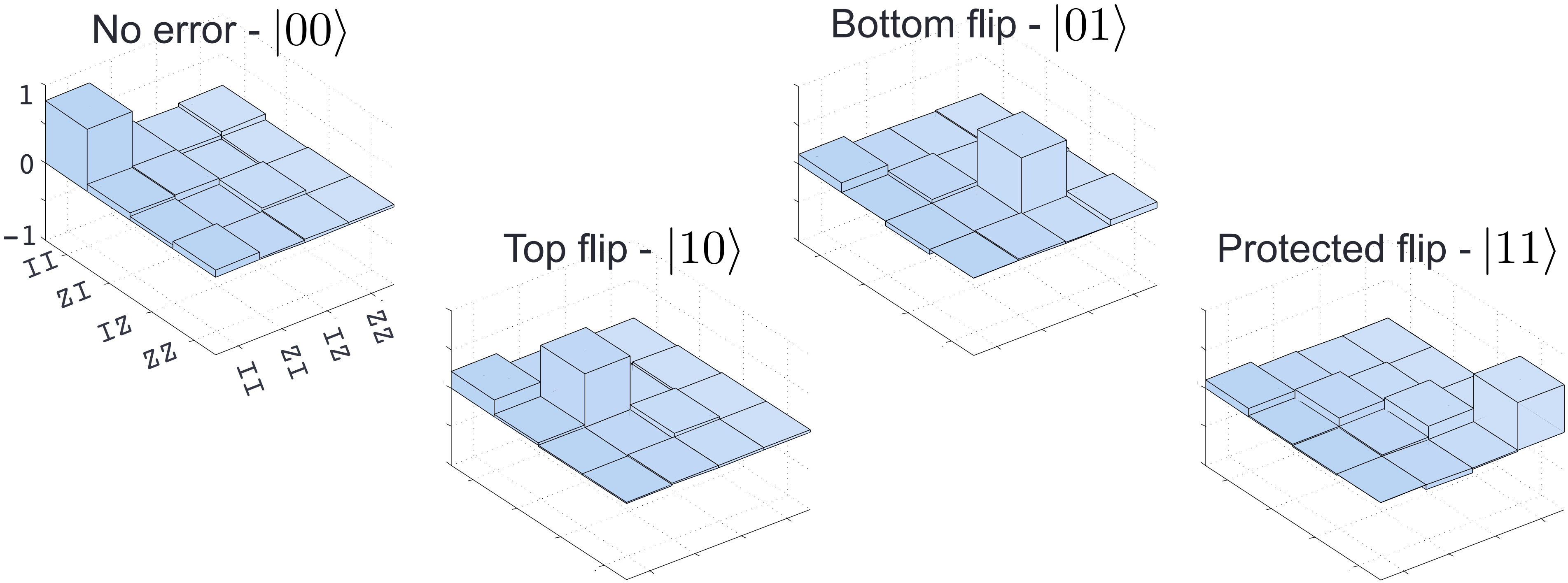


Error syndromes



Is the algorithm really doing what we think?

Look at **two-qubit density matrices of** $|\mathrm{junk}\rangle$ after a full flip

