

Long-range entanglement generation via frequent measurements

L.-A. Wu, D. A. Lidar, and S. Schneider

*Chemical Physics Theory Group, Chemistry Department, and Center for Quantum Information and Quantum Control,
University of Toronto, Toronto, Ontario, Canada M5S 3H6*

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A method is studied wherein two noninteracting quantum subsystems, that each interact with a third subsystem, are entangled via repeated projective measurements of the state of the third subsystem. A variety of condensed matter and quantum optical examples are presented. The method can be used to establish long range entanglement between distant parties in one parallel measurement step, thus obviating the need for entanglement swapping.

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I. INTRODUCTION

Entanglement is at the heart of quantum information processing [1], and is a resource that allows quantum processes to outperform their classical counterparts for tasks such as computation [2], communication [3], and cryptography [4]. The standard way to generate entanglement between distinguishable and initially separable particles is to let them interact directly for a certain amount of time: any nontrivial two-body Hamiltonian is capable of generating entanglement in this manner [5]. However, the direct interaction method poses limitations in the context of the generation of long-range entanglement, since in many systems the interaction strength typically decreases at least as fast as some power of the distance between particles. Here we consider a different paradigm for entanglement generation: we show that it is possible to entangle two particles that never interact directly by means of repeated (noncontinuous) measurements of a third subsystem that interacts with both. In addition to its conceptual interest, we show that this scheme offers practical advantages for long-range entanglement generation.

Our theoretical framework for measurement-generated entanglement is inspired by the recent work by Nakazato, Takazawa, and Yuasa (NTY) [6], who investigated the effects of repeated rapid projective measurements on one subsystem of a bipartite system. Under appropriate conditions, NTY showed that the unobserved subsystem is gradually projected into a pure state, independent of the initial state. This result leads to the question of whether a similar measurement scheme is capable of generating entanglement when the unobserved subsystem is itself multipartite, noninteracting, and in an arbitrary initial state. We answer this question in the affirmative for a wide range of model systems, and establish general conditions for validity of the method. We note that an additional advantage of this measurement-based method for generating entanglement over the direct-interaction based method is that it does not depend on sensitive timing of interactions: instead, entanglement is gradually purified as the number of measurements increases, and (under appropriate conditions) can be made arbitrarily high. An important conclusion from our study is that the method can be used to establish long range entanglement between distant particles, by measuring along a chain of intermediate particles. This provides an alternative to entanglement swapping that does

not scale with the chain length. A similar result—long range entanglement from local (but nonrepeated) measurements—was obtained in Ref. [7] for the ground state of an antiferromagnetic spin chain (see also Ref. [8]).

A second main source of inspiration for our work are recent quantum optics proposals for measurement-generated entanglement. It has been shown there that entanglement can be generated conditional on the (null) detection of a light mode that has interacted with two or more atoms or ions. The first such proposal showed that atoms in a leaky cavity can become entangled conditioned on null detection at a photodetector placed outside the cavity [9] (decoherence-induced entanglement; see also Ref. [10]). In Refs. [11–15] entanglement is generated in a similar manner conditioned instead upon detection of the spontaneously emitted photons. This method is particularly suitable for generation of long-range entanglement, a subject we address here. The schemes described in Refs. [13–15] are largely insensitive to control errors, another important aspect that is addressed in our work. Measurements of the phase of light transmitted through a cavity have also been proposed in Ref. [16] as a method to prepare entangled states and implement quantum computation in the case of atoms in optical cavities.

In the quantum optics setting we have just mentioned, the measurement typically consists of a photodetector monitoring radiation from a cavity or a trap. In this case the measurement is an act of continuous observation. In contrast, in the scheme we study here, it is important that the measurement is discontinuous, namely, is repeated often, but with nonvanishing time interval. A second important distinction is that the aforementioned quantum optical studies typically use the Monte Carlo wave-function (or “quantum-jump”) approach [17,18], and hence apply the Markovian approximation. In contrast, in our case the Markovian limit is never assumed.

The structure of this paper is as follows. In Sec. II we briefly review the results of NTY [6] on purification via repeated measurements. In Sec. III we develop our general theory of entanglement-generation via repeated measurements. We show that both pure and mixed-state entanglement can be generated. In Sec. IV we study a number of physically relevant examples, borrowed from condensed matter and quantum optics models. In Sec. V we develop a perturbative treatment of the general measurement-based scheme. It is

here that the distinction between our method and method of continuous observation [9–16] becomes particularly transparent, as the short-time limit in our case yields the quantum Zeno effect, which cannot be captured in the pure Markovian limit [19]. In Sec. VI we show how our method can be used to prepare long range entanglement via repeated local measurements, executed in parallel. We conclude in Sec. VII.

