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Topology-changing first order phase transition and the dynamics of flavor

Tameem Albash,[*] Veselin Filev,[†] Clifford V. Johnson,[‡,‖] and Arnab Kundu[§]

Department of Physics and Astronomy, wUniversity of Southern California, Los Angeles, California 90089-0484, USA

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In studying the dynamics of large N_c, $SU(N_c)$ gauge theory at finite temperature with fundamental quark flavors in the quenched approximation, we observe a first order phase transition. A quark condensate forms at finite quark mass, and the value of the condensate varies smoothly with the quark mass for generic regions in parameter space. At a particular value of the quark mass, there is a finite discontinuity in the condensate's vacuum expectation value, corresponding to a first order phase transition. We study the gauge theory via its string dual formulation using the AdS/CFT conjecture, the string dual being the near-horizon geometry of N_c D3-branes at finite temperature, AdS_5-Schwarzschild $\times S^5$, probed by a D7-brane. The D7-brane has topology $\mathbb{R}^4 \times S^3 \times S^1$ and allowed solutions correspond to either the S^3 or the S^1 shrinking away in the interior of the geometry. The phase transition represents a jump between branches of solutions having these two distinct D-brane topologies. The transition also appears in the meson spectrum.

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I. INTRODUCTION

String theory is a powerful tool for probing the strongly coupled dynamics of gauge theory, physics which is of vital importance especially in the context of the strong nuclear interactions. Gauge/string correspondences have enlarged and refined the toolbox available for such studies, and there is a large literature on the subject, with several powerful examples such as the AdS/CFT correspondence [1–3] and deformations thereof [4].

However, we are still some way from describing the ''realistic'' dynamics of QCD using this approach. The main challenges that remain include getting access to low N_c, fully including dynamical quarks in the fundamental representation of $SU(N_c)$, and getting reliable control of the nonsupersymmetric regime.

It is to be hoped (if not expected) that even if we cannot obtain a controllable string dual of QCD, there may be considerable progress to be made in capturing physical phenomena that are in the same universality class as those of QCD. This is the motivation of the present work, which is part of a series of studies upon which we hope to report new and interesting results.

We study the geometry of AdS_5-Schwarzschild $\times S^5$, which is the decoupled/near-horizon geometry of N_c D3-branes, where N_c is large and set by the (small, for reliability) curvature of the geometry. The physics of closed type IIB string theory in this background is dual to the physics of $\mathcal{N} = 4$ supersymmetric $SU(N_c)$ gauge theory in four dimensions, with the supersymmetry broken by being at finite temperature [5]. The temperature is set by the horizon radius of the Schwarzschild black hole, as we will recall below. Since the background geometry includes a black hole, we are in the deconfined phase of the Hawking-Page transition. Therefore, the dual gauge theory is in the deconfined phase, i.e. the adjoint matter forms a plasma. This transition does not appear in our analysis since we are not working in global coordinates.

We introduce a D7-brane probe into the background. Four of the brane's eight world-volume directions are parallel with those of the D3-branes, and three of them wrap an $S^3 \subset S^5$. The remaining direction lies in the radial direction of the asymptotically AdS_5 geometry.

Such a D3-D7 configuration controls the physics of the $SU(N_c)$ gauge theory with a dynamical quark in the fundamental representation [6]. The configuration (at zero temperature) preserves $\mathcal{N} = 2$ supersymmetry in $D = 4$, and the quark is part of a hypermultiplet. Generically, we will be studying the physics at finite temperature, so supersymmetry will play no explicit role here.

We are studying the D7-brane as a probe only, corresponding to taking the $N_c \gg N_f$ limit, and therefore there is no backreaction on the background geometry. This is roughly analogous to the quenched approximation in lattice QCD. The quark mass and other flavor physics—such as the vacuum expectation value (vev) of a condensate and the spectrum of mesons that can be constructed from the quarks—are all physics which are therefore invisible in the background geometry. We will learn nothing new from the background; our study is of the response of the probe D7-branes to the background, and this is where the new physics emerges from.

We carefully study the physics of the probe itself as it moves in the background geometry. The coordinates of the probe in the background are fields in an effective D7-brane world-volume theory, and the geometry of the background enters as couplings controlling the dynamics of those

[*]albash@usc.edu
[†]filev@usc.edu
[‡]johnson1@usc.edu
[§]akundu@usc.edu
[‖]Also, Visiting Professor at the Centre for Particle Theory, Department of Mathematical Sciencs, University of Durham, Durham DH1 3LE, United Kingdom.

fields. One such coupling in the effective model represents the local separation, $L(u)$, of the D7-brane probe from the D3-branes, where u is the radial AdS_5-Schwarzschild coordinate.

In fact, the asymptotic value of the separation between the D3-branes and D7-brane for large u yields the bare quark mass m_q and the condensate vacuum expectation value $\langle\bar{\psi}\psi\rangle$ as follows [7,8]:

$$\lim_{u\to\infty} L(u) = m + \frac{c}{u^2} + \ldots \tag{1}$$

where $m = 2\pi\alpha' m_q$ and $-c = \langle\bar{\psi}\psi\rangle/(8\pi^3\alpha' N_f \tau_7)$, where in this paper $N_f = 1$. (The fundamental string tension is defined as $T = 1/(2\pi\alpha')$ here, and the D7-brane tension is $\tau_7 = (2\pi)^{-7}(\alpha')^{-4}$.) The zero temperature behavior of the D7-branes in the geometry is simple. The D7-brane world-volume actually *vanishes* at finite u, corresponding to the part of the brane wrapped on the S^3 shrinking to zero size. The location in u where this vanishing happens encodes the mass of the quark, or equivalently, the separation of the probe from the D3-branes. In addition, in the zero temperature background, the only value of c allowed is zero, meaning no condensate is allowed to form, as is expected from supersymmetry.

