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Perspective

APPLIED MATHEMATICS

Is the Geometry of Nature Fractal?

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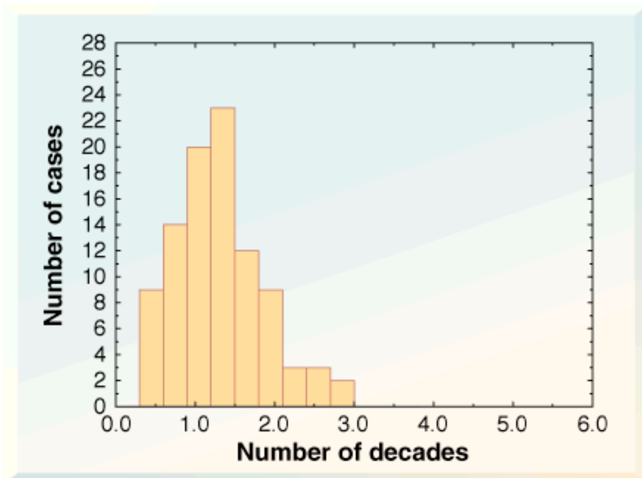
Fractals are beautiful mathematical contructs characterized by a never-ending cascade of similar structural details that are revealed upon magnification on all scales. Over the past two decades, the notion has been intensively put forward that fractal geometry describes well the irregular face of nature. But does it? Consider the recent Perspective in Science by Marder (1). Marder summarizes a simulation study of fractured silicon nitride by Kalia et al. (2) that successfully mimics experimental data, and he generally emphasizes the role of fractal geometry in describing physical structures of complex geometry. Specifically, the results of Kalia et al. were interpreted as “showing that this mechanism … leads to fractal fracture surfaces.” However, upon examining Kalia's results [figure 4 in (2)], one finds that Marder's statement is based on four exponents, all of which hold over less than one order of magnitude. A fractal object, in the purely mathematical sense, requires infinitely many orders of magnitude of power-law scaling, and a consequent interpretation of experimental results as indicating fractality requires “many” orders of magnitude. In the celebrated fractal Koch curve, which resembles a symmetric snow-flake with many edges, one order of magnitude means that one stops its construction after about two iterations; a two-iterations Koch curve is not a fractal object. Marder, like many others in the scientific community, may have been swayed by the widespread image and belief that fractality has been found over many orders of magnitude in experimental documentation.

We have reason to believe that this is not the case (3). In fact, reported experimental fractality in a wide range of physical systems is typically based on a scaling range that spans only 0.5 to 2.0 decades (factors of 10). To assess this, we surveyed all experimental papers reporting fractal analysis of data that appeared over a period of 7 years in all Physical Review journals (Phys. Rev. A to E and Phys. Rev. Lett., 1990 to 1996). In these papers, an empirical fractal dimension D was calculated from various relations between a property P and the resolution r of the general form

$$P = kr^{f(D)} \quad (1)$$

where k is the prefactor for the power law and the exponent f(D) is a simple function of D. In most cases, fitting the data to Eq. 1 was done through its linear log-log presentation. Typically, the range of linear behavior terminated on both sides either because further data was not accessible or because of crossover bends. A histogram of the number of orders of magnitude used to declare fractality, covering all 96 reports, reveals a clear picture (see figure): The scaling range of experimentally declared fractality is extremely limited, centered around 1.3 orders of magnitude, spanning mainly between 0.5 and 2.0 (4). This limited range stands in stark contradiction to the public image of the status of experimental fractals.



Figure

Limited scaling range

The number of decades (factors of 10) spanned by experimentally derived scaling exponents that led to the labeling of the studied systems as fractal (4).

The most acute questions posed by these data are if the limited range is inherent, if these limited-range power-law objects are fractal, and if, in fact, nature is describable in terms of fractality. The question of fractality is actually secondary to the benefits of carrying out a multiple resolution analysis (Eq. 1); these benefits outweigh the perhaps erroneous fractal label.

The existence of cutoffs is inherently associated with experimentation on real physical objects. The lower cutoff is dictated typically by the basic building block unit (such as an atom, molecule, microcrystal, or small aggregate) of the system. The upper cutoff is, at most, of the order of the system size but is usually far below it. It is bounded either by the mechanical strength, by growth rates (which drop sharply with time), by the emergence of background effects (such as nonisotropic fields), or by the depletion of resources. Temporal self-affine trails scale over many orders of magnitude, but this is a completely different issue: the time axis can be extended at will.

