How to Teleport Superpositions of Chiral Amplitudes

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Chiral molecules may exist in superpositions of left- and right-handed states. We show how the amplitudes of such superpositions may be teleported to the polarization degrees of freedom of a photon and thus measured. Two experimental schemes are proposed, one leading to perfect, the other to state-dependent teleportation. Both methods yield complete information about the amplitudes. [S0031-9007(98)08006-5]

"Quantum teleportation," proposed in 1993 by Bennett *et al.* [1], has become a reality [2,3]. However, to date all of the teleportation work deals with intraspecies teleportation (e.g., atom-atom or photon-photon). In this Letter we propose an interspecies teleportation scheme. Specifically, we outline experiments in which the information contained in a superposition of chiral amplitudes, $|\phi_M\rangle = a|L\rangle + b|R\rangle$, may be teleported to a photon. Here, $|L\rangle$ and $|R\rangle$ are the left- and right-handed states of a chiral molecule. In special cases, the teleportation scheme presented here can also be used to teleport the state of more general molecular superpositions such as superpositions of *cis*- and *trans*-isomers. While methods for creating and detecting a molecular state of the form $|\phi_M\rangle$ have already been discussed [4,5], the corresponding experiments have not been performed. Indeed, no chiral superposition has ever been measured. By teleporting the information contained in the amplitudes a and b to the polarization vector of a photon, the superposition becomes easy to detect via standard photon polarization measurements. Then by performing another teleportation to a spin $1/2$ nucleus or a trapped ion (as envisioned in [2]) one can imprint the chirality information onto a much stabler state, suitable for further manipulations: the nucleus or ion acts as a *quantum memory* device.

To teleport the chiral superposition state $|\phi_M\rangle$ we take our entangled pair to be two photons in the state $|\Psi_{12}^-\rangle = (1/\sqrt{2})(|l_1\rangle|r_2\rangle - |r_1\rangle|l_2\rangle)$. Here, $|l\rangle$ and $|r\rangle$ denote left and right circularly polarized photons. Photon 1 will become entangled with the molecule, and photon 2 will be the one whose polarization state will receive the amplitudes a and b. Initially, the total molecule-photon state is unentangled: $|\psi\rangle = |\phi\rangle|\Psi_{12}^-\rangle$. As in [1], this state can be rewritten as

$$|\psi\rangle = \tfrac{1}{2}(|\Psi_{M1}^-\rangle|1\rangle + |\Psi_{M1}^+\rangle|2\rangle + |\Phi_{M1}^-\rangle|3\rangle + |\Phi_{M1}^+\rangle|4\rangle). \qquad (1)$$

Here the four maximally entangled "Bell states" are $|\Psi_{M1}^\pm\rangle = (1/\sqrt{2})(|L\rangle|r_1\rangle \pm |R\rangle|l_1\rangle)$, $|\Phi_{M1}^\pm\rangle = (1/\sqrt{2})(|L\rangle|l_1\rangle \pm |R\rangle|r_1\rangle)$, and the four states involving photon 2 are

$$|1\rangle = -\begin{pmatrix} a \\ b \end{pmatrix} = -a|l_2\rangle - b|r_2\rangle, \qquad (2)$$

$$|2\rangle = -\sigma_z\begin{pmatrix} a \\ b \end{pmatrix} = -a|l_2\rangle + b|r_2\rangle, \qquad (3)$$

$$|3\rangle = \sigma_x\begin{pmatrix} a \\ b \end{pmatrix} = b|l_2\rangle + a|r_2\rangle, \qquad (4)$$

$$|4\rangle = -i\sigma_y\begin{pmatrix} a \\ b \end{pmatrix} = -b|l_2\rangle + a|r_2\rangle. \qquad (5)$$

The σ's are the Pauli matrices and $|l_2\rangle = \binom{1}{0}$, $|r_2\rangle = \binom{0}{1}$. As emphasized in [1], the above form implies that the "teleportee" photon (2) can be transformed into the state $\binom{a}{b}$ by one of four simple unitary operations. Which of the four operations needs to be applied depends *only* on the measurement outcome of the projection onto the Bell states. The scheme thus requires two bits of classical communication to transfer the information regarding the *continuum* of quantum states $\binom{a}{b}$, at the price of establishing prior entanglement. However, this is not to say that a unitary transformation of the original teleported photon state is the only way to obtain complete information. Below we give an example of "state-dependent" teleportation which nonetheless does yield complete information. Note further that the circular polarized basis plays no special role in the above discussion. We now show how to extend the previous work by proposing an apparatus which can be used to teleport the superposition of chiral amplitudes.