The finite temperature physics introduces an important new feature. As is standard [9], finite temperature is studied by Euclideanizing the geometry and identifying the temperature with the period of the time coordinate. The horizon of the background geometry is the place where that S^1 shrinks to zero size. The D7-brane is also wrapped on this S^1, so it can vanish at the horizon, if it has not vanished due to the shrinking of the S^3. For large quark mass compared to the temperature (horizon size), the S^3 shrinking will occur at some finite $u > u_H$, and the physics will be similar to the zero temperature situation. However, for small quark mass, the world-volume will vanish due to the shrinking of the S^1 corresponding to the D7-branes going into the horizon. This is new physics of the flavor sector.

The authors of Ref. [10] explored some of the physics of this situation (the dependence of the condensate and of the meson mass on the bare quark mass), and predicted that a phase transition should occur when the topology of the probe D7-brane changes. However, they were not able to explicitly see this transition because of poor data resolution in the transition region, coming from using UV boundary conditions on the scalar fields on the D7-brane world-volume. The origin of this phase transition, as we shall see, is as follows: The generic behavior of an allowed solution for $L(u)$ as in Eq. (1), is not enough to determine whether the behavior corresponds to an S^3-vanishing D7-brane or an S^1-vanishing D7-brane. Each branch of solutions has different values of c, generically. In other words, for a given value of m there can be more than one value of c. There are therefore two or more candidate solutions potentially controlling the physics. The actual physical solution is the one which has the lowest value for the D7-brane's free energy. The key point is that, at a certain value of the mass, the lowest energy solution may suddenly come from a different branch, and, as the corresponding value of the condensate changes discontinuously in moving between branches, we find that the system therefore undergoes a first order phase transition. On the gauge theory side, we can imagine a similar situation occurring; two different branches of solution are competing, and the lowest energy branch is always picked. The two phases/branches that are competing in this case are a "melted" meson phase and a stable meson phase. In the former, the mesons break apart in the plasma, whereas in the latter they remain stable. We are able to uncover this physics by doing a careful numerical analysis of the equations of motion for the probe dynamics on the gravity side of the AdS/CFT correspondence, which goes well beyond that carried out in previous papers. By using IR boundary conditions instead of UV boundary conditions, our analysis allows us to obtain vastly more data points, allowing us to complete the picture described above.[1] In addition, we are able to study the meson spectrum in some detail, and we find that the phase transition also manifests itself as a first order discontinuity in that physics.

II. THE PROBE COMPUTATION

We begin by reviewing the physics of the D7-brane probe in the AdS_5-Schwarzschild background solution [10]. The metric is given by

$$\begin{aligned} ds^2 = &-\frac{f(u)}{R^2}dt^2 + \frac{R^2}{f(u)}du^2 + \frac{u^2}{R^2}d\vec{x}\cdot d\vec{x} + R^2 d\theta^2 \\ &+ R^2\cos^2\theta d\Omega_3^2 + R^2\sin^2\theta d\phi^2, \end{aligned} \tag{2}$$

where $\vec{x}$ is a three vector,

$$f(u) = u^2 - \frac{b^4}{u^2},$$

and the quantity R^2 is given by

$$R^2 = \sqrt{4\pi g_s N_c}\,\alpha',$$

where g_s is the string coupling (which, with the inverse string tension α' sets, for example, Newton's constant). The quantity b is related to the mass of the black hole, $b^2 = 8G_5 m_{\mathrm{b.h.}}/(3\pi)$. The temperature of the black hole can be extracted using the standard Euclidean continuation and requiring regularity at the horizon. Doing this in the metric given by Eq. (2), we find that $\beta^{-1} = b/\pi R^2$. Therefore, by picking the value of b, we are choosing at which temperature we are holding the theory. We choose to embed the

[1] In the process of writing this paper, similar work was published by Mateos *et al.* [11]. In addition, after this paper appeared, I. Kirsch pointed out to us some related work in his thesis [12].

D7-brane probe transverse to θ and ϕ. In order to study the embeddings with the lowest value of the on-shell action (and hence the lowest free energy), we choose an ansatz of the form $\phi = 0$ and $\theta = \theta(u)$. The asymptotic separation of the D3 and D7-branes is given by $L(u) = u\sin\theta$. Given this particular choice of embedding, the world-volume of the D7 brane is given by

$$\sqrt{-g} = u^2\cos^3\theta(u)\sqrt{\det S^3}\sqrt{u^2 + (u^4 - b^4)\theta'(u)^2}, \quad (3)$$

where g is the determinant of the induced metric on the D7-brane given by the pullback of the space-time metric $G_{\mu\nu}$. We are interested in two particular cases. First, there is the case where u goes to b, which corresponds to the D7-brane probe falling into the event horizon. In the Euclidean section, this case corresponds to the shrinking of the S^1 of periodic time. This corresponds to what in Ref. [10] were called condensate solutions. Second, there is the case where θ goes to $\pi/2$, which corresponds to the shrinking of the S^3. This corresponds to what were called Karch-Katz-like solutions in Ref. [10]. It is this change in topology (S^1 versus S^3 shrinking) between the different solutions that will correspond to a phase transition. The classical equation of motion for $\theta(u)$ is

$$\frac{d}{du}\left(\frac{u^2(u^4 - b^4)\theta'(u)\cos^3(\theta(u))}{\sqrt{u^2 + (u^4 - b^4)\theta'(u)^2}}\right) + 3u^2\cos^2\theta(u)\sin\theta(u)\sqrt{u^2 + (u^4 - b^4)\theta'(u)^2} = 0. \quad (4)$$