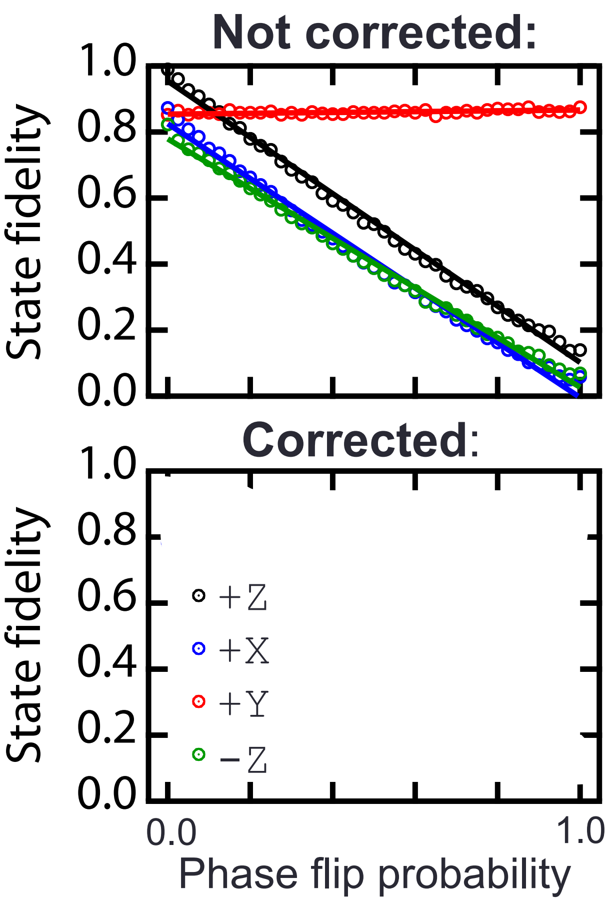


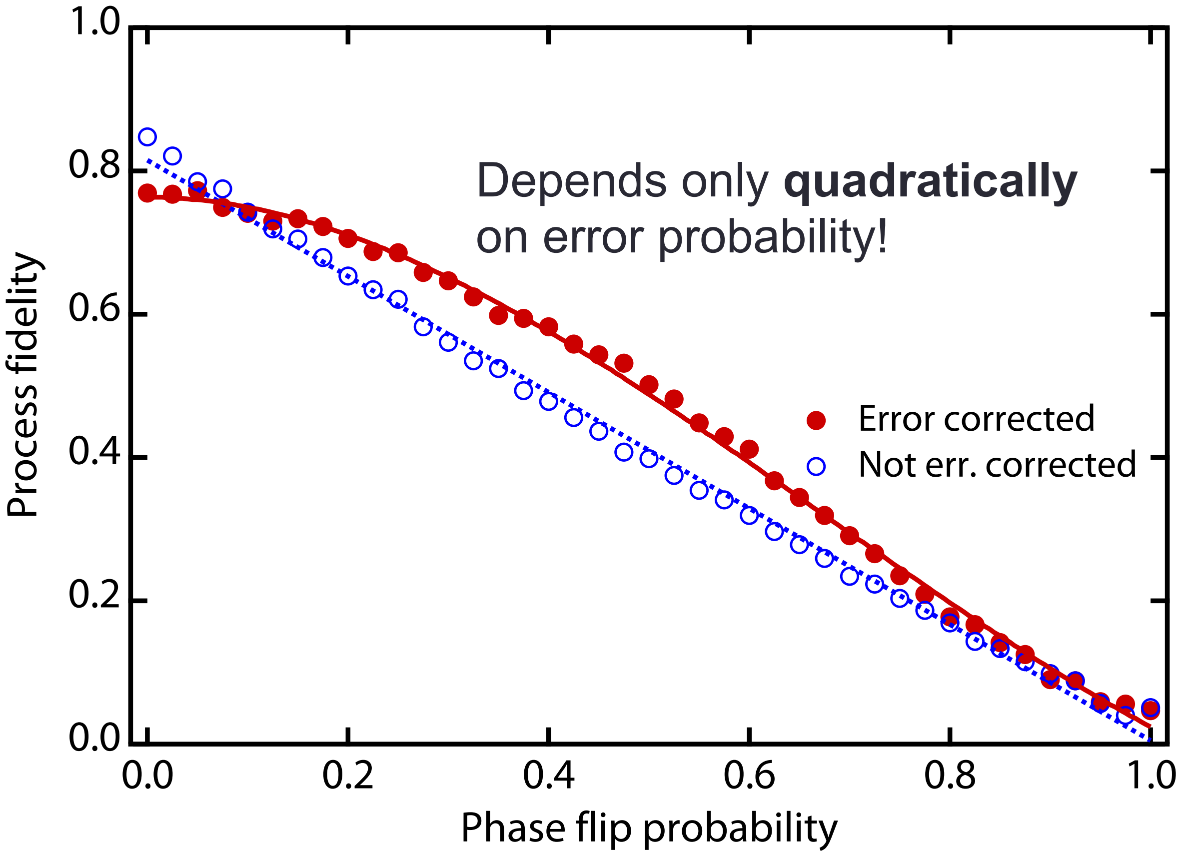
It is also clear here why you need at least three qubits!