Do power laws that are limited in range represent fractals? Is it justified to term them as such (5)? Regardless of the question of fractality, a more basic question should be asked: Is this presentation useful? The very existence of so many reports by competent researchers who are well aware of the problematics of declaring fractality for experimental results that span only one order of magnitude suggests that experimentalists seem to gain from the resolution analysis and from the fact that the result of such analysis is often a power law. The usefulness is in the following points: (i) The power law condenses the description of a complex geometry. (ii) It allows one to correlate in a simple way properties and performances of a system to its structure and to the dynamics of its formation. (iii) In many instances, the choice is either to use the limited-range data or to discard it altogether and not have even an approximate picture of the studied object. Opting for the former can be emphatically understood. (iv) Fractal geometry provides a proper language and symbolism for studies of ill-defined geometries.

It is important to reiterate, however, that the ability to fit data to Eq. 1 does not imply fractality and that the label "fractal" is not needed. So should one refer to such results in terms of a fractal object? If by "fractal" one refers to the original Mandelbrot teaching of many orders of magnitude, then the data we collected do not seem to support it in an unequivocal way. If by "fractal" one means an object that obeys Eq. 1 over a limited range, then the use of this label may be acceptable, not only because of its usefulness, but because of the following additional reasons: (i) Interestingly, the sense of self-similarity in irregular objects is comprehended visually even for a limited range. (ii) In some cases, experimentally derived objects resemble simulated objects obtained from fractal models. (iii) The empirical values of D for spatial objects fall in the fractal regime of $0 < D < 3$. (iv) And, it may be too late to make any changes in a terminology that, at this stage, seems to be deeply rooted in practice. A drift from an original meaning of a concept is common in science, representing adaptability of the original ideal definition to realistic restrictions that emerge when put to practice.

We arrive at our final question: Is the geometry of nature fractal? Several key processes involving equilibrium-critical phenomena (in magnets, liquids, percolations, and phase transitions, for example) and some nonequilibrium growth models (such as aggregation) are backed by intrinsically scale-free theories and lead therefore to power-law scaling behavior on all scales. However, the majority of the data that was interpreted in terms of fractality in the surveyed Physical Review journals does not seem to be linked (at least in an obvious way) to existing models and, in fact, does not have theoretical backing. Most of the data represent results from nonequilibrium processes. The common situation is this: An experimentalist performs a resolution analysis and finds a limited-range power law with a value of D smaller than the embedding dimension. Without necessarily resorting to special underlying mechanistic arguments, the experimentalist then often chooses to label the object for which she or he finds this power law a "fractal." This is the fractal geometry of nature.

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For more detail, see O. Malcai et al., Phys. Rev. E 56, 2817 (1997).

Our earlier version of the histogram (3) had two cases with a range of 3.7 to 3.8 decades. It turns out that we were too "liberal" in our interpretation: One case was a deterministically built exact Koch fractal [B. Sapoval et al., ibid. 48, 3333 (1993)], and the other was described by the authors as representing "almost no deviations … for the first three decades" [T. Holten et al., ibid. 50, 754 (1994)].

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Full Text

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Letters

Is Nature Fractal?

David Avnir et al. (Science's Compass, 2 Jan., p. 39) report on the high proportion of hasty claims of fractality in Physical Review journals and end by saying that “[t]his is the fractal geometry of nature.” When assessing a field, other authors might not dwell so much on the statistics of implied and possible failures, but on the variety and quality of the best work. In the case of fractal geometry, it is outstanding.

As I have stressed (1, p. 3) fractals are not a panacea; they are not everywhere. But many investigations in numerous fields started with few decades of experimental data and later moved to many. For example, the fractality of metal fractures was reported (1, p. 461) over a few decades, and this produced the first appropriate measurement of roughness. E. Bouchaud has now confirmed fractality over five decades (2). In another example [references and discussion in (1), chapter 8], in 1963, Berger and I postulated the fractality of transmission errors on the basis of data ranging from seven to nine decades. Even in finance, my new multifractal model (3) covers data ranging from three to four decades. In a multitude of other instances, repeated analysis, based on abundant data and distinct methods, yields the same result, or a well-understood theory explains why upper and lower cutoffs are both unavoidable, or both.

Those examples do not exhaust the usefulness of careful fractal modeling. Many claims that are questioned by Avnir et al. are best understood as unfortunate side effects of enthusiasm, imperfectly controlled by refereeing, for a new tool that was (incorrectly) perceived as simple.

Since 1983, Avnir has published ex tensively on data that cover one decade or less (4), and his claims of fractality have become widely known and disputed. This work is not mentioned in the article. It appears, then, that Avnir is withdrawing his earlier claims.