When u goes to infinity and the background metric becomes asymptotically $\mathrm{AdS}_5 \times S^5$, the equation of motion reduces to

$$\frac{d}{du}(u^5\theta'(u)) + 3u^2\theta(u) = 0, \quad (5)$$

which has solution:

$$\theta(u) = \frac{1}{u}\left(m + \frac{c}{u^2}\right). \quad (6)$$

These two terms are exactly the non-normalizable and normalizable terms corresponding to a dimension 3 operator ($\bar{\psi}\psi$) in the dual field theory with source m and vacuum expectation value c.

III. EMBEDDING SOLUTIONS

We solve Eq. (4) numerically using a shooting technique. Shooting from infinity towards the horizon, physical solutions are those that have a finite value at the horizon. This will only be accomplished for a particular m and c value from Eq. (1), which would have to be delicately chosen by hand. Therefore, if we instead start from the horizon with a finite solution, it will shoot towards the physical asymptotic solutions we desire. In order to be able to analyze the phase transition between the condensate and Karch-Katz-like solutions, we shoot from the horizon for the condensate solutions and from $\theta = \pi/2$ for the Karch-Katz-like solutions. This technique avoids having to correctly choose the boundary conditions at infinity, which is a sensitive procedure, allowing us to have many more data points to analyze the phase transition. We impose the boundary condition that, at our starting point—the horizon—we have

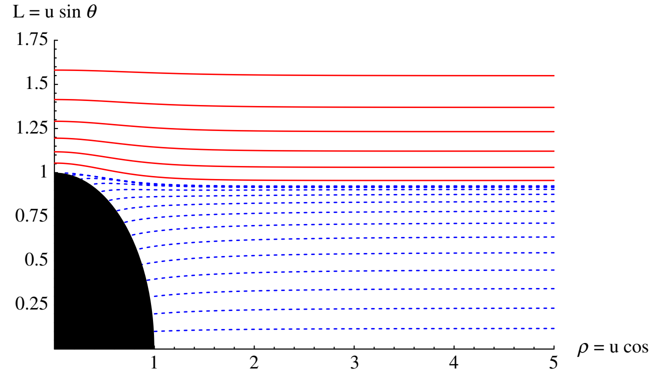

FIG. 1 (color online). Solutions for the D7-brane probe in the black hole background.

$$\theta'(u)|_{S^1\to 0} = \frac{3b^2}{8}\tan\theta(b)\theta'(u)|_{S^3\to 0} = \infty. \quad (7)$$

We argue that this is the physical boundary condition to take; the first is simply a result of taking the limit of $u \to b$ in the equation of motion in Eq. (4), whereas the second is a result of requiring no conifold singularity as the S^3 shrinks to zero size [13].

We solve the equation of motion, Eq. (4), numerically using $b = 1$ and $R = 1$. These numerical choices correspond to fixing the temperature and to measuring lengths in units of the radius of the AdS space; the latter condition also means we are choosing a particular relationship between the t'Hooft coupling and the dual quantities:

$$\lambda = g_{\mathrm{YM}}^2 \qquad N_c = \frac{1}{2\alpha'^2}, \qquad \beta^{-1} = \frac{1}{\pi}. \quad (8)$$

Several D7-brane embedding solutions are shown in Fig. 1. The red (solid) lines correspond to Karch-Katz-like solutions, and the blue (dashed) lines correspond to condensate solutions. From each of these solutions, we can extrapolate the bare quark mass and quark condensate vev.

IV. THE PHASE TRANSITION

We plot the c values as a function of m in Fig. 2(a). When enlarged, as shown in Fig. 2(b), we find, as anticipated in the introductory remarks, the multivaluedness in c for a given m. Physics will choose just one answer for c. There is therefore the possibility of a transition from one branch to another as one changes m.

In order to determine exactly where the transition takes place, we have to calculate the free energy of the D7-brane.