Simultaneous phase-flip errors

More realistic error model: Flip happens with probability $p = \sin^2(\theta/2)$

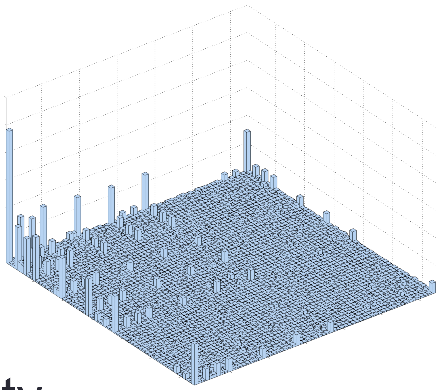
Correction only works for **single errors**. Probability of two or three errors: $3p^2 - 2p^3$





Conclusions

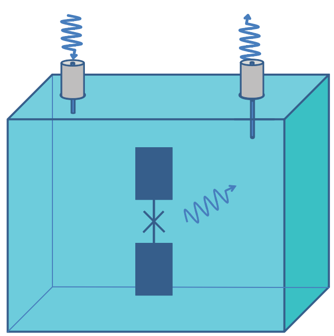
- Demonstrated the simplest version of gate-based QEC
  - Both bit- and phase-flip correction
  - Not fault-tolerant (gate based, un-encoded)
- Based on new three-qubit phase gate
  - Adiabatic interaction with transmon third excited state
  - Works for any three nearest-neighbor qubits
  - 86% classical fidelity and 77% quantum process fidelity

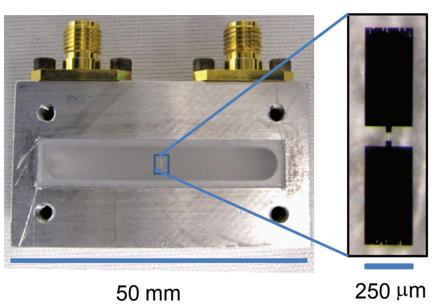


Preprint available at arXiv:1109.4948 (accepted to *Nature*)

Outlook

- Concatenating bit and phase flip codes gives full QEC
  - But requires nine qubits
  - A logical qubit per cavity with intra-cavity coupling?
- Planar qubits are not coherent enough
  - But we've made huge progress on that front with a parallel experiment (Paik, *et. al.* arXiv:1105.4652, in the press at *PRL*)
  - **Three-dimensional** architecture yields ~**40 times** longer qubit lifetimes
  - Need to re-integrate control knobs (e.g. FBLs) and scale up