II. PRELIMINARIES

NTY considered the following scenario: Consider a bipartite system composed of subsystems A , B , initially in the separable state $\rho_{AB}(0) = |\phi\rangle\langle\phi| \otimes \rho_B$, where A is in a pure state $|\phi\rangle$ and ρ_B is arbitrary. Subsystem A is subject to projective measurements $P_A = |\phi\rangle\langle\phi|$ applied with period τ . In between measurements the system evolves under the Hamiltonian

$$H = H_A + H_B + H_{AB} \quad (1)$$

(respectively, the sum of free Hamiltonians and an interaction). It can be shown [6] that after M such measurements the state of subsystem B , given that all outcomes were $|\phi\rangle$, is

$$\rho_B(M\tau) = \frac{V_B(\tau)^M \rho_B V_B^\dagger(\tau)^M}{P_M}, \quad (2)$$

where P_M is the survival probability, i.e., the probability of finding subsystem A in its initial state,

$$P_M = \text{Tr}_B[V_B(\tau)^M \rho_B V_B^\dagger(\tau)^M], \quad (3)$$

where $U = \exp(-i\tau H)$ (we use units where $\hbar=1$), and where

$$V_B(\tau) \equiv \langle\phi|U|\phi\rangle \quad (4)$$

is an operator on subsystem B that is in general not Hermitian. However, one may still find left- and right-eigenvectors with complex eigenvalues, whose modulus can be shown to be bounded between 0 and 1. The central result of NTY is that in the limit of large M and small but finite τ , $\rho_B(M\tau)$ tends to a pure state independent of B 's initial state. This result assumes that the largest eigenvalue λ_{\max} of $V_B(\tau)$ is nondegenerate. The final pure state $\rho_B(M\tau)$ is then the corresponding right eigenvector $|u_{\max}\rangle$ and this outcome is found with probability

$$P_M \rightarrow |\lambda_{\max}|^2 \langle u_{\max} | u_{\max} \rangle \langle v_{\max} | \rho_B | v_{\max} \rangle, \quad (5)$$

where $|v_{\max}\rangle$ is the corresponding left eigenvector. Note that because of τ 's finiteness the dynamics here is distinct from the Zeno effect and “quantum Zeno dynamics” [20].

We now extend NTY's model by allowing B itself to be composed of multiple noninteracting subsystems, $B = \{B_1, B_2, \dots, B_N\}$, and pose the following question: is the NTY measurement procedure capable of generating entanglement amongst B 's subsystems, assuming that they all interact with the “station” A ? The situation is illustrated in Fig. 1. To answer this questions we consider the following model: We set all internal Hamiltonians $H_A = H_{B_i} = 0$, so that $H = H_{AB}$ (equivalently, we can always transform to an interaction picture rotating with the internal Hamiltonians; this will make H_{AB} time dependent, but it will not introduce cou-

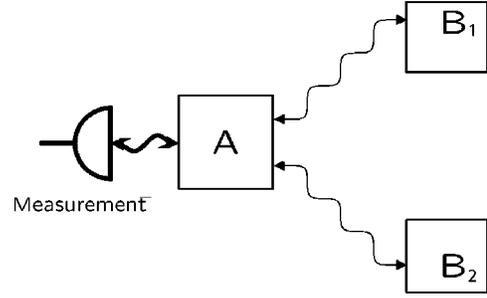


FIG. 1. Schematic illustration of our model: Systems B_1 and B_2 are coupled to system A ; system A can be measured with a projective measurement. Note that no direct coupling exists between systems B_1 and B_2 .

plings between subsystems B_i). We assume that the B subsystems are all qubits and the interaction between A and all the B_i is identical. We can then define total quasispin operators $\sigma_\beta \equiv \sum_{i=1}^N \sigma_\beta^i$ and write the interaction as $H_{AB} = \sum_{\alpha \in \{x,y,z\}} h_\alpha A_\alpha \otimes \sigma_\alpha$ (h_α real) or, more simply,

$$H_{AB} = \vec{A} \cdot \vec{\sigma}, \quad (6)$$

where the parameters h_α are included in the vector \vec{A} . We do not restrict subsystem A .