Teleportation of Chirality.—Consider the apparatus shown in Fig. 1. Before photon 1 reaches the interferometer, the system is in the direct product state $|\phi\rangle|\Psi_{12}^-\rangle$. At some later time, photon 1 will have reached the beam splitter and thus will have some amplitude to be found in the top arm and some amplitude to be found in the bottom arm where it will interact with the molecule.

Now it is well known that left and right circularly polarized photons acquire different phase shifts when scattering through a chiral molecule [6]. Ordinarily the phase shifts due to a single molecule are undetectably small. However, it has been shown that by utilizing a high-finesse optical resonator (cavity), phase shifts due
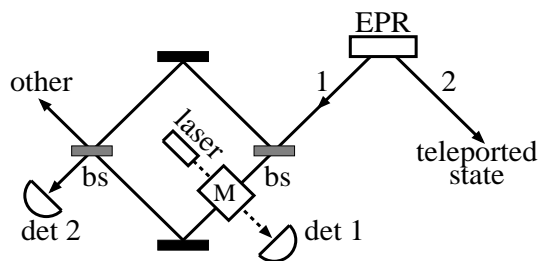
FIG. 1. A source produces an entangled pair of photons, 1 and 2. Photon 1 enters an interferometer and interacts with the molecule, labeled M. A π pulse, produced by the box labeled laser, excites the molecule to a state of definite parity and the resulting fluorescence is detected by detector 1. Detector 2 is set to detect photons of a definite polarization. A coincidence measurement then teleports the state of the molecular superposition to the polarization state of photon 2.

to the coupling of a single photon to a single cesium atom may be as large as 16° [7]. The interaction which gives rise to this phase shift is an electric dipole-dipole scattering process, whereas optical activity is mediated by an electric dipole-magnetic dipole interaction. Since molecular magnetic dipole moments are about 10^{-2} smaller than electric dipole moments, we expect that with current technology the phase shift due to optical activity in a high-finesse cavity would be on the order of a tenth of a degree, perhaps too small to be useful in our scheme. On the other hand, the last decade has seen tremendous progress in the fabrication of high-finesse optical cavities [8], and we expect that our proposed experiment will be feasible in the future.

Furthermore, natural optical activity cannot be enhanced by a standing wave cavity because it is erased upon reflection back through the optically active medium [9]. Thus, natural optical activity is enhanced only by the use of a *ring* cavity. Another possibility is to utilize the effect of *E*-field optical activity [5]. This type of optical activity does not vanish upon reflection through the medium so a standing wave cavity may be used. The formalism which we present here is applicable to natural optical activity. For the case of *E*-field optical activity, the superposition of chiral states should be written in the "false chirality" basis made from the states $|L\rangle \pm i|R\rangle$ [10]. These states are converted into one another by inversion, and they are to *E*-field optical activity what the states $|L\rangle$ and $|R\rangle$ are to natural optical activity [5].

The photon-molecule scattering implements a *conditional phase-shift* mechanism. Specifically,

$$|l\rangle|L\rangle \rightarrow e^{i(kz-\varphi)}|l\rangle|L\rangle, \qquad |r\rangle|R\rangle \rightarrow e^{i(kz-\varphi)}|r\rangle|R\rangle, \tag{6}$$

$$|l\rangle|R\rangle \rightarrow e^{i(kz+\varphi)}|l\rangle|R\rangle, \qquad |r\rangle|L\rangle \rightarrow e^{i(kz+\varphi)}|r\rangle|L\rangle. \tag{7}$$

Here $e^{\pm i\varphi}$ is the phase shift due to optical activity and z is the free-space optical path length. For arbitrary molecular superpositions, the Faraday effect [6] gives rise to a table similar to the one above. However, with the

Faraday effect, the two different molecular states give rise to different rotations and spin-independent indices of refraction (related to the quantities ϕ and kz in the above equations). For the special case where the two states give equal and opposite rotations of the polarization vector and also give rise to the same spin-independent phase shift, the Faraday effect gives rise to a set of equations identical to the ones above but where $|L\rangle$ and $|R\rangle$ are replaced by kets representing the two molecular states which are superposed. For this special situation, the Faraday effect can be used in place of optical activity and the teleportation scheme can be used to teleport more general types of molecular superpositions. As an added bonus, the Faraday effect does not vanish upon reflecting the beam back through the medium and can therefore be enhanced in a standing wave cavity.