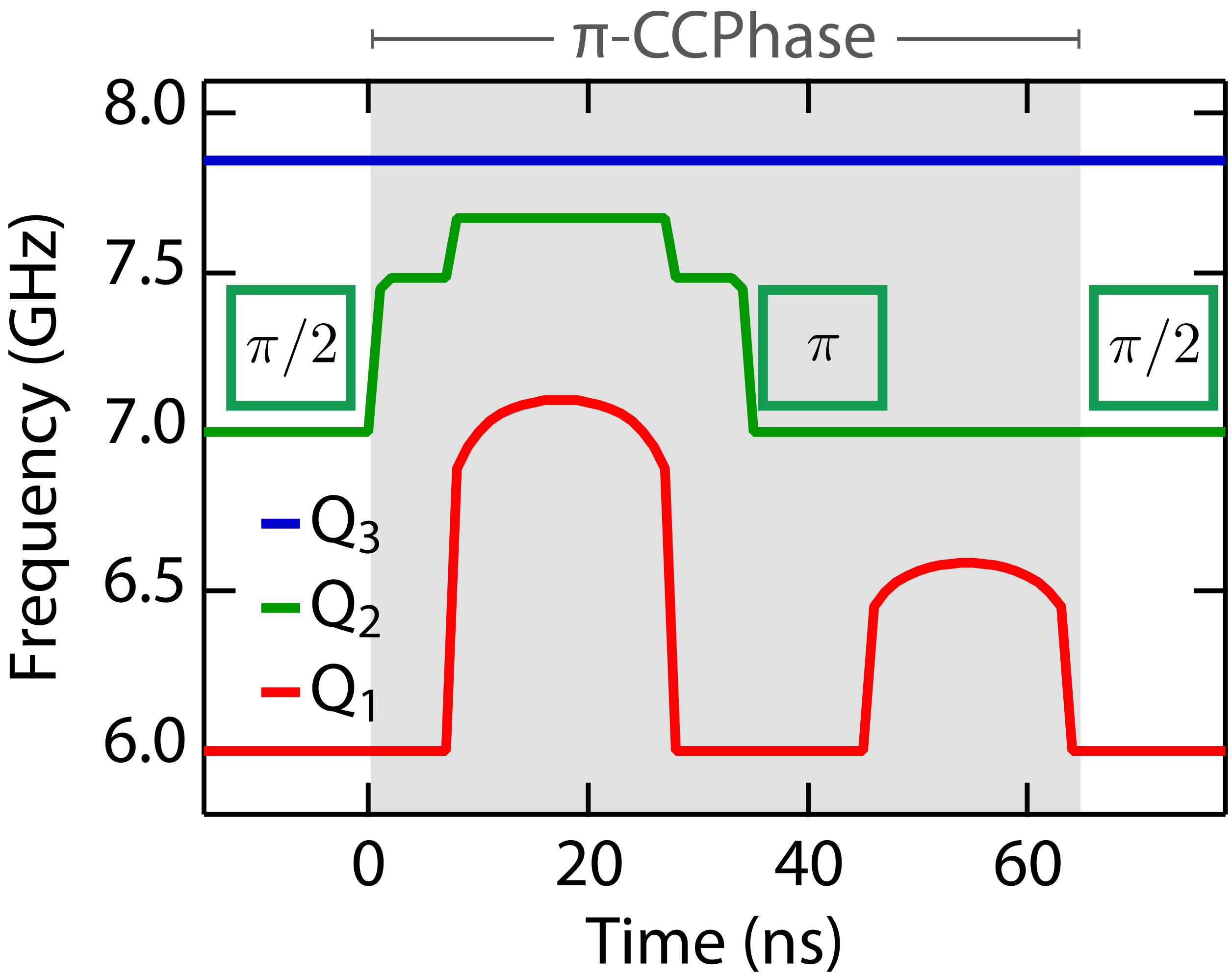




Questions?

Preprint: Reed, *et al.* arXiv:1109.4948
(accepted to *Nature*)

CCNot gate pulse sequence

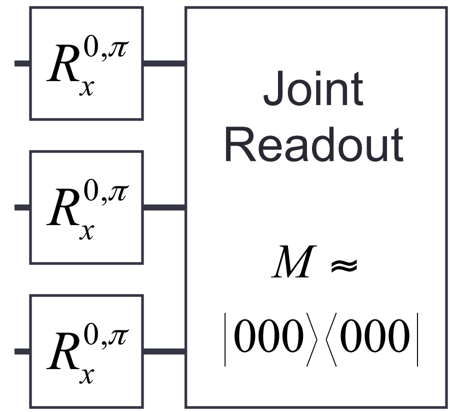


More than two times faster than equivalent two-qubit gate sequence

Three qubit state tomography

$$M = |000\rangle\langle 000|$$

$$\propto ZII + IZI + IIZ + ZZI + ZIZ + IZZ + ZZZ$$



Example: extract $\langle ZZZ \rangle$

no pre-rotation: $+\langle ZII\rangle + \langle IZI\rangle + \langle IIZ\rangle + \langle ZZI\rangle + \langle ZIZ\rangle + \langle IZZ\rangle + \langle ZZZ\rangle$

$R_x(\pi)$ on Q1 and Q2: $-\langle ZII\rangle - \langle IZI\rangle + \langle IIZ\rangle + \langle ZZI\rangle - \langle ZIZ\rangle - \langle IZZ\rangle + \langle ZZZ\rangle$

$R_x(\pi)$ on Q1 and Q3: $-\langle ZII\rangle + \langle IZI\rangle - \langle IIZ\rangle - \langle ZZI\rangle + \langle ZIZ\rangle - \langle IZZ\rangle + \langle ZZZ\rangle$

$R_x(\pi)$ on Q2 and Q3: $+\langle ZII\rangle - \langle IZI\rangle - \langle IIZ\rangle - \langle ZZI\rangle - \langle ZIZ\rangle + \langle IZZ\rangle + \langle ZZZ\rangle$

$$4\langle ZZZ\rangle$$

DiCarlo, *et al. Nature* **467** 574 (2010)

Toffoli gate with noncomputational states

Two-qubit gate is conditional because the interaction requires two excitations

$$|11\rangle \circlearrowleft |02\rangle$$

A three-qubit interaction would address a third excited state

$$|111\rangle \circlearrowleft |003\rangle$$

This is the essence of the gate!