III. GENERAL THEORY

A. A good basis

We construct a basis that block diagonalizes the effective post-measurement evolution operator $V_B(\tau)$. Let a basis for the A subsystem be the measurement state $|\phi\rangle$ and any set of states orthonormal to it, denoted $|\phi_l^\perp\rangle$, $l=1, \dots, d-1$. Let N be even and let a basis for the B subsystem be constructed out of the usual spin basis $|S, M_S\rangle$, where S is the total quasispin of the N particles and M_S its projection along the z axis. We denote the (orthonormal) singlet states by $|s_j\rangle$, $j=1, \dots, D_N = N! / [(N/2+1)!(N/2)!]$, and the remaining (orthonormal) states with $S > 0$ by $|t_k\rangle$, $k=1, \dots, K_N$, $K_N = 2^N - D_N$.

Proposition 1. Consider the ordered basis $\{|\phi\rangle, |\phi_1^\perp\rangle, \dots, |\phi_{d-1}^\perp\rangle\} \otimes \{|s_1\rangle, \dots, |s_{D_N}\rangle, |t_1\rangle, \dots, |t_{K_N}\rangle\}$. In this basis we have the block-diagonal representation

$$V_B(\tau) = \begin{pmatrix} I_{D_N} & \mathbf{0} \\ \mathbf{0} & V_B^s \end{pmatrix}, \quad (7)$$

where I_{D_N} is a $D_N \times D_N$ -dimensional identity matrix. The maximal eigenvalue of $V_B(\tau)$ is 1, and is at least D_N -fold degenerate.

Proof. The singlets states, $|S, M_S\rangle = |0, 0\rangle$, are annihilated by the σ_β when N is even (since they are states with zero total quasispin). Therefore $H_{AB}|s_j\rangle = 0$, independent of the state of subsystem A . In the ordered basis H_{AB} is thus represented as

$$H_{AB} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & H'_{AB} \end{pmatrix}, \quad (8)$$

where the dimension of the upper-left block is $D_N \times D_N$ and that of the lower-right block is $K_N \times K_N$. Then

$$U_{AB} = e^{-i\tau H_{AB}} = \begin{pmatrix} I_{D_N} & \mathbf{0} \\ \mathbf{0} & U'_{AB} \end{pmatrix}, \quad (9)$$

where $U'_{AB} = W' e^{-i\tau \Lambda} W'^{\dagger}$ is unitary and W' is the unitary matrix that diagonalizes H'_{AB} : $W'^{\dagger} H'_{AB} W' = \Lambda = \text{diag}(\lambda_1, \dots, \lambda_{K_N})$. Taking the expectation value with respect to $|\phi\rangle$ we then find

$$V_B(\tau) = \sum_{j=1}^{D_N} |s_j\rangle\langle s_j| + V_B^s, \quad (10)$$

which is the claimed result, with $V_B^s \equiv \sum_{k,k'=1}^{K_N} |t_k\rangle\langle t_{k'}| (U'_{AB})_{\phi k, \phi k'} \langle t_{k'}|$ a $K_N \times K_N$ -dimensional matrix. That the maximal eigenvalue is 1 follows from unitarity of U'_{AB} [6], and that it is at least D_N -fold degenerate is immediate from the representation (7). Q.E.D.

Proposition 2 [6]. Nondegeneracy of $V_B(\tau)$ is a necessary condition for obtaining a pure state in the limit of large M .

Proof. In the degenerate case it follows from Eq. (2) that the method yields an equally weighted sum of degenerate pure states corresponding to the maximum eigenvalue of $V_B(\tau)$. Q.E.D.

Note that $D_2=1$. Hence in the totally symmetric case we have been considering so far, for $N=2$ the method will generate a pure (maximally entangled) singlet state, provided V_B^s has maximal eigenvalue with modulus smaller than 1. We give examples of corresponding Hamiltonians below. On the other hand, $D_4=2$ and hence due to degeneracy the method will not produce a pure entangled state for $N \geq 4$. However, we can still generate pure-state entanglement in the $N=4$ case by breaking the total permutation symmetry and only preserving the symmetry between 1,2 and 3,4 [e.g., having a coupling to subsystem A such that $\sigma_\beta = a_1(\sigma_\beta^1 + \sigma_\beta^2) + a_2(\sigma_\beta^3 + \sigma_\beta^4)$, $a_1 \neq a_2 \neq 0$]. We can then project out a one-dimensional subspace $|s\rangle_{12|34}, |s\rangle_{ij} = (|0_i 1_j\rangle - |1_i 0_j\rangle) / \sqrt{2}$ is the singlet state, which is entangled for 1,2 and 3,4 but separable across the 12:34 partition. Similarly, for $N=2n \geq 6$ and $\sigma_\beta = \sum_{i=1}^n a_i(\sigma_\beta^{2i-1} + \sigma_\beta^{2i})$, $a_i \neq a_j \neq 0$ the projected state is $|s\rangle_{12|34} \cdots |s\rangle_{n-3,n-2|s\rangle_{n-1,n}}$.