In any case, using the above equations, it is easy to show that after the interaction with the chiral superposition the amplitude on the bottom arm of the interferometer (Fig. 1) is proportional to

$$|\psi_{\text{bot}}\rangle \propto \frac{e^{i\varphi}}{2}[|\Psi_{M1}^-\rangle|1\rangle + |\Psi_{M1}^+\rangle|2\rangle]$$
$$+ \frac{e^{-i\varphi}}{2}[|\Phi_{M1}^-\rangle|3\rangle + |\Phi_{M1}^+\rangle|4\rangle]. \tag{8}$$

The amplitude for going through the upper arm is still described by Eq. (1), and the full state is a superposition of the amplitude on the top and bottom arms. Now, by adjusting the path length and thus the phase of the amplitude in the top arm of the interferometer, we can arrange it so that only one pair of the states $|\Psi_{M1}^\pm\rangle$ and $|\Phi_{M1}^\pm\rangle$ has nonzero amplitude to reach the detector 2. Suppose we adjust the top arm so that only the $|\Psi_{M1}^\pm\rangle$ reach detector 2. Then after the photon has left the interferometer the state can be written (up to an overall phase) $|\psi\rangle = [\sin(\varphi)/\sqrt{2}][|\Psi_{M1}^-\rangle_{D_2}|1\rangle + |\Psi_{M1}^+\rangle_{D_2}|2\rangle] + |\psi'\rangle$. Here the subscript D_2 indicates that the photon in that state is heading for detector 2. The state $|\psi'\rangle$ is the amplitude which in Fig. 1 is now traveling vertically away from this detector (denoted "other"). We are not concerned with the precise form of $|\psi'\rangle$; for our purposes it suffices to know that this state involves photon amplitude which will never intersect D_2. Using the basis of parity eigenstates $|\pm\rangle$ and linear polarizations $|x\rangle$, $|y\rangle$,

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|L\rangle \pm |R\rangle), \tag{9}$$

$$|x\rangle = \frac{1}{\sqrt{2}}(|l\rangle + |r\rangle), \qquad |y\rangle = \frac{i}{\sqrt{2}}(|l\rangle - |r\rangle), \tag{10}$$

we can rewrite the postinterferometer state as

$$|\psi\rangle = \frac{\sin(\varphi)}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|+\rangle|x\rangle_{D_2} - \frac{i}{\sqrt{2}}|-\rangle|y\rangle_{D_2}\right)|1\rangle$$
$$+ \left(\frac{1}{\sqrt{2}}|-\rangle|x\rangle_{D_2} - \frac{i}{\sqrt{2}}|+\rangle|y\rangle_{D_2}\right)|2\rangle + |\psi'\rangle. \tag{11}$$

The laser in Fig. 1 is used to produce a π pulse, tuned to a transition between the ground state of the molecule and an excited state of definite parity which fluoresces. Suppose that the excited state is of odd parity; in the electric dipole approximation the parity must change upon electronic excitation, so only the state $|+\rangle$ will be excited. Therefore, after excitation and fluorescence, we arrive at the state

$$|\psi\rangle = \frac{\sin(\varphi)}{\sqrt{2}}\left[\left(\frac{1}{\sqrt{2}}|+\rangle|\nu_1\rangle|x\rangle_{D_2} - \frac{i}{\sqrt{2}}|-\rangle|\nu_0\rangle|y\rangle_{D_2}\right)|1\rangle\right.$$
$$\left. + \left(\frac{1}{\sqrt{2}}|-\rangle|\nu_0\rangle|x\rangle_{D_2} - \frac{i}{\sqrt{2}}|+\rangle|\nu_1\rangle|y\rangle_{D_2}\right)|2\rangle\right] + |\psi'\rangle. \quad (12)$$