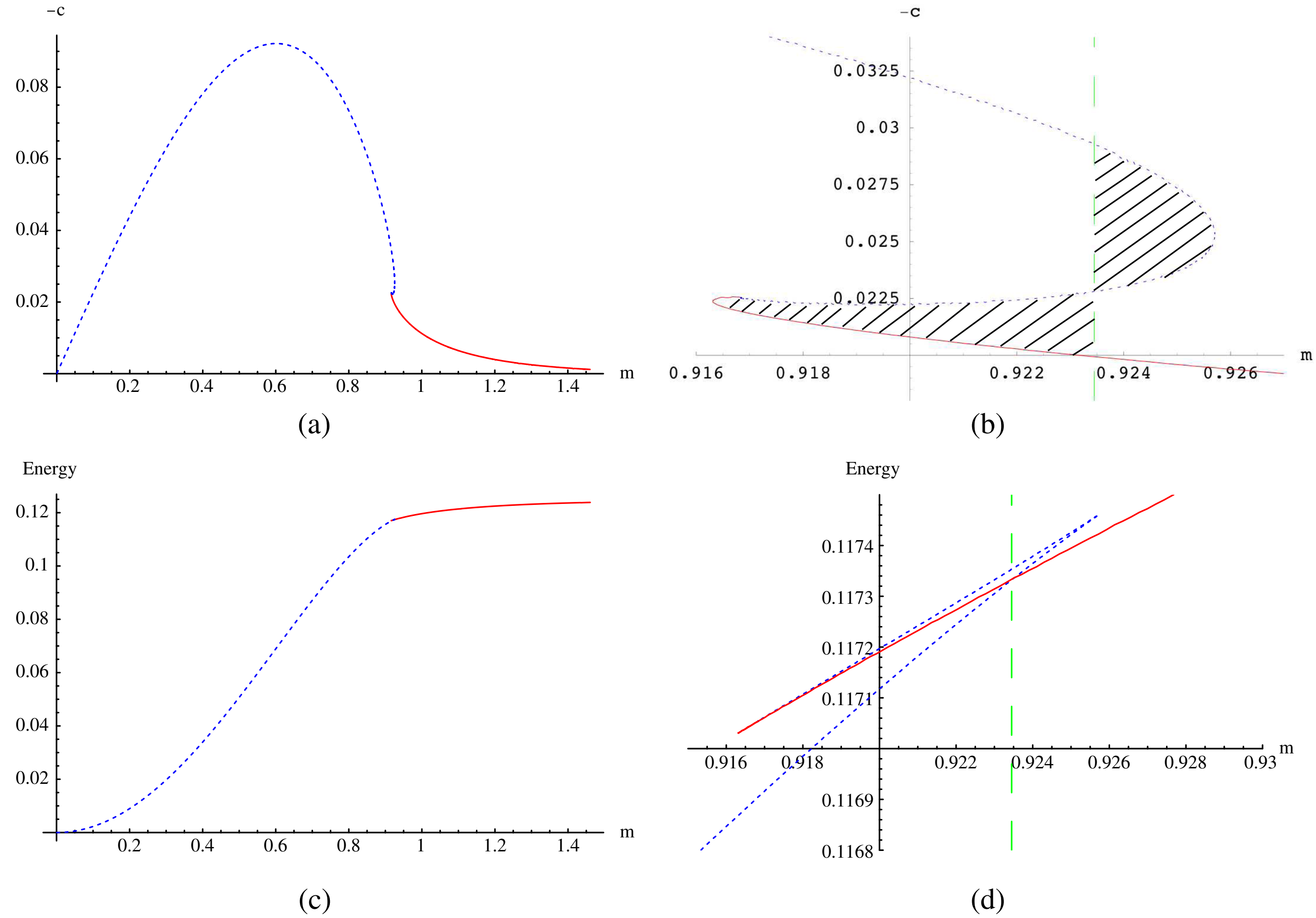

FIG. 2 (color online). Probe free energy (in units of $2\pi\tau_7 N_f b/R^2$) and condensate vev at the phase transition. See text for more details.

In the semiclassical limit that we are considering, the free energy is given by the on-shell action times β^{-1}. For our case, this is simply given by [8]

$$\mathcal{F} = \beta^{-1}\tau_7 N_f \int d^4x d\Omega_3 du \sqrt{-\det g}, \tag{9}$$

where here $N_f = 1$. We calculate this integral numerically using our solutions for $\theta(u)$, and we plot the results for the energy in Fig. 2(c) after we regulate the result by subtracting off the energy from the $\theta = 0$ solution. There is again multivaluedness at the same value of m as before, and we zoom in on this neighborhood in Fig. 2(d). If one follows the solutions of lowest energy for a given m, one can clearly see that there is a crossover from one branch to another as m changes. This was also observed in Ref. [14]. Therefore, we find a first order phase transition—at $m \approx 0.92345$—where the condensate's vev jumps discontinuously. This had been deduced on other grounds in Ref. [15]. We show where the jump between curves occurs with the dashed green line in Figs. 2(b) and 2(d). We note that the shaded areas in Fig. 2(b) are equal; this is to be expected since c and m are thermodynamically conjugate to each other, and the area of the graph has the interpretation as a free energy difference $d\mathcal{E} \sim -cdm$.

V. MESON SPECTRUM

Next we study the meson spectrum. This can be read off from the physics of the fluctuations in θ and ϕ about our classical solutions $\theta_{(0)} = \theta(u)$ and $\phi_{(0)} = 0$ described above [16], corresponding to scalar and pseudoscalar fields, respectively, in the gauge theory [17]. As a reminder, in Ref. [17], the exact meson spectrum for the $AdS_5 \times S^5$ background was found to be given by

$$M(n, \ell) = \frac{2m}{R^2}\sqrt{(n+\ell+1)(n+\ell+2)}, \tag{10}$$

where ℓ labels the order of the spherical harmonic expansion, and n is a positive integer that represents the order of the mode. In our meson spectrum calculations, we have only considered the $n = \ell = 0$ solutions for the fluctuations in θ and ϕ. We begin by considering the four-dimensional mass of the meson corresponding to fluctuations in ϕ. By considering an ansatz of the form