B. Invariance

The results of the above discussion are invariant as long as the Hamiltonians belong to the family $\{U_B H_{AB} U_B^\dagger\}$, where U_B is an arbitrary unitary transformation of the Hamiltonian of subsystem B . This enables the method to generate entangled states equivalent under local transformations. The invariance of $H_{AB}|s_j\rangle=0$ is $H'_{AB}|s'_j\rangle=0$, where $H'_{AB} = U_B H_{AB} U_B^\dagger$ and $|s'_j\rangle = U_B |s_j\rangle$. Thus, e.g., in the case of $N=2$, we can generate the other three Bell states by applying $U_B = X_2, Y_2, Z_2$ (where X_2 is the Pauli matrix σ_x acting on subsystem B_2 , etc.), which results, respectively, in $|s'\rangle = (1/\sqrt{2})(|11\rangle + |00\rangle)$, $(1/\sqrt{2})(|00\rangle - |11\rangle)$, $(1/\sqrt{2})(|01\rangle + |10\rangle)$ as the outcome of the method.

C. Degenerate maximal eigenvalue

What happens when there are other states, either singlets or coming from V_B^s , that also have an eigenvalue with modulus 1? In this case NTY's result does not apply and entanglement may or may not be spoiled (though it is always reduced from maximally entangled). Two examples will illustrate this. (i) $N=2$, suppose there is a triplet state $|00\rangle$ that also has $|\lambda_{\max}|=1$. Then the resulting state is the mixture $|s\rangle\langle s| + |00\rangle\langle 00|$. This state is entangled as its partial transpose [21] has a negative eigenvalue of -0.207 . We will encounter this case in the Heisenberg model below. (ii) $N=4$, suppose the two singlet states $|s_1\rangle = |s\rangle_{12}|s\rangle_{34}$ and $|s_2\rangle = (1/\sqrt{3})[|t_+\rangle_{12}|t_-\rangle_{34} + |t_-\rangle_{12}|t_+\rangle_{34} - 2|t_0\rangle_{12}|t_0\rangle_{34}]$ (where $|t_\alpha\rangle$ are triplets with projection quantum number α) appear with eigenvalue one, but no other states do. With respect to the 12:34 cut $|s_1\rangle$ is a product state but $|s_2\rangle$ is clearly entangled. This state, just like in the previous example, has negative partial transpose, and we have entanglement across the 12:34 cut. These examples illustrate that degeneracy still allows for mixed state entanglement to be generated by our method. This is useful entanglement in the sense that it can be used for teleportation and all other QIP primitives [22].

IV. EXAMPLES

We now discuss examples, limiting ourselves to $N=2$. Our task then reduces to calculating the eigenvalues of the 3×3 matrix $V_B^s = \langle \phi | U'_{AB} | \phi \rangle$. However, this requires diagonalization of U'_{AB} , a 6×6 matrix, so it cannot be done analytically in complete generality. Our basis for A is $\{|\phi\rangle, |\phi^\perp\rangle\}$ and for B is

$$\vec{\beta} \equiv \left\{ \begin{aligned} |s\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle), |t_-\rangle = |00\rangle, \\ |t_0\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), |t_+\rangle = |11\rangle \end{aligned} \right\}, \quad (11)$$

without loss of generality (recall the invariance discussion above).

A. Axial symmetry model

Suppose $A_z=0$ and A_x, A_y satisfy $[A_x, A_y^2] = [A_y, A_x^2] = 0$ (or A_x or $A_y=0$), whence the Hamiltonian reads $H = [A_x(X_1 + X_2) + A_y(Y_1 + Y_2)]/2$. We first consider as a special case the XY model: $A_x = JX$ and $A_y = JY$, relevant for quantum information processing in solid state [23–25] and atomic [26] systems. It follows after some algebra that $H^3 = |A|^2 H$, where $|A|^2 = A_x^2 + A_y^2$. Therefore, the evolution is

$$U_{AB} = e^{-i\tau H} = I - 2 \left(\frac{H}{|A|} \right)^2 \sin^2 \frac{\tau|A|}{2} - i \frac{H}{|A|} \sin \tau|A|, \quad (12)$$

which means that V_B^s is a function only of $\langle \phi | H | \phi \rangle$ and $\langle \phi | H^2 | \phi \rangle$. In particular, for the XY model we find, using $|A| = \sqrt{2}J$,