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Avnir et al. turn the question of whether experimental power laws scale over many or few decades of length into a litmus test for well or poorly established fractals in nature. There are several problems with this viewpoint. Avnir and I have presented, inter alia, scaling ranges of less than a decade as fractals (1); I developed criteria to distinguish tentatively fractal power laws from crossover effects (2); in critical cases, he and I tested the fractal hypothesis extensively—and successfully—for consistency with all available data (3). Thus, the article leaves unmentioned Avnir's own contributions to what he classifies as "perhaps erroneous fractal label." It also leaves unmentioned that the discovery of fractals requires a lot more than fitting a power law through a set of points and asking how many decades of length it spans. To discredit limited scaling ("[t]he scaling range of experimentally declared fractality is extremely limited") panders to the skeptic; to allow that "the use of this label [fractal] may be acceptable" caters to the enthusiast; to state that "the question of fractality is … secondary" and "the label 'fractal' is not needed" says the issue is not important. One can't have it all three ways. To assess the fractality of nature, one can't just take a histogram of 96 power laws and compute the mean. It's too much like fitting a power law through a set of points.

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Response

Mandelbrot's reaction to the outcome of our analysis is uncalled for. Our papers (1, 2) reported on the most comprehensive survey of experimental measurements of fractals done thus far. This survey allows one both to assess the abundance of fractals in various types of physical systems and to examine the dimensions and the scaling range of empirical fractals. The answer to the critical question of “the abundance of fractals” determines either their central relevance to all fields of natural sciences or their esotericity.

Mandelbrot's main point is that there are some examples of many decades of fractality, and he suggests that we simply were looking at the wrong data. However, the data we analyzed is not junk and cannot be dismissed: it comes from a prestigious set of journals in the physics community, and they represent beyond doubt the status of fractals in the natural sciences. The main problem is that the “best data,” according to Mandelbrot's own criteria, is exceptionally rare, which at the very least raises the need for a serious reexamination of the explicit book-title claim (3). It is in order, then, to reexamine some of the best known experimental examples, beginning with the flag question of the whole field, “How long is the coast of Britain?” (3). The answer, given by Mandelbrot in (3) in terms of the original study of Richardson, is that various coastlines exhibit power-law behavior, spanning between one and two orders of magnitude, with an average of about 1.3 orders (conforming with the average we found in our survey). If these limited power-law correlations represent legitimate fractals according to Mandelbrot, then by the same token so are all of the 96 limited-range examples of fractals we analyzed.

It was not suggested in (1) or (2) that many-orders fractal objects do not exist. However, one must use an extremely fine sieve to search through the scientific literature for a meager handful of examples. Even this handful is, in many cases, problematic. Let us take, for instance, Mandelbrot's metal fracture study (4), cited in his comment and cited also by Marder (5) and by Kalia et al. (6) as a classical example. The four orders of magnitude, shown in figure 1 of (4), are in fact only two orders. This is due to the method used there to extract the fractal dimension, namely the perimeter-area relationship. The yardstick used in this type of resolution analysis is of area units, and not the relevant linear extent. This leads to an artificial doubling of the number of decades. Another classical example for a many-orders physical fractal has been Lovejoy's report on the fractality of clouds (7), also determined from perimeter-area relations. Again, the six orders shown in figure 1 of that reference are actually only three, for the same reason, as is indeed stated in the text. Even these three decades are composed of two different experiments (radar data sensitive to rainfall and satellite pictures of clouds), covering each about two orders of magnitude, with some overlap. The other two examples mentioned by Mandelbrot are temporal self-affine trails. As stated in (2), such trails fall outside the domain of our discussion, because the time axis can be extended at will. Moreover, the eight cases in (1) and (2) with a scaling range extending beyond two decades are dominated by spatial self-affine fractals, such as sections of rough surfaces and fronts (8). This further lowers the average number of decades in isotropic self-similar fractals. As in temporal self-affine trails, an experiment leading to spatial self-affinity can in principle start with as long a front as desirable and is thus not limited in scaling range.

In conclusion, it appears that the limited-range empirical fractals (9) are the dominant justification for "the fractal geometry of nature." Rather than sweeping them under the carpet as "bad data," their limited range should be carefully studied and understood. An intriguing and fundamental question that remains open is, Why are these limited-range fractals so common?

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The usefulness and authenticity of the limited range empirical fractals, which is a central theme in (2), is not addressed by Mandelbrot. Thus, there is no cause for alarm that "Avnir is withdrawing his earlier claims." On the contrary, not only does Avnir stand behind the usefulness of the many limited-range fractals he, P. Pfeifer, and their many co-workers have found over the years, but he continues to detect them and reported recently on a method of controlling the effective small-angle x-ray scattering surface fractality of modified silicas