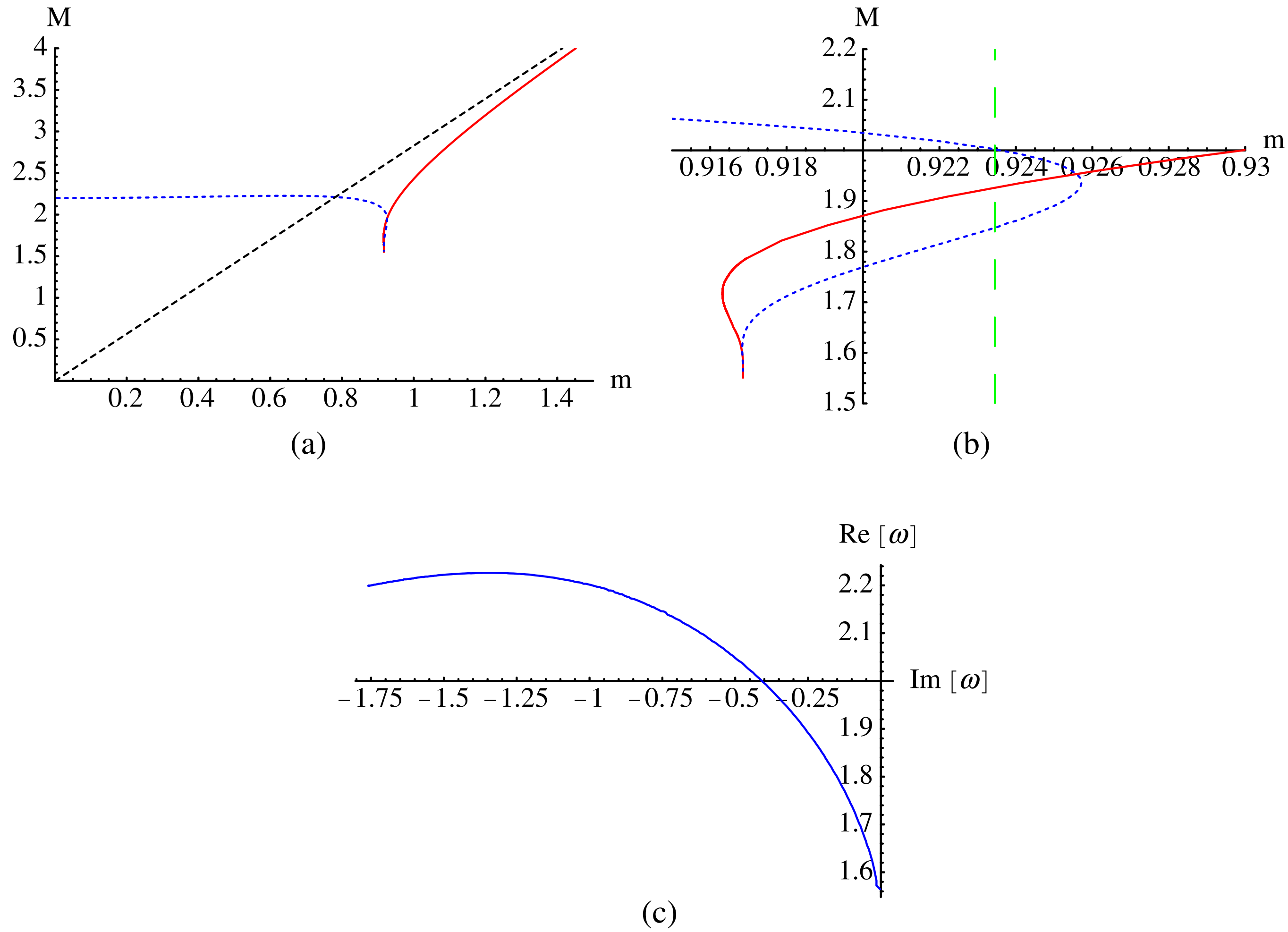

FIG. 3 (color online). Four-dimensional meson mass M, as a function of fundamental quark mass m, from fluctuations in ϕ. Only the first level of the discrete spectrum is shown. The blue (dashed) curves in (a) and (b) correspond to the meson mass before the meson melts.

$$\phi(z, t) = 0 + \delta\phi(z, t) = f(z)e^{-i\omega t}, \tag{11}$$

where $z = u^{-2}$ in terms of our previous coordinates, and by expanding to second order in $\delta\phi(u, t)$, we can calculate an equation of motion for $f(z)$. In particular, we find that near the event horizon, the equation of motion becomes

$$f''(z) + \frac{f'(z)}{z - b^{-2}} + \frac{R^4\omega^2}{16b^2}\frac{f(z)}{(z - b^{-2})^2} = 0. \tag{12}$$