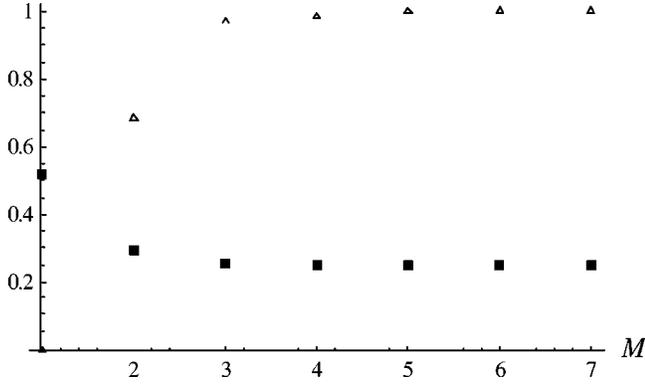


FIG. 2. Concurrence (Δ) and success probability (\blacksquare) P_M for $|\phi\rangle=(1/\sqrt{2})(|0\rangle+|1\rangle)$ and $\tau J=\pi/2$, for the XY model.

$$V_B(\tau) = \cos^2 \frac{\tau J}{\sqrt{2}} - \frac{X_1 X_2 + Y_1 Y_2 - \langle Z \rangle (Z_1 + Z_2)}{2} \sin^2 \frac{\tau J}{\sqrt{2}} - i \frac{\langle X \rangle (X_1 + X_2) + \langle Y \rangle (Y_1 + Y_2)}{2\sqrt{2}} \sin \sqrt{2} \tau J, \quad (13)$$

where $\langle X \rangle \equiv \langle \phi | X | \phi \rangle$, etc. It is simple to check that, as required from our general result, $V_B |s\rangle = |s\rangle$, i.e., the singlet state has eigenvalue 1. Whether this eigenvalue is degenerate is now seen to depend on the measurement of the A subsystem: If we choose $|\phi\rangle$ to be a σ_z -eigenstate then the additional eigenvalues of V_B are found to be $\{1, \cos \sqrt{2} \tau J, \cos \sqrt{2} \tau J\}$, while if we choose $|\phi\rangle$ to be a σ_x or σ_y eigenstate then we find that the additional eigenvalues are $\{\cos^2 q, 1 + 3 \cos 2q \pm \sqrt{\cos 4q + \sin^4 q - 1}\}$, where $q = \tau J / \sqrt{2}$. Thus, if we measure along z then there is never pure state entanglement, but at times τ other than $n\pi / \sqrt{2} J$ we have a mixed entangled state, since the eigenstate corresponding to the additional eigenvalue 1 is found to be $|11\rangle$ (and recall the discussion above). In the case of measurement along x or y the eigenvalues periodically have modulus 1, and with the exception of those times degeneracy is avoided and we do obtain a pure entangled state [in particular, it is simple to show that this is true for all $\tau < \sqrt{2} \arccos(1/3) / J$].

Next we consider the performance of the scheme after a finite number of steps (Fig. 2). To do so we calculate the concurrence [27] after M steps of evolution/measurement for $J\tau = \pi/2 < \sqrt{2} \arccos(1/3)$, in the case of $|\phi\rangle$ a σ_x eigenstate. In this case, as shown above, the singlet state is generated by the scheme. We also plot the survival probability P_M as given by Eq. (3). After as few as three measurements the concurrence is essentially unity, indicating that the system is already in the desired singlet state. The scheme does, however, not work with unit probability: The probability P_M of having projected system A on the desired σ_x eigenstate converges to ~ 0.2 for the values above, a value reached for $M=3$.

As another special case we consider a simplified form of the d -wave grain boundary qubit [28]: $H = \Delta(X_1 + X_2) + JZ(Z_1 + Z_2)$ (Δ is a tunneling parameter and J is the Josephson coupling). Thus $\vec{A} = 2(\Delta, 0, JZ)$, and we may use the general result (12), replacing A_y by A_z . If we now choose $|\phi\rangle$ to be a σ_z eigenstate then V_B turns out to be unitary and hence

all its eigenvalues have modulus 1, and no entanglement is generated. However, if we choose $|\phi\rangle$ to be a σ_x eigenstate, we find

$$V_B = \cos^2 \phi - \sin^2 \phi (\cos^2 \theta X_1 X_2 + \sin^2 \theta Z_1 Z_2) - \frac{i}{2} \sin 2\phi \cos \theta (X_1 + X_2), \quad (14)$$

where $\phi = \tau \sqrt{\Delta^2 + J^2}$ and $\tan \theta = J / \Delta$. It is again easy to check that $V_B |s\rangle = |s\rangle$. One of the other three eigenvalues is $1 - 2 \sin^2 \phi \sin^2 \theta$. The other two are complex conjugates and for short times τ such that $\tan^2 \phi < 4 \cos^2 \theta / \sin^4 \theta$ have the same amplitude $(1 - 2 \sin^2 \phi \sin^2 \theta)^{1/2}$. For instance, when $\theta = \pi/4$, the amplitudes of the three eigenvalues are $\cos^2 \phi, \cos \phi, \cos \phi$, which vanish rapidly when raised to the power of the number of measurements.