The $|+\rangle$ molecular state has become coupled to a spontaneously emitted photon ($|\nu_1\rangle$), whereas the $|-\rangle$ state has not (the vacuum state $|\nu_0\rangle$). Teleportation can now be performed by projecting onto an *unentangled* state [11]: one places a Polaroid oriented in the x direction in front of the detector 2 and looks for coincidences with detector 1 (which detects the spontaneous emission $|\nu_1\rangle$). A co-incidence measurement then constitutes a projection onto the state $|\nu_1\rangle|x\rangle_{D_2}$. This implies that the teleportee photon is in the state $|1\rangle = -a|l_2\rangle - b|r_2\rangle$. Teleportation has been achieved. By altering the length of the top arm of the interferometer and/or exciting the molecule to an even parity state, we can teleport to any of the four photon states in Eq. (1).

State-Dependent Teleportation.—The latter scheme accomplishes perfect teleportation: an *unknown* amplitude of the chiral superposition appears in the polarization vector of photon 2. We will next consider a simplified experiment which avoids the use of interferometry, and accomplishes *state-dependent*, imperfect teleportation.

Suppose we remove the upper arm of the interferometer. Then the entire apparatus is represented by $|\psi_{bot}\rangle$ as in Eq. (8). We again rewrite the amplitude in the $|+\rangle$, $|-\rangle$, $|x\rangle$, and $|y\rangle$ representation, yielding $|\psi'_{bot}\rangle = [|-\rangle|y\rangle|1'\rangle + |+\rangle|y\rangle|2'\rangle + |-\rangle|x\rangle|3'\rangle + |+\rangle|x\rangle|4'\rangle]$ where the four unnormalized teleportee photon states are

$$|1'\rangle \equiv \frac{i}{2\sqrt{2}}(e^{-i\varphi}|2\rangle - e^{i\varphi}|4\rangle), \quad (13)$$

$$|2'\rangle \equiv \frac{i}{2\sqrt{2}}(e^{-i\varphi}|1\rangle - e^{i\varphi}|3\rangle), \quad (14)$$

$$|3'\rangle \equiv \frac{1}{2\sqrt{2}}(e^{-i\varphi}|1\rangle + e^{i\varphi}|3\rangle), \quad (15)$$

$$|4'\rangle \equiv \frac{1}{2\sqrt{2}}(e^{-i\varphi}|2\rangle + e^{i\varphi}|4\rangle). \quad (16)$$

Denoting $a = \alpha e^{i\theta_a}$ and $b = \beta e^{i\theta_b}$, the norms are

$$\text{Prob}(1') = |\langle\psi'_{bot}|-,y\rangle|^2 = \text{Prob}(3') = |\langle\psi'_{bot}|-,x\rangle|^2$$
$$= \frac{1}{4}[1 - 2\alpha\beta\cos(\theta_a - \theta_b)\cos(2\varphi)], \quad (17)$$

$$\text{Prob}(2') = |\langle\psi'_{bot}|+,y\rangle|^2 = \text{Prob}(4') = |\langle\psi'_{bot}|+,x\rangle|^2$$
$$= \frac{1}{4}[1 + 2\alpha\beta\cos(\theta_a - \theta_b)\cos(2\varphi)]. \quad (18)$$

We notice that the transformation matrices that send $\binom{a}{b}$ into $|1'\rangle$ through $|4'\rangle$ are not unitary. For example,

$$|1'\rangle = \frac{1}{i2\sqrt{2}}\begin{pmatrix} e^{-i\varphi} & -e^{i\varphi} \\ e^{i\varphi} & -e^{-i\varphi} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}.$$

The resulting states are, however, *pure*. Appropriately renormalized, they may be represented by polarization vectors on the unit Bloch sphere. Hence there is a unitary rotation which carries each teleported vector into the original state, $\binom{a}{b}$. However, as seen from Eqs. (17) and (18), the transformation depends upon the values of a, b, and the phase-shift angle φ. In this sense the present scheme constitutes a state-dependent, imperfect teleportation, since it cannot be used to teleport an unknown quantum state [12]. Nevertheless, it is possible to obtain *full* information about a and b by standard optical methods. One can measure the relative phase and relative magnitude of the polarization components of photon 2. The relative phases and separately, the relative magnitudes, are equal in pairs. Thus by transmission of a single bit of classical information, it is possible to tell cases $1', 3'$ from $2', 4'$ and to obtain complete information on the chiral superposition. Of course, each measurement which contributes to this process destroys the superposition of polarizations of photon 2. This loss cannot be prevented in the perfect teleportation scheme either if the actual values of a and b are needed. From this perspective there is no real advantage to the perfect scheme. Indeed, the perfect scheme is better only if photon 2 is put to use in a later quantum information processing stage, such as an input to a quantum computer.

