HYBRID DECOHERENCE-FREE ERROR-CORRECTING CODES VIA QUANTUM TRAJECTORIES

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Generalizing a proposal by Alfer et al. (quant-ph/0103042), we develop a hybrid decoherence-free subspace (DFS) quantum error-correcting code (QECC) approach, utilizing the methodology of quantum trajectories. The DFS acts as a first layer of protection against errors due to the conditional evolution, while the QECC acts as a second layer and uses the DFS encoding to offer protection against random quantum jumps. We also study the effect of incorporation of quantum logic gates and hence the possibility of fault tolerant quantum computation. Finally we give an example of the method in cavity QED systems.

In this work we use a hybrid of decoherence free subspaces (DFS) and quantum error correction codes (QECC), along with the concept of quantum trajectories, to devise a coding scheme for qubits that protects them against spontaneous emissions and collective dephasing. This construction also allows for relatively easy means of universal computation with physically available Hamiltonians. It is also shown that this construction is fault tolerant in the sense that, for sufficiently low error rates, the errors do not propagate and quantum computation with this code does not add to the decoherence.

Suppose we have a physical system composed of n qubits (two-level quantum systems, with $|0\rangle$ corresponding to the ground state and $|1\rangle$ corresponding to the excited state). Spontaneous emission, induces a transition from $|1\rangle$ to $|0\rangle$, but leaves $|0\rangle$ unchanged. The operator generating this error on the ith qubit is $F_i = |0_i\rangle\langle 1_i|$. In the quantum trajectories picture, the decoherence cycle has two ingredients: The unitary evolution is replaced by a conditional evolution dictated by a non-Hermitian conditional Hamiltonian, which is defined in terms of the system Hamiltonian $H_s$ and the error generators $F_i$, as:

$$H_s = H_s - \frac{i}{2} \sum_i \lambda_i F_i^\dagger F_i,$$

where $\lambda_i$ is some measure of the strength of errors acting on the ith qubit. The second ingredient is random occurrence/application of the errors $F_i$, at random times on random qubits, for our case. In our system we assume all the
never make them susceptible to spontaneous emissions. Second, the proposed operations satisfy the fault-tolerant property in terms of the stabilizer formalism and its extension to continuous stabilizers. The stabilizer formalism is used to construct operators that encode the logical qubits, which are represented by the set of operators $D^p_c$. The stabilizer formalism also provides a method to construct fault-tolerant quantum circuits and state preparation.

To finally discuss the feasibility of the proposed approach for preparing logical states in our code words in proposals such as quantum data, or carry QED with atoms.

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References


In the next paragraphs, we discuss the logical state preparation and its error minimization. We assume that one can use any of the above techniques to prepare the logical state in the code words. Once the logical state is prepared, we can use the error correction protocols to minimize the error probability.

### Footnotes

- The logical state preparation protocol is based on the idea of using a set of stabilizer operators to encode the logical state.
- The error correction protocol is based on the measurement of the stabilizer operators and the application of the appropriate logical correction operations.

### Notes

- The logical state preparation protocol is robust against single-qubit errors.
- The error correction protocol is robust against arbitrary single-qubit and two-qubit errors.

### References