Finally, note that the results presented so far are not restricted to A being a qubit: the unitary operator (12) is valid even when A is a multilevel system, as long as the condition $[A_x, A_y^2] = [A_y, A_x^2] = 0$ is satisfied.

B. Heisenberg model

We now consider the Heisenberg interaction $H = J \vec{\sigma}_1 \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)$. After showing that $(H + J \vec{\sigma}_1 \cdot \vec{\sigma}_2)^2 = 9J^2$ and $[H, J \vec{\sigma}_1 \cdot \vec{\sigma}_2] = 0$, it is simple to compute $e^{-i\tau(H + J \vec{\sigma}_1 \cdot \vec{\sigma}_2)}$, from which we directly obtain

$$U_{AB} = e^{-i\tau H} = e^{i\tau J \vec{\sigma}_1 \cdot \vec{\sigma}_2} \{ \cos(3\tau J) - i \sin(3\tau J) [\vec{\sigma}_1 \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) + \vec{\sigma}_1 \cdot \vec{\sigma}_2] / 3 \}. \quad (15)$$

V_B then is identical to U_{AB} provided one replaces with $\langle \phi | \vec{\sigma} | \phi \rangle$ the operator $\vec{\sigma}$ in U_{AB} . The four eigenvalues are $\{1, e^{-2i\tau J}, e^{i\tau J} (\cos 3\tau J \pm i \frac{1}{3} \sin 3\tau J)\}$, the last two having the same amplitude $1 - \frac{8}{9} \sin^2 3\tau J < 1$. The corresponding eigenvectors are, respectively, β [Eq. (11)]. Since the first two eigenvalues both have magnitude 1, we have the case discussed above: a pure entangled state cannot be projected out by our method, but we can prepare a mixed entangled state that is useful for all QIP protocols. Note further that these Heisenberg model results hold for the entire class of Hamiltonians $H = \vec{A}_\sigma \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)$, when \vec{A}_σ is generated from an arbitrary two-dimensional unitary transformation U_A , $\vec{A}_\sigma = U_A J \vec{\sigma} U_A^\dagger$ (this is different from the invariance under a rotation of the B system considered above).

C. Bosonic media

We now consider a photon or phonon (A) interacting symmetrically with two identical qubits (B). The Jaynes-Cummings Hamiltonian is

$$H = \epsilon b^\dagger b + g(\sigma_1^z + \sigma_2^z) + \frac{J}{2} [b(\sigma_1^+ + \sigma_2^+) + b^\dagger(\sigma_1^- + \sigma_2^-)], \quad (16)$$

where b is a bosonic annihilation operator, and whence $\vec{A} = (J(b + b^\dagger), iJ(b - b^\dagger), g)$. Such a model can easily be realized

using microwave cavity QED [29] (two atoms in one or two cavities) or trapped ions [30]. We can exactly diagonalize H by writing it as a direct sum over three-dimensional matrices in the basis $|n-1, t_+\rangle$, $|n, t_0\rangle$, and $|n+1, t_-\rangle$, where $|n\rangle$ are number states and $|t_\alpha\rangle$ are triplet states of the two qubits. Note that $\mathcal{N} \equiv [b^\dagger b + (\sigma_1^z + \sigma_2^z)]$ is a conserved quantity and

$$V_B(\tau) = \text{diag} \left\{ 1, e^{-2i\epsilon\tau} \frac{3 + 2 \cos(\sqrt{10}\pi J)}{5}, e^{-i\epsilon\tau} \cos(\sqrt{6}\pi J), \cos(\sqrt{2}\pi J) \right\} \quad (17)$$

in the ordered basis $\{|1\rangle|s\rangle, |1, t_+\rangle, |1, t_0\rangle, |1, t_-\rangle\}$. Thus, as long as we make sure that $\tau < 2\pi/\sqrt{10}J$ the method will project out a pure singlet state. It follows from our discussion of invariance above that another Bell state $(1/\sqrt{2})(|01\rangle + |10\rangle)$ can be projected out if the two qubits couple to the photon or phonon with opposite signs. We note that a similar problem was considered in Ref. [9], using a different formalism.