Discussion.—This work has, for the first time, considered in detail the possibility of interspecies teleportation. This led us to propose a concrete scheme by which the quantum chiral state of a single molecule could be measured, by means of transferring the chiral information to an easily measurable photon polarization state. We have outlined two experiments, one leading to perfect, or unitary, teleportation of the amplitudes of a chiral superposition, the other to state-dependent teleportation. However, we have shown that the latter is able to transmit, in general, full amplitude information as well. The key to the schemes proposed here is the use of the parity-conserving *symmetry* governing the interaction between light and a chiral molecule, Eqs. (6) and (7). This symmetry leads to the possibility of implementing a conditional phase shift, without which teleportation cannot take place. We conjecture that it is possible to exploit other

symmetries in order to affect interspecies teleportation in other cases. Indeed, we have discussed how, using the Faraday effect, the state of a more general molecular superposition can be teleported. In the same vein, any other spin $1/2$ particle can be used to replace the photons in our scheme, but with a different interaction, such as spin-orbit coupling [13].

A virtue of the method of chiral teleportation is that it provides a genuinely new way of measuring chiral superpositions of chiral amplitudes. Indeed, even if the original molecular state is $|L\rangle$, successful teleportation is a manifestation of at least one pair of superpositions, $|+\rangle$ and $|-\rangle$.

Model calculations show that chiral superpositions in media are extremely short lived: they decohere on a time scale of picoseconds to femtoseconds [14]. Hence one might wonder whether the chiral superposition will not decohere over the time scale needed for the photon to interact with it. Collisions with the walls and asymmetries of the cavity are the chief decoherence agents as they lead to fluctuating chiral environments [15]. In the case of a high-finesse cavity we estimate that this should lead to decoherence times of the order of 1 sec. Decoherence thus does not present a significant obstacle in accomplishing the proposed experiment.

Another issue brought up by this work is the possibility of probing quantum properties in "large" objects, and thus the transition (be it continuous or sharp) as a function of object size to classical behavior. The emergence of the latter is one of the most fascinating unsolved problems of present-day physics. At which point is the object "too large" to enable teleportation of its chiral superposition? The number of degrees of freedom of the object will set the decoherence time scale, since it determines the coupling to the bath degrees of freedom. It thus controls the extent of "environmental monitoring," leading to classical behavior, i.e., the absence of quantum interference in large objects [16]. In principle, as long as the object is chiral and there exist excited nondegenerate states of definite parity, our schemes apply. *Ceteris paribus*, failure to teleport may thus be taken as an indication of classicality of the chiral object.

Finally, the extension of quantum teleportation to superpositions of molecular states has yielded an entirely new way of measuring given superpositions of chiral amplitudes. It is therefore tempting to consider *reverse* interspecies teleportation. Consider, e.g., diasterioisomers: these are two molecules connected by a chemical bond or a van der Waals complex. The molecules are mirror images of one another. That is, one is $|L\rangle$, and the other is $|R\rangle$. For example, a left-handed oligomer of diphenyl alanine (DA) is connected by a disulphide bridge (DB) to a right-handed oligomer of DA. The DB bridge is easily

cleaved. Let us imagine that the dimer is cooled down to a $J = 0$ state of total angular momentum. When cleaved the dimer state breaks up into monomer states of equal total angular momentum and equal and opposite z component of angular momentum, M. The fragments also move in opposite directions. Clearly then there is a sum of entangled states in M. Using the same scheme as described above for a simultaneous measurement on a chiral molecule and a photon, we can now teleport a superposition of *photon* polarization states to create a superposition of handed states in one of the molecules. Thus we can create arbitrary superpositions of chiral amplitudes through interspecies teleportation.

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