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Universal quantum computation using exchange interactions and measurements of single- and two-spin observables

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We show how to construct a universal set of quantum logic gates using control over exchange interactions and single- and two-spin measurements only. Single-spin unitary operations are teleported between neighboring spins instead of being executed directly, thus potentially eliminating a major difficulty in the construction of several of the most promising proposals for solid-state quantum computation, such as spin-coupled quantum dots, donor-atom nuclear spins in silicon, and electrons on helium. Contrary to previous proposals dealing with this difficulty, our scheme requires no encoding redundancy. We also discuss an application to superconducting phase qubits.

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I. INTRODUCTION

Quantum computers (QCs) hold great promise for inherently faster computation than is possible on their classical counterparts, but so far progress in building a large-scale QC has been slow. An essential requirement is that a QC should be capable of performing ''universal quantum computation'' (UQC): a set of quantum logic gates (unitary transformations) is said to be ''universal'' if any unitary transformation can be approximated to arbitrary accuracy by a quantum circuit involving only those gates [1]. One of the chief obstacles in constructing large-scale QCs is the seemingly innocuous, but in reality very daunting set of requirements must be met for universality, according to the standard circuit model [1]: (1) preparation of a fiducial initial state (*initialization*), (2) a set of single and two-qubit unitary transformations generating the group of all unitary transformations on the Hilbert space of the QC (*computation*), and (3) single-qubit measurements (*read out*). Since initialization can often be performed through measurements, requirements (1) and (3) do not necessarily imply different experimental procedures and contraints. Until recently it was thought that computation is irreducible to measurements, so that requirement (2) would appear to be an essential component of UQC. However, unitary transformations are sometimes very challenging to perform. Two important examples are the exceedingly small photon-photon interaction that was thought to preclude linear optics QCs, and the difficult-to-execute single-spin gates in certain solid-state QC proposals, such as quantum dots [2] and donor atom nuclear or electron spins in silicon [3,4]. The problem with single-spin unitary gates is that they impose difficult demands on g-factor engineering of heterostructure materials, and require strong and inhomogeneous magnetic fields or microwave manipulations of spins, that are often slow and may cause device heating [5]. For this reason there has recently been a great deal of theoretical activity involving various qubit encoding schemes, which allow for UQC without invoking difficult-to-control single-spin gates. Specifically, in the case of exchange Hamiltonians, it was shown that when qubits are encoded into states of two or more spins, the exchange interaction, possibly supplemented by static Zeeman splittings, is sufficient to construct a set of universal gates; see, e.g., Ref. [5] and references therein. In the linear optics case, it was shown that photon-photon interactions can be induced indirectly via *gate teleportation* [6]. This idea has its origins in earlier work on fault-tolerant constructions for quantum gates [7] and stochastic programmable quantum gates [8]. The same work inspired more recent results showing that, in fact, measurements and state preparation *alone* suffice for UQC [9–11].

Experimentally, retaining only the absolutely essential ingredients needed to construct a universal QC may be an important simplification. Since read-out is necessary, *measurements are inevitable*. Here, we propose a minimalistic approach for universal quantum computation that is particularly well suited to the important class of spin-based QC proposals governed by exchange interactions [2–4], and other proposals governed by *effective* exchange interactions [12]. In particular, we show that *UQC can be performed using only single- and two-qubit measurements and controlled exchange interactions, via gate teleportation*. We hasten to add that in our case teleportation involves only nearest-and next-nearest-neighbor spins, so that no coherent manipulations between macroscopically separated spins are required. In our approach, which offers a new perspective on the requirements for UQC, two important advantages are obtained: (i) we require no encoding redundancy, (2) the need to perform the aforementioned difficult single-spin unitary operations is obviated, and replaced by measurements, which are anyhow necessary. The tradeoff is that the implementation of gates becomes probabilistic (as in all gate-teleportation-based approaches), but this probability can be boosted arbitrarily close to 1, exponentially fast in the number of measurements.

II. SUPERCONDUCTING PHASE QUBITS EXAMPLE

We begin our discussion with a relatively simple example of the utility of measurement-aided UQC, in the context of the proposal to use d-wave grain boundary (dGB) phase qubits for QC [13]. In this proposal, it is important to reduce the constraints on fabrication by removing the need to apply a bias on individual qubits [14]. This bias requires, e.g., the possibility of applying a local magnetic field on each qubit, and is a major experimental challenge. The effective system Hamiltonian that we consider is then $H_S = H_X + H_{ZZ}$, where

$H_X = \Sigma_i \Delta_i X_i$ describes phase tunneling, and $H_{ZZ} = \Sigma_{i,j} J_{ij} Z_i Z_j$ represents Josephson coupling of qubits; X_i, Y_i, Z_i denote the Pauli matrices $\sigma^x, \sigma^y, \sigma^z$ acting on the ith qubit. We assume continuous control over J_{ij}. In Ref. [14] it was shown how UQC can be performed given this Hamiltonian, by encoding a logical qubit into two physical qubits, and using sequences of recoupling pulses. Instead, we now show how to implement Z_i using measurements, which together with H_S is sufficient for UQC. Suppose we start from an unknown state of qubit 1: $|\psi\rangle = a|0\rangle + b|1\rangle$. By cooling in the idle state (only H_X on) we can prepare an ancilla qubit 2 in the state $(|0\rangle + |1\rangle)/\sqrt{2}$. Then the total state is $a|00\rangle + b|10\rangle + a|01\rangle + b|11\rangle$. Letting the Josephson-gate $e^{-i\phi Z_1 Z_2/2}$ act on this state, we obtain $a|00\rangle + e^{i\phi} b|10\rangle + e^{i\phi} a|01\rangle + b|11\rangle \propto e^{-i\phi Z_1/2}|\psi\rangle|0\rangle + e^{i\phi Z_1/2}|\psi\rangle|1\rangle$. We then measure Z_2. If we find $+1$ (with probablity 1/2) then the state has collapsed to $e^{-i\phi Z_1/2}|\psi\rangle|0\rangle$, which is the required operation on qubit 1. If we find -1 then the state is $e^{i\phi Z_1/2}|\psi\rangle|1\rangle$, which is an erred state. To correct it we apply the pulse $e^{-i\phi Z_1 Z_2}$, which takes the erred state to the correct state
$-e^{-i\phi Z_1/2}|\psi\rangle|1\rangle$. We then reinitialize the ancilla qubit. This method for implementing Z_i succeeds with certainty after one measurement, possibly requiring (with probability 1/2) one correction step.