This interaction is very small, so we use an intermediate state

$$|111\rangle \circlearrowleft |102\rangle \circlearrowleft |003\rangle$$

Difficulty comes from doing this fast and getting all the two-qubit phases correct

- One of the two qubit phases isn't 0, but doesn't matter for QEC

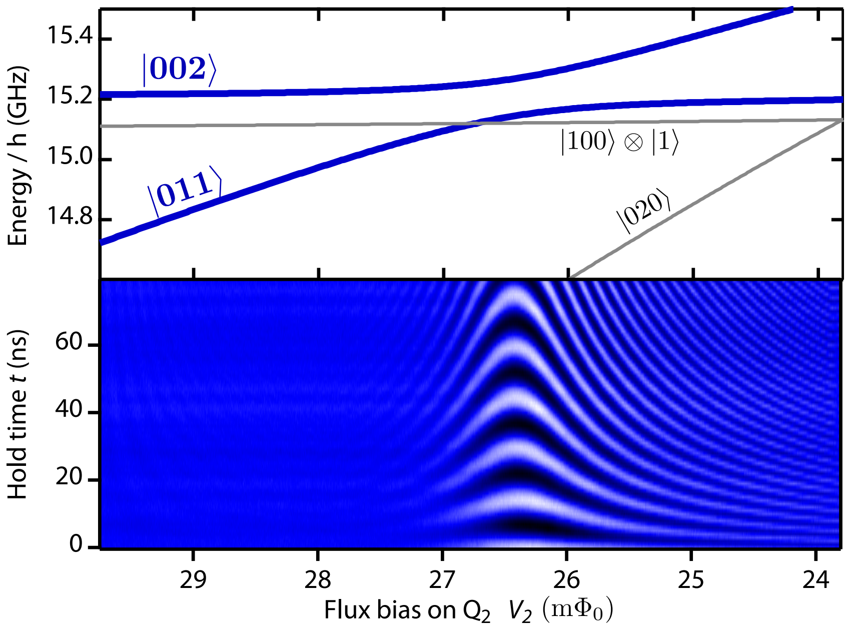
B. P. Layton, et al. Nat. Phys. 5 134 (2008)

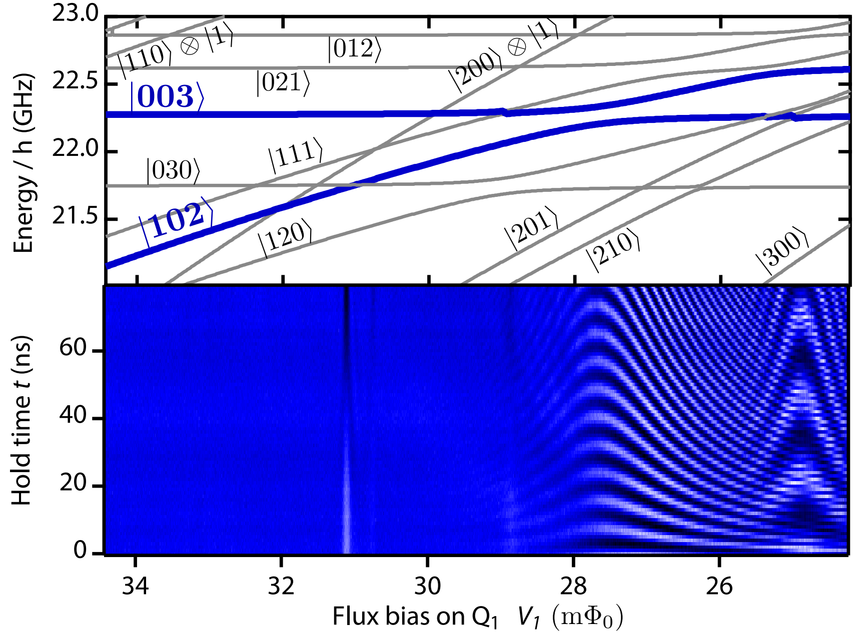
T. Monz, et al. PRL 040501 (2009)

Santa Barbara Group

$$|111\rangle \rightarrow -|111\rangle$$
$$|\mathrm{others}\rangle \rightarrow |\mathrm{others}\rangle$$