This equation has solutions given by in-falling and outgoing waves:

$$f(z) = A(1 - zb^2)^{i((R^2\omega)/4b)} + B(1 - zb^2)^{-i((R^2\omega)/4b)}, \tag{13}$$

where A and B are constants. This analysis of the quasinormal mode behavior has been shown in Refs. [18–20], and the (correct) boundary condition of in-falling waves corresponds to the fact that fluctuations about condensate solutions do not allow for mesons with a discrete mass spectrum.[2] The mesons "melt" away into the plasma, from the gauge theory perspective. However, as was shown in Ref. [20], we can still find the mass of the meson (which we shall denote as M) for these fluctuations before they melt, in addition to the meson's lifetime (τ) with the identification

$$\mathrm{Re}[\omega] = M, \quad -\mathrm{Im}[\omega] = (2\tau)^{-1}. \tag{14}$$

The fluctuations about the Karch-Katz-like solutions have a discrete mass spectrum; therefore the mesons are stable, and ω will be purely real. To search for the condensate fluctuations, we do a convenient field redefinition:

[2]Refs. [19,20] appeared after the original version of this manuscript appeared, and were useful in refining this section's discussion.

$$f(z) = y(z)(1 - zb^2)^{-i((R^2\omega)/4b)}. \tag{15}$$

Since at the boundary, we want $f(z)$ to be exactly an incoming-wave solution, this means that our boundary condition is simply

$$y(b^{-2}) = 1 \qquad y'(b^{-2}) \sim (1 - zb^2)^{-1}|_{z\to b^{-2}} \to \infty. \tag{16}$$

For the fluctuations about Karch-Katz-like solutions, we do not have the incoming-wave requirement. Instead, we have a shrinking S^3, and the derivative of any smooth function on the world-volume should vanish by symmetry arguments. Therefore, we choose Neumann boundary conditions to get smooth solutions:

$$f(z)|_{\theta(z)=\pi/2} = \epsilon \qquad f'(z)|_{\theta(z)=\pi/2} = 0, \tag{17}$$

where ϵ is sufficiently small. (We used $\epsilon \sim 10^{-2}$ here.) In order to find the correct value of ω for both sets of fluctuations, we note that in the limit that z approaches zero, where the background becomes asymptotically $AdS_5 \times S^5$, the field $f(z)$ must only be comprised of a normalizable term, i.e. the coefficient in front of the non-normalizable term must be zero. We show the first level of the discrete four-dimensional meson mass dependence on the bare quark mass in Figs. 3(a) and 3(b) (zoomed). In Fig. 3(c) we show the behavior of the real and imaginary part of ω. For large bare quark mass, our solutions match those of the $AdS_5 \times S^5$ case discussed in Ref. [17] (shown as the black lines with long dashes). We note that, for large bare quark mass, the meson mass at finite temperature is bounded from above by the meson mass at zero temperature (i.e. in pure $AdS_5 \times S^5$). This is to be expected since the coupling in the finite temperature gauge theory goes as

$$\frac{1}{g^2_{4\text{d-YM}}} \sim \left(1 - \frac{b}{u}\right)^{-1/2} \frac{1}{g^2_{8\text{d-YM}}}\bigg|_{\beta=\infty}. \tag{18}$$

In addition, we see that at zero bare quark mass, the meson has a finite mass. This is simply the mass gap of the three-

