Comment on "Conservative Quantum Computing"

In [1], Ozawa considers the limitations imposed by conservation laws on the possibility of accurately generating quantum logic gates. In particular, he argues that conservation of angular momentum imposes a fundamental limit on how accurately the controlled-NOT (CNOT) gate can be performed. His argument runs roughly as follows. Since CNOT does not commute with total angular momentum, it cannot be implemented directly in a system in which angular momentum is conserved. Therefore one must enlarge the two-qubit Hilbert space by attaching ancilla qubits, perform a unitary operation on the resulting enlarged Hilbert space, then trace out the ancillae. This defines a completely positive dynamical quantum map, which can only yield the CNOT gate on the original two qubits *probabilistically.* Ozawa calculates upper bounds on this probability of success, and concludes that the accuracy of quantum logic gates is fundamentally limited in the presence of conservation laws.

The purpose of this Comment is to point out that this is an overly restrictive conclusion. Indeed, as Ozawa notes at the end of his paper: "The present investigation suggests that the current choice of the computational basis should be modified so that the computational basis commutes with the conserved quantity." "... we may find such a computational basis comprised of orthogonal entangled states over a multiple-qubit system." Such a modification of the computational basis is already well known. This is possible since, while it is true that the CNOT gate sometimes does not commute with the total angular momentum operator, it does *not* follow that CNOT cannot be generated from an interaction that is rotationally invariant. It can, by using an encoding into multiqubit states that define an *invariant subspace* with respect to the interaction at hand. As first shown in [2], using an encoding of a logical qubit into four physical qubits, *the isotropic Heisenberg exchange interaction* $H_{\mathrm{Heis}} = J(X_iX_j + Y_iY_j + Z_iZ_j)$, which commutes with total angular momentum, *is universal for quantum computing.* Here X_i is the Pauli σ_x matrix acting on qubit i, etc. This means that Heisenberg exchange can be used to generate the group $U(2^K)$ on K encoded qubits, and, in particular, it can be used to generate an encoded CNOT gate, which treats only two out of the 16 dimensions of the four-physical-qubit Hilbert space as computational basis states of an encoded qubit. This result was soon followed by a general proof that the Heisenberg exchange interaction is by itself universal over encodings into any number of qubits $n \geq 3$ [3]. Specific gate sequences were then proposed for the $n = 3$ qubit encoding [4]. Furthermore, by supplementing Heisenberg exchange with a Zeeman splitting $\varepsilon(Z_i - Z_j)$ (another interaction that commutes with the total angular momentum operator), a simple encoding into $n = 2$ qubits, $|0_L\rangle = |0\rangle_i|1\rangle_j$ and $|1_L\rangle = |1\rangle_i|0\rangle_j$, already suffices for universal quantum computation [5,6], and thus to generate an encoded CNOT gate. In fact, other exchange interactions that admit conserved quantities [e.g., the Hamiltonians $H_{\mathrm{XY}} = J(X_iX_j + Y_iY_j)$ and $H_{\mathrm{XXZ}} = J(X_iX_j + Y_iY_j) + J_zZ_iZ_j$, that have axial symmetry] are also universal for quantum computation, with [6] or without Zeeman splitting [7].

The reason that encoding helps is that it enables the quantum logic gates to be executed on an invariant subspace of the full Hilbert space of the original qubits plus ancillae. The exchange Hamiltonians, and, hence, the encoded logic gates, preserve this subspace and, as is evident from their universality, commute with all the symmetries of the system. Thus, Ozawa's result can be seen as a confirmation of the advantage that may be had by allowing a flexible definition of the computational basis: Conservation laws do not impose fundamental limitations on the accuracy of quantum logic operations, as long as one uses Hamiltonians that act on an encoding of logical qubits into subspaces that are invariant under the symmetries generating the conservation laws.

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[1] M. Ozawa, Phys. Rev. Lett. **89**, 057902 (2002).
[2] D. Bacon, J. Kempe, D. A. Lidar, and K. B. Whaley, Phys. Rev. Lett. **85**, 1758 (2000).
[3] J. Kempe, D. Bacon, D. A. Lidar, and K. B. Whaley, Phys. Rev. A **63**, 042307 (2001).
[4] D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, and K. B. Whaley, Nature (London) **408**, 339 (2000).
[5] J. Levy, quant-ph/0101057.
[6] L.-A. Wu and D. A. Lidar, Phys. Rev. A **65**, 042318 (2002); D. A. Lidar and L.-A. Wu, Phys. Rev. Lett. **88**, 017905 (2002).
[7] J. Kempe, D. Bacon, D. P. DiVincenzo, and K. B. Whaley, Quantum Inform. Comput. **1**, 33 (2001); L.-A. Wu and D. A. Lidar, quant-ph/0109078.

