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- **LETTERS**

Fractality in Nature

David Avnir *et al.* (*Science*'s Compass, 2 Jan., p. 39) pose the question, “Is the geometry of nature fractal?” By considering results from 96 reports that have claimed fractality in natural systems, they show that the declared fractality spans on the average only about 1.5 decades (orders of magnitude). Accordingly, they question the practice of associating power laws over such a limited range of scales with fractal processes that by definition extend to an infinite range of scales.

This is a legitimate question, and I agree that natural systems are not described by simple scale-free models and that limited scaling may arise from random processes. There is, however, another possibility. Natural systems involve mechanisms and processes operating at different ranges of space-time scales. The climate system is a good example. The processes at those different scales may or may not interact with one other, but if they are fractal, their properties and scaling may be different and limited to the corresponding scales.

This kind of result can often provide useful insights about the physics and processes underlying the physical system in question. As such, the real issue is not whether or not we label the power law a fractal, but whether or not it is the appropriate fit to the data. In most studies, the power law is determined by the slope of a linear region in a log-log plot. In such plots, it is easy to visually identify narrow regions that appear linear. In effect, in most cases a power law is not proved, but is a priori assumed to exist. Very few studies (including those in the 96 reports) ask whether or not the data in the range of scales of the alleged power law are consistent with the corresponding family of true fractals or even if the power law is indeed the best fit to the data. This question is not always easy to answer, but in certain cases (self-affine processes, for example) statistical tests can be used to provide an answer (1).

Response: Tsonis raises two important issues—the possible origin of the limited range of empirical fractals and the practice of fitting data to a power law when there is no theoretical model that suggests the suitability of such analysis. Indeed, we also believe that the question of the origin and abundance of the limited-range fractals is the central issue. Theories that predict power-law scaling have been proposed and studied extensively for both equilibrium critical phenomena and nonequilibrium processes.

It seems to us, however, that the diversity of experiments and phenomena and the fact that most observations are as yet without a solid theoretical background call for a fresh look at the general phenomenon of empirical fractals with a limited range. Without claiming that we have an answer at hand, we do mention, as an example, an interesting finding we made, namely, that randomness, either in its elementary, uncorrelated forms or superimposed with internal correlations, generates apparent fractal structures below medium densities over one to two decades 1.

The question then, as correctly raised by Tsonis, is how can one distinguish between power laws that are the result of such non-mechanistic phenomena, and inherent power laws that fully justify the use of this analysis. We reiterate here the usefulness of the limited-range apparent fractals, even in the absence of an underlying theoretical justification (as we detailed in our *Science* piece). The detection and interpretation of inherent power laws requires scale-free theories and models. For some systems and processes, such theories and models exist, as mentioned above, while for others they are still needed. In any event, this task is far from being complete. We are currently developing guidelines and recommendations for the detection and analysis of fractal structures that are bound within cutoffs; these will be reported separately.

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