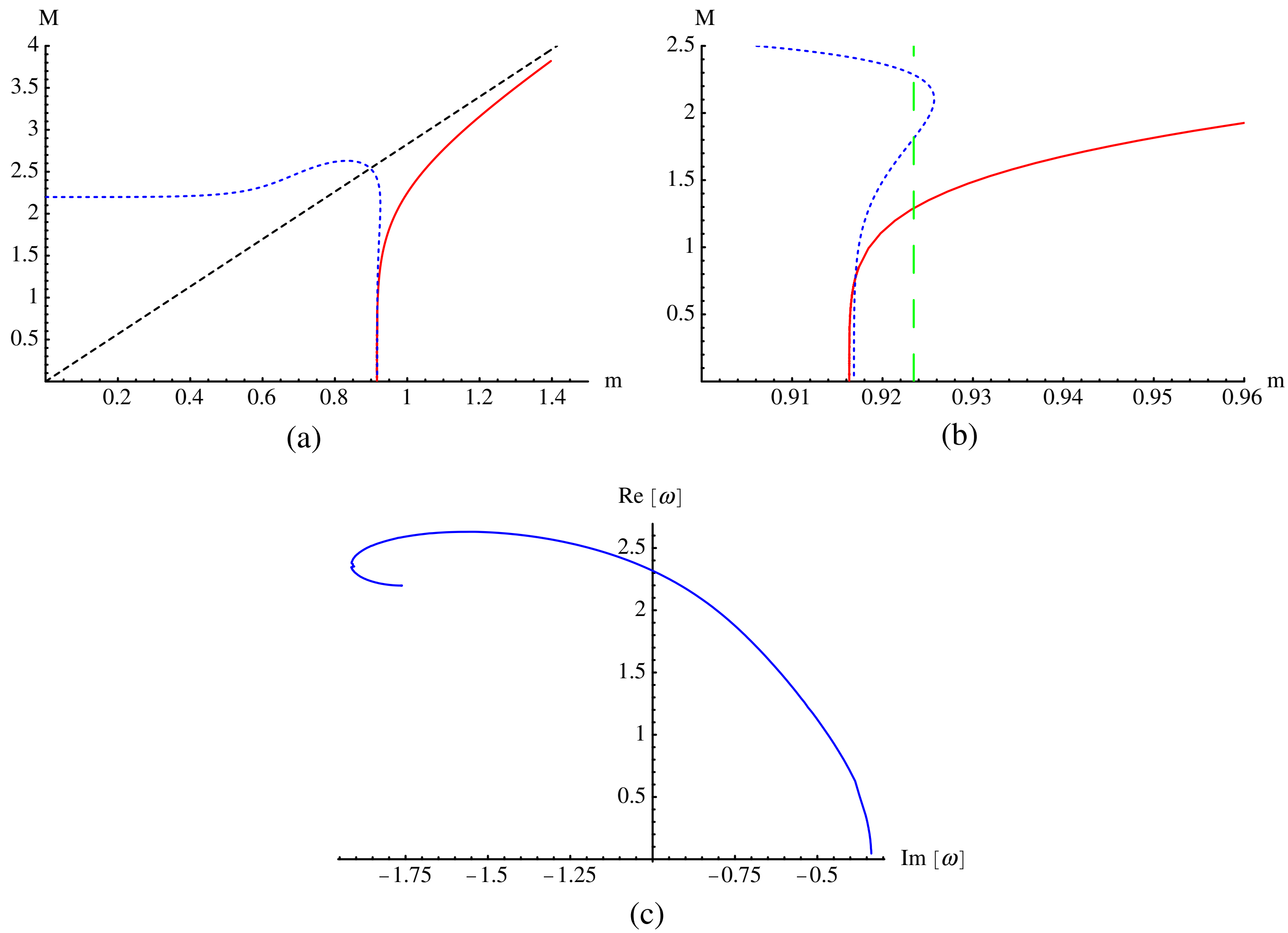

FIG. 4 (color online). Four-dimensional meson mass M, as a function of fundamental quark mass m, from fluctuations in θ. Only the first level of the discrete spectrum is shown. The blue (dashed) curves in (a) and (b) correspond to the meson mass before the meson melts.

dimensional gauge theory: In the Euclidean section, the four-dimensional gauge theory is compactified on an S^1, whose radius is set by the energy scale of the theory and whose periodicity is set by the temperature. In this case, the energy scale is set by the bare quark mass, so at zero bare quark mass, the circle shrinks to zero size, making the gauge theory effectively three-dimensional. For fluctuations in the θ direction, we can follow a similar analysis as for the fluctuations in the ϕ direction. We begin by searching for the four-dimensional mass by taking an ansatz $\phi = 0$ and $\theta = \theta_{(0)} + \delta\theta(z, t)$, where $\delta\theta(z, t)$ has the form

$$\delta\theta(z, t) = f(z)e^{-i\omega t}. \qquad (19)$$

The equation of motion shows the same behavior as the ϕ-meson, so we choose the same boundary condition as in Eq. (16) for the condensate solutions. For the Karch-Katz-like solutions, we use similar boundary conditions for the first derivative as for the classical embeddings:

$$f(z)|_{\theta(z)=\pi/2} = \epsilon, \qquad f'(z)|_{\theta(z)=\pi/2} = \infty, \qquad (20)$$

where ϵ is suitably small. (We chose $\epsilon \sim 10^{-2}$.) The spectrum is shown in Figs. 4(a) and 4(b). In Fig. 4(b), the disconnect between the condensate and Karch-Katz-like solutions is due to there being tachyonic modes that extend beyond the zero meson mass [19], but we have not shown them in our plots. They occur well away from the physical embeddings that are determined by the phase transition. We may also consider fluctuations in the gauge field living on the D7-brane; such fluctuations would correspond to vector mesons in the gauge theory. In order to study their behavior, we must begin by including the field strength term (from the world-volume gauge field) to the Dirac-Born-Infeld action. The action is given by