D. Multilevel systems

Let us now consider a rather general, though abstract multilevel case. Assume that subsystem B consists of two particles that have $M \geq 2$ levels each, and that the interaction Hamiltonian with subsystem A (also an M -level system) is of the form

$$H = \sum_{i,j=1}^M A_{ij}(O_{ij}^1 + O_{ji}^2), \quad (18)$$

where $O_{ij} = E_{ij} - E_{ji}$ and $E_{ij} = |i\rangle\langle j|$ is a matrix whose elements are zero everywhere except for a 1 at position (i, j) . Namely, there is an $SO(M)$ symmetry for odd M or an $Sp(M)$ symmetry for even M . In this case H has a single nondegenerate eigenvector with zero eigenvalue. This state is entangled. For instance, when $N=3$, we have two qutrits, which can be represented in the spherical basis $\{|-1\rangle, |0\rangle, |+1\rangle\}$. The state with zero eigenvalue is

$$|\Psi\rangle = (|1\rangle_1|-1\rangle_2 - |0\rangle_1|0\rangle_2 + |-1\rangle_1|1\rangle_2)/\sqrt{3}, \quad (19)$$

and is maximally entangled. In general, the one-dimensional subspace containing the state with zero eigenvalue is the irreducible representation $(0, 0, \dots, 0)$ of $SO(M)$ or $Sp(M)$. This state is the generalization of the singlet state that arose in the general theory and examples treated above when the subsystems were qubits.

V. PERTURBATIVE TREATMENT

It is apparent from the above examples that the success of our method depends on keeping the period τ between two measurements short. For instance, in the bosonic example, we require $\tau < 2\pi/\sqrt{10}J$ in order to prevent the appearance

hence H is block diagonal in its eigenvalues. For simplicity we consider the case with $g = \epsilon$ and measure the single photon state $|\phi\rangle = b^\dagger|0\rangle = |1\rangle$ (this projects onto a single block of the infinite matrix U_{AB}). In this case we readily find the already diagonal

of another eigenvalue with amplitude 1. Let us therefore consider a short-time expansion of $V_B(\tau)$. To first order

$$V_B(\tau)^M = (I - i\tau\langle\phi|H_{AB}|\phi\rangle)^M \xrightarrow{M \rightarrow \infty} \exp(-it\langle\phi|H_{AB}|\phi\rangle), \quad (20)$$

where $M\tau = t$ (constant), i.e., the evolution is unitary. This is the quantum Zeno effect, which completely decouples the interaction among all parties of subsystem B , so that no entanglement can be generated in this limit. Thus the dominant contribution to entanglement generation originates from the second-order term, which contains the self-correlation of subsystem B . Consider for simplicity the case $\langle\phi|H_{AB}|\phi\rangle = 0$ (as in our last example). Letting $M\tau^2/2 = t^2$ (constant), we have

$$V_B(\tau)^M \xrightarrow{M \rightarrow \infty} e^{-t^2\langle\phi|H_{AB}^2|\phi\rangle}. \quad (21)$$

All eigenstates of $\langle\phi|H_{AB}^2|\phi\rangle$ with nonzero eigenvalues are rapidly suppressed as t (in practice M) increases, while those with eigenvalue zero survive. Since it is much simpler to analytically calculate $\langle\phi|H_{AB}^2|\phi\rangle$ than $V_B(\tau)$, this perturbative method provides a relatively simple tool for estimating the possibility of entanglement generation via our method, for complicated systems. We consider spin-orbital coupling as another illustrative example: $H = \hbar L_x(X_1 + X_2) + \hbar L_y(Y_1 + Y_2)$, denoting two spin-qubits that couple with the same orbital angular momentum. If we measure the eigenstate $|\phi\rangle = |l, 0\rangle$ of the orbital component, we find $\langle\phi|H|\phi\rangle = 0$ and $\langle\phi|H^2|\phi\rangle = \hbar l(l+1)(2 + X_1X_2 + Y_1Y_2)$. The eigenvalues are $\hbar l(l+1)\{0, 2, 4, 2\}$ with respective eigenvectors $\tilde{\beta}$, meaning that the singlet state $|s\rangle$ is projected out in the second-order analysis. Note that this example is qualitatively different from the previous one since subsystem A here does not refer to a physically separate particle.

