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Is Nature Fractal?

David Avnir *et al.* (*Science's* Compass, 2 Jan., [p. 39](#)) report on the high proportion of hasty claims of **fractality in** *Physical Review* journals and end by saying that "[t]his is the fractal geometry of **nature**." When assessing a field, other authors might not dwell so much on the statistics of implied and possible failures, but on the variety and quality of the best work. **In** the case of fractal geometry, it is outstanding.

As I have stressed ([1](#), p. 3) fractals *are not* a panacea; they *are not* everywhere. But many **investigations in** numerous fields started with few decades of experimental data and later moved to many. For example, the **fractality** of metal fractures was reported ([1](#), p. 461) over a few decades, and this produced the first appropriate measurement of roughness. E. Bouchaud has now confirmed **fractality** over five decades ([2](#)). **In** another example [references and discussion **in** ([1](#)), chapter 8], **in** 1963, Berger and I postulated the **fractality** of transmission errors on the basis of data ranging from seven to **nine** decades. Even **in finance**, my new multifractal model ([3](#)) covers data ranging from three to four decades. **In** a multitude of other **instances**, repeated analysis, based on abundant data and **distinct** methods, yields the same result, or a well-understood theory **explains** why upper and lower cutoffs are both unavoidable, or both.

Those examples do not exhaust the usefulness of *careful* fractal modeling. Many claims that are questioned by Avnir *et al.* are best understood as unfortunate side effects of enthusiasm, imperfectly controlled by refereeing, for a new tool that was (**incorrectly**) perceived as simple.

Since 1983, Avnir has published extensively on data that cover one decade or less ([4](#)), and his claims of **fractality** have become widely known and disputed. This work is not mentioned **in** the article. It appears, then, that Avnir is withdrawing his earlier claims.

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Avnir *et al.* turn the question of whether experimental power laws scale over many or few decades of length **into** a litmus test for well or poorly established fractals **in nature**. There are several problems with this viewpoint. Avnir and I have presented, *inter alia*, scaling ranges of less than a decade as fractals (1); I developed criteria to distinguish tentatively fractal power laws from crossover effects (2); **in** critical cases, he and I tested the fractal hypothesis extensively--and successfully--for consistency with all available data (3). Thus, the article leaves unmentioned Avnir's own contributions to what he classifies as "perhaps erroneous fractal label." It also leaves unmentioned that the discovery of fractals requires a lot more than fitting a power law through a set of points and asking how many decades of length it spans. To discredit limited scaling ("[t]he scaling range of experimentally declared **fractality** is extremely limited") panders to the skeptic; to allow that "the use of this label [fractal] may be acceptable" caters to the enthusiast; to state that "the question of **fractality** is ... secondary" and "the label 'fractal' is not needed" says the issue is not important. One can't have it all three ways. To assess the **fractality** of **nature**, one can't just take a histogram of 96 power laws and compute the mean. It's too much like fitting a power law through a set of points.

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Response: Mandelbrot's reaction to the outcome of our analysis is uncalled for. Our papers (1, 2) reported on the most comprehensive survey of experimental measurements of fractals done thus far. This survey allows one both to assess the abundance of fractals **in** various types of physical systems and to examine the dimensions and the scaling range of empirical fractals. The answer to the critical question of "the abundance of fractals" determines either their central relevance to all fields of natural sciences or their esotericity.

Mandelbrot's main point is that there are some examples of many decades of **fractality**, and he suggests that we simply were looking at the wrong data. However, the data we analyzed is not junk and cannot be dismissed: it comes from a prestigious set of journals **in** the physics community, and they represent beyond doubt the status of fractals **in** the natural sciences. The main problem is that the "best data," according to Mandelbrot's own criteria, is exceptionally rare, which at the very least raises the need for a serious reexamination of the explicit book-title claim (3). It is **in** order, then, to reexamine some of the best known experimental examples, beginning with the flag question of the whole field, "How long is the coast of Britain?" (3). The answer, given by Mandelbrot **in** (3) **in** terms of the original study of Richardson, is that various coastlines exhibit power-law behavior, spanning between one and two orders of magnitude, with an

average of about 1.3 orders (conforming with the average we found in our survey). If these limited power-law correlations represent legitimate fractals according to Mandelbrot, then by the same token so are all of the 96 limited-range examples of fractals we analyzed.

It was not suggested in (1) or (2) that many-orders fractal objects do not exist. However, one must use an extremely fine sieve to search through the scientific literature for a meager handful of examples. Even this handful is, in many cases, problematic. Let us take, for instance, Mandelbrot's metal fracture study (4), cited in his comment and cited also by Marder (5) and by Kalia *et al.* (6) as a classical example. The four orders of magnitude, shown in figure 1 of (4), are in fact only two orders. This is due to the method used there to extract the fractal dimension, namely the perimeter-area relationship. The yardstick used in this type of resolution analysis is of area units, and not the relevant linear extent. This leads to an artificial doubling of the number of decades. Another classical example for a many-orders physical fractal has been Lovejoy's report on the fractality of clouds (7), also determined from perimeter-area relations. Again, the six orders shown in figure 1 of that reference are actually only three, for the same reason, as is indeed stated in the text. Even these three decades are composed of two different experiments (radar data sensitive to rainfall and satellite pictures of clouds), covering each about two orders of magnitude, with some overlap. The other two examples mentioned by Mandelbrot are temporal self-affine trails. As stated in (2), such trails fall outside the domain of our discussion, because the time axis can be extended at will. Moreover, the eight cases in (1) and (2) with a scaling range extending beyond two decades are dominated by spatial self-affine fractals, such as sections of rough surfaces and fronts (8). This further lowers the average number of decades in isotropic self-similar fractals. As in temporal self-affine trails, an experiment leading to spatial self-affinity can in principle start with as long a front as desirable and is thus not limited in scaling range.

In conclusion, it appears that the limited-range empirical fractals (9) are the dominant justification for "the fractal geometry of nature." Rather than sweeping them under the carpet as "bad data," their limited range should be carefully studied and understood. An intriguing and fundamental question that remains open is, Why are these limited-range fractals so common?

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8. See also P. Daguier, S. Henaux, E. Bouchaud, F. Creuzet, *Phys. Rev.* **E53**, 5637 (1996), although an explicit fractal dimension is not mentioned.
9. The usefulness and authenticity of the limited range empirical fractals, which is a central theme **in** (2), is not addressed by Mandelbrot. Thus, there is no cause for alarm that "Avnir is withdraw**ing** his earlier claims." On the contrary, not only does Avnir stand beh**ind** the usefulness of the many limited-range fractals he, P. Pfeifer, and their many co-workers have found over the years, but he contin**ues** to detect them and reported recently on a method of controll**ing** the effective small-angle x-ray scatter**ing** surface **fractality** of modified silicas [C. Rottman, G. S. Grader, Y. De Hazan, D. Avnir, *Langmuir* **12**, 5505 (1996)].

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Volume 279, Number 5352, Issue of 6 Feb 1998, p. 783.

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