Sudden and adiabatic interactions





Sudden transfer of

$|011\rangle \rightarrow |002\rangle$ and

$|111\rangle \rightarrow |102\rangle$

Adiabatic interaction of

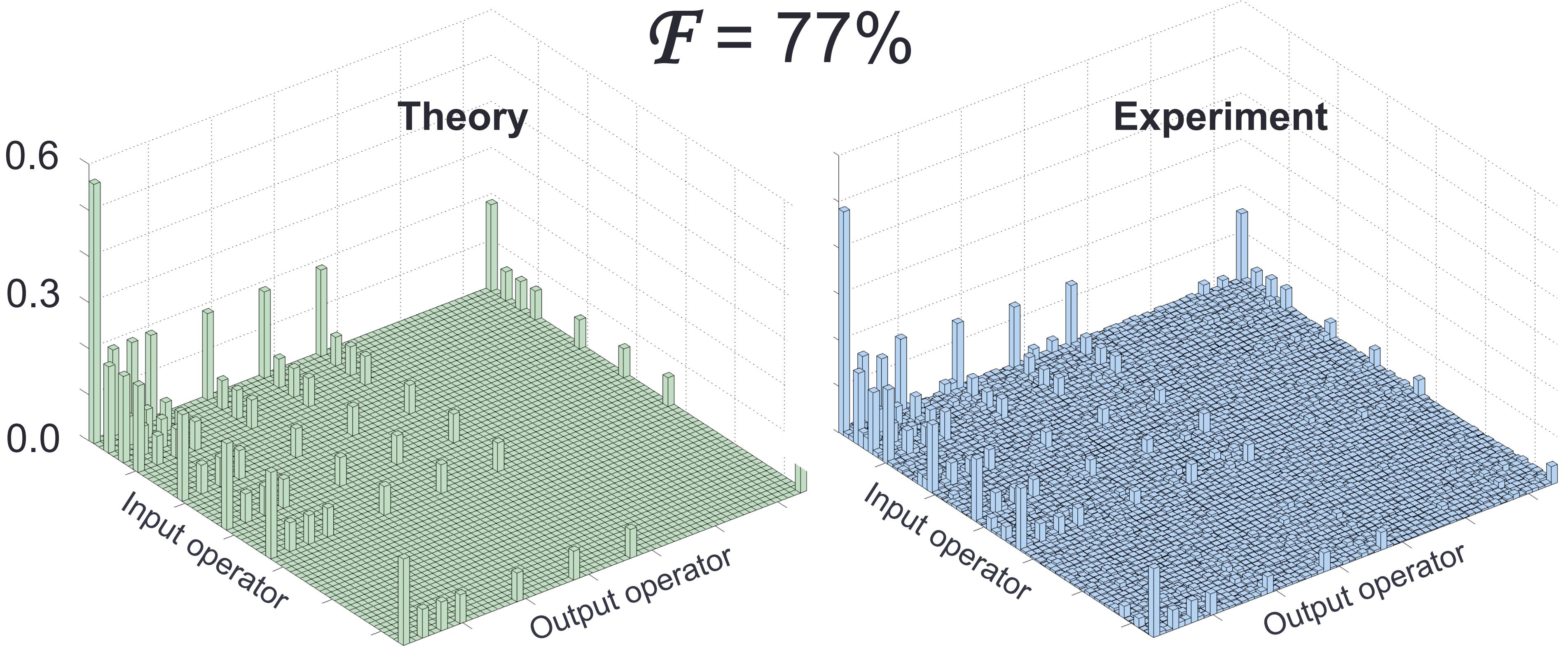
$|102\rangle$ with $|003\rangle$

generates three-qubit phase

Must engineer interaction times correctly and correct a 2Q phase

How do we prove the gate works?

Quantum process tomography of CCPhase



4032 Pauli correlation measurements (90 minutes)

Operator order: III, XII, YII, ZII, IXI, IYI, IZI, IIX, IIY, IIZ, XXI, XYI, XZI, YXI, YYI, YZI, ZXI, ZYI, ZZI, XIX, XIY, XIZ, YIX, YIY, YIZ, ZIX, ZIY, ZIZ, IXX, IXY, IXZ, IYX, IYY, IYZ, IZX, IZY, IZZ, XXX, XXY, XXZ, XYX, XYY, XYZ, XZX, XZY, XZZ, YXX, YXY, YXZ, YYX, YYY, YYZ, YZX, YZY, YZZ, ZXX, ZXY, ZXZ, ZYX, ZYY, ZYZ, ZZX, ZZY, ZZZ.

Quantum process tomography

Want to **fully** characterize the gate process – e.g. the action on superpositions

QPT tells you **everything that can be known** about a process, given a Hilbert space

$$|\mathrm{input}\rangle \rightarrow \hat{O} \rightarrow |\mathrm{output}\rangle$$

Needs 64 input states, instead of just 8

$$\rho_{\mathrm{out}} = P(\rho_{\mathrm{in}}) = \sum_{m,n=1}^{4^N} \chi_{m,n} A_m \rho_{\mathrm{in}} A_n^{\dagger}$$

Invert this equation to find χ

Nielsen & Chuang *Cambridge Univ. Press*