VI. PREPARATION OF LONG-DISTANCE ENTANGLEMENT

Finally we come to our main result: the generation of long-distance bipartite entanglement. Since in our discussion above there was no restriction on the size of the A sub-

system, in a sense long-distance entanglement generation already follows from the results above. However, it is important to specify how the measurements on A can be carried out. Assume that the A subsystem is composed of $N-2$ qubits $2, 3, \dots, N-1$ and we wish to entangle the two B -subsystem qubits $1, N$. We consider again as an illustrative example a simplified form of the d -wave grain boundary qubit [28]:

$$U_{AB} = \exp[-iJ\tau(X_1 + X_N) - i\Delta\tau(Z_1Z_2 + Z_2Z_3 + h_3 + Z_{N-2}Z_{N-1} + Z_{N-1}Z_N)], \quad (22)$$

where $h_3 = \sum_{i=3}^{N-3} Z_i Z_{i+1}$. We choose $|\phi\rangle$ to be the state $|+\rangle_2 |R\rangle |+\rangle_{N-1}$, where $|R\rangle = |+\rangle_3 |+\rangle_4 \cdots |+\rangle_{N-2}$, and where $|+\rangle = (|0\rangle + |1\rangle) / \sqrt{2}$. It follows after some calculations that

$$V_B = \frac{1}{2} \left\{ a \left[\cos^2 \phi - \sin^2 \phi (\cos^2 \theta X_1 X_N + \sin^2 \theta Z_1 Z_N) - i \frac{1}{2} \sin 2\phi \cos \theta (X_1 + X_N) + b \left[\cos^2 \phi - \sin^2 \phi \times (\cos^2 \theta X_1 X_N - \sin^2 \theta Z_1 Z_N) - i \frac{1}{2} \sin 2\phi \times \cos \theta (X_1 + X_N) \right] \right], \quad (23)$$

where

$$a = \langle R | e^{-i\Delta\tau(Z_3 + Z_{N-2} + h_3)} | R \rangle = \langle R | e^{-i\Delta\tau(-Z_3 - Z_{N-2} + h_3)} | R \rangle, \quad (24)$$

$$b = \langle R | e^{-i\Delta\tau(Z_3 - Z_{N-2} + h_3)} | R \rangle = \langle R | e^{-i\Delta\tau(-Z_3 + Z_{N-2} + h_3)} | R \rangle \quad (25)$$

(and are real numbers for odd N), $\phi = \tau \sqrt{J^2 + \Delta^2}$, $\tan \theta = \Delta / J$. Two of the eigenvalues and eigenstates are

$$E_1 = \frac{1}{2} \{ a + b(1 - 2 \sin^2 \phi \sin^2 \theta) \}; \frac{1}{\sqrt{2}} (|1_1 0_N\rangle - |0_1 1_N\rangle), \quad (26)$$

$$E_2 = \frac{1}{2} \{ b + a(1 - 2 \sin^2 \phi \sin^2 \theta) \}; \frac{1}{\sqrt{2}} (|0_1 0_N\rangle - |1_1 1_N\rangle). \quad (27)$$

Thus, as desired, entanglement is generated between the B subsystem particles $1, N$. Note that since the measurement of all the particles of the A subsystem is carried out in parallel

this method for long-range entanglement generation is independent of the distance N .

The amplitudes of the other two eigenvalues are $\sqrt{E_1 E_2}$ in the case of odd $N \geq 5$. Since $\max(|E_2|, |E_1|) > \sqrt{E_1 E_2}$ for sufficiently short time, and $E_1 - E_2 = 2(\sin \phi)^2 (\sin \theta)^2 (a - b)$ as long as $a \neq b$, the procedure prepares either $(1/\sqrt{2})(|1_1 0_N\rangle - |0_1 1_N\rangle)$ or $(1/\sqrt{2})(|0_1 0_N\rangle - |1_1 1_N\rangle)$. We find, e.g, for $N=5$, $a = \cos 2\Delta\tau$, $b=1$ and $(1/\sqrt{2})(|00\rangle - |11\rangle)$ is projected out, while for $N=7$, $a = \frac{1}{4}(3 + \cos 4\Delta\tau)$, $b = \cos 2\Delta\tau$, and $(1/\sqrt{2})(|01\rangle - |10\rangle)$ is projected out. The case of even N is more complicated since the amplitudes of two other eigenvalues depend on the values of a and b . For example, in the $N=4$ case, V_B has the same form as above but $a = e^{-i\Delta\tau}$ and $b = e^{i\Delta\tau}$; all roots have the same amplitude and no pure state will be projected out.

VII. CONCLUSIONS

We have studied a general framework for measurement-based method for entangling two systems that only interact indirectly, via a third system. Applications range from solid state to atomic quantum information processing. One of the advantages of this measurement-based method of entanglement generation is that it does not depend on precisely timed interactions, as are interaction-based schemes. The method can be used to prepare arbitrarily long-distance entanglement via a chain of intermediate particle in repeated parallel measurement steps. This may have useful applications in reducing the latency overhead in quantum communication in quantum computing architectures. An interesting open question which we leave for future research is whether the same method can be used to apply quantum logic gates between noninteracting particles.

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