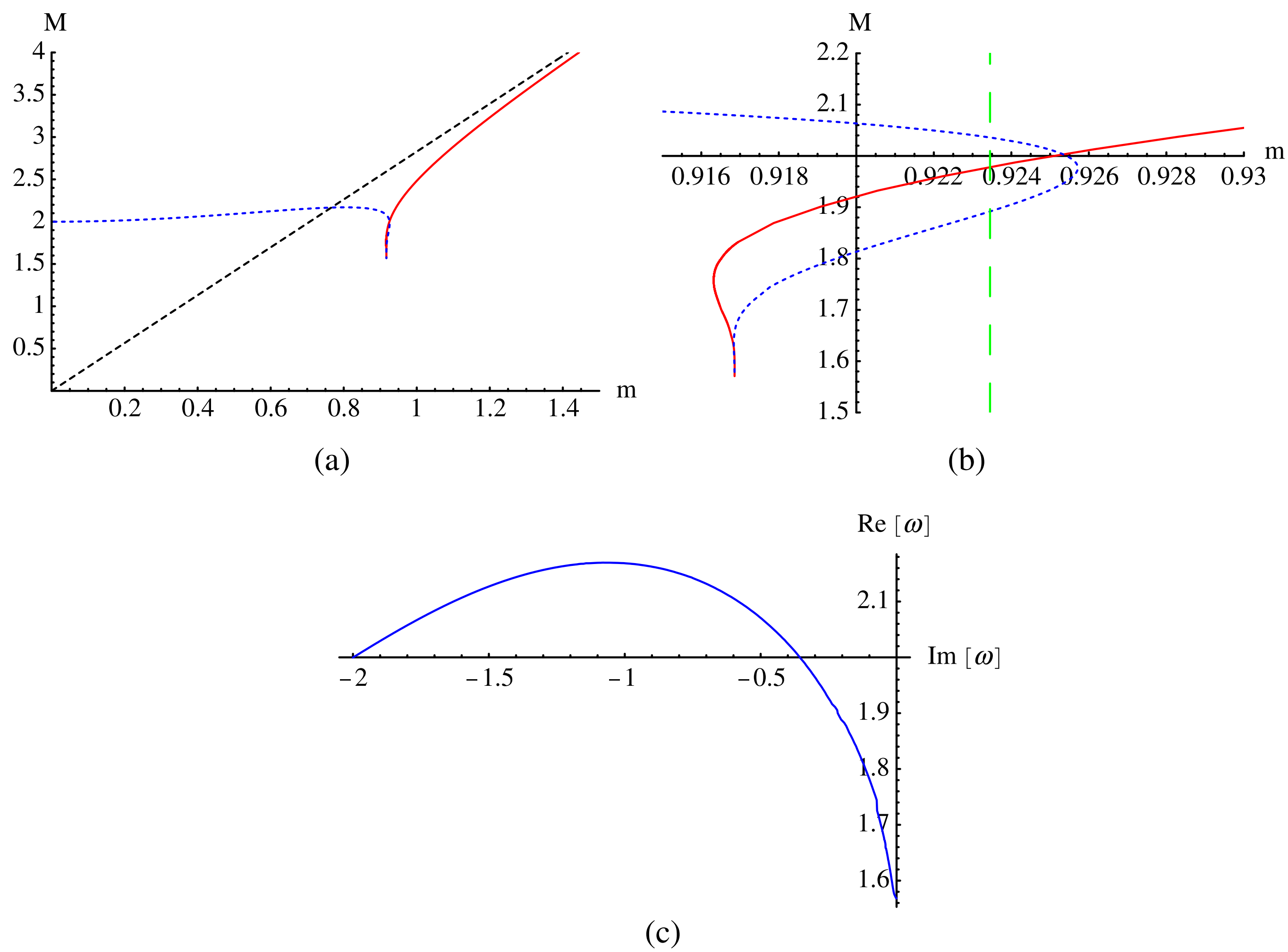

FIG. 5 (color online). Four-dimensional meson mass M, as a function of fundamental quark mass m, from fluctuations in A_b. Only the first level of the discrete spectrum is shown. The blue (dashed) curves in (a) and (b) correspond to the meson mass before the meson melts.

$$S = -\tau_7 \int d^8\xi (-\det(g_{ab} + 2\pi\alpha' F_{ab}))^{1/2} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C_{(4)}] \wedge F \wedge F, \tag{21}$$

where $P[C_{(4)}]$ is the pullback of the 4-form potential sourced by the N_c D3-branes. The action is Abelian since we are taking $N_f = 1$. Our previous solutions assumed a solution $A_b = 0$, so we will consider quadratic fluctuations about this solution. Expanding this action to second order in A_b, we have

$$S = -\tau_7 \int d^8\xi \bigg[(-\det(g_{ab}))^{-1/2} \bigg(1 - \frac{1}{4}(2\pi\alpha')^2 F_{ab}F^{ba}\bigg) - \frac{(2\pi\alpha')^2}{8}\frac{u^4}{R^4} F_{ij}F_{kl}\epsilon^{ijkl}\bigg], \tag{22}$$

where the subscripts a, b run over the world-volume coordinates and i, j, k, l run over the world-volume coordinates transverse to the D3-brane. We have dropped the first order term since it will just give the classical equation of motion for A_b. Since we are only interested in the components of the gauge field along the D3-brane world-volume, the Wess-Zumino term will not contribute to the equations of motion that we are interested in. Therefore, the equation of motion for the quadratic fluctuations is given by

$$\partial_a(\sqrt{-g}F^{a\mu}) = 0. \tag{23}$$

To compute the four-dimensional mass of the vector meson, we consider an ansatz of the form

$$A_\mu = V_\mu(z)e^{-i\omega t}. \tag{24}$$

We want to consider the case where the condition $k_\mu A^\mu = 0$ still holds, which requires us to set $A_t = 0$ since we are working in the rest frame of the vector meson. The equations of motion for the three V_{x^i}'s are identical, so we only need to solve for one of them. The equations of motion show the same behavior as the ϕ-mesons, so we follow the same procedure as before with the same boundary conditions as in Eqs. (16) and (17). The spectrum is shown in Figs. 5(a) and 5(b). We see again that the mass is bounded from above by the meson mass at zero temperature (shown as the black lines with long dashes).

VI. CONCLUSION

We expect that, while the details of this construction will not persist in a realistic QCD string dual, the phase transition itself represents strongly coupled dynamics that may well persist as part of the full story of the QCD phase diagram. The transition is exciting in itself, of course (particularly since its dual involves a change of topology in the D7-brane world-volume), but much further work is needed on several questions. For example, the robustness of the phase transition against $1/N_c$ corrections would be interesting to study. It would also be interesting to see if the backreaction can be included, allowing us to study $N_f \sim N_c$ and follow the phase structure to this regime. Although there are complications involving conical deficits in otherwise flat directions when dealing with the backreacted geometry in the case of a D3-D7 system, some progress has been made in studying the backreacted geometry of other more stable $Dp - D(p+4)$ systems [21–27]. However, many of these calculations have not considered black hole or other finite temperature configurations. These, and several other questions, are exciting matters for further study.

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