III. EXCHANGE BASED PROPOSALS

We now turn to the QC proposals based on exchange interactions, e.g., Refs. [2–4,12]. In these systems, that are some of the more promising candidates for scalable QC, the qubit-qubit interaction can be written as an axially symmetric exchange interaction of the form

$$H_{ij}^{\mathrm{ex}}(t) = J_{ij}^{\perp}(t)(X_i X_j + Y_i Y_j) + J_{ij}^{z}(t) Z_i Z_j, \qquad (1)$$

where $J_{ij}^{\alpha}(t)$ $(\alpha = \perp, z)$ are controllable coupling constants. The XY (XXZ) model is the case when $J_{ij}^{z} = 0$ $(\neq 0)$. The Heisenberg interaction is the case when $J_{ij}^{z}(t) = J_{ij}^{\perp}(t)$. See Ref. [5] for a classification of various QC models by the type of exchange interaction. In agreement with the QC proposals [2–4,12], we assume here that $J_{ij}^{\perp}(t)$ is competely controllable and allow that $J_{ij}^{z}(t)$ may not be controllable. The method we present here works equally well for all three types of exchange interactions, thus unifying all exchange-based proposals under a single universality framework. Since all terms in $H_{\mathrm{ex}}(t)$ commute, it is simple to show that, in the ordered basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ it generates a unitary two-qubit evolution operator of the form

$$U_{ij}(\varphi^{\perp}, \varphi^{z}) = \exp\left[-i \int^{t} dt' H_{ij}^{\mathrm{ex}}(t')\right] = \begin{pmatrix} e^{-i\varphi^z} & & & \\ & e^{i\varphi^z}\cos 2\varphi^{\perp} & -ie^{i\varphi^z}\sin 2\varphi^{\perp} & \\ & -ie^{i\varphi^z}\sin 2\varphi^{\perp} & e^{i\varphi^z}\cos 2\varphi^{\perp} & \\ & & & e^{-i\varphi^z} \end{pmatrix} \qquad (2)$$

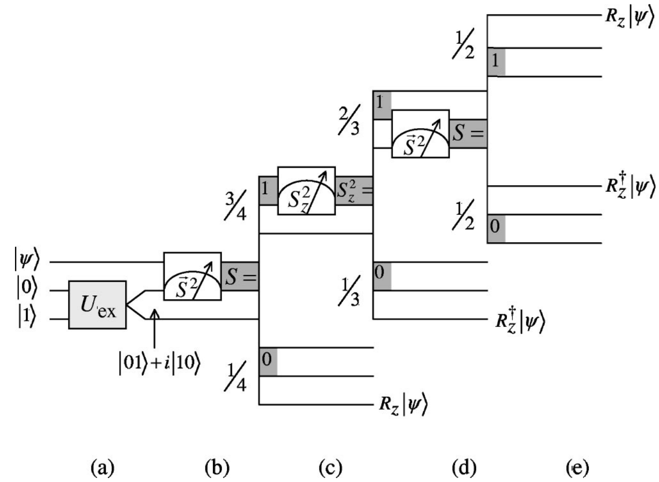

FIG. 1. Gate teleportation of single-qubit operation R_z. Initially Alice has $|\psi\rangle_1$ and $|0\rangle$. Bob has $|1\rangle$. Time proceeds from left to right. Starting from the three-qubit state $|\psi\rangle|01\rangle$, the task is to obtain $R_z|\psi\rangle$. The protocol shown succeeds with probability 1/2. When it fails the operation $R_z^{\dagger}$ is applied instead. Fractions give the probability of a branch; 0 and 1 in a gray box are possible measurement outcomes of the observable in the preceding gray box. See text for full details.

(we use units where $\hbar = 1$), where $\varphi^{\alpha} = \int^t dt' J^{\alpha}(t')$, and we have suppressed the qubit indices for clarity. In preparation of our main result, we first prove the following:

Proposition. The set $\mathcal{G} = \{U_{ij}(\varphi^{\perp}, \varphi^{z}),\ R_{j\beta} \equiv \exp(i\pi/4\sigma_j^{\beta})\}$ $(\beta = x, z)$ is universal for quantum computation.

Proof. A set of continuous one-qubit unitary gates and any two-body Hamiltonian entangling qubits are universal for quantum computation [15]. The exchange Hamiltonian H_{ij}^{ex} clearly can generate entanglement, so it suffices to show that we can generate all single-qubit transformations using $\mathcal{G}$. Two of the Pauli matrices are given simply by $\sigma_j^{\beta} = -iR_{j\beta}^2$. Now, let $C_A^{\theta} \circ \exp(i\varphi B) \equiv \exp(-i\theta A)\exp(i\varphi B)\exp(+i\theta A)$; two useful identities for anticommuting A, B with $A^2 = I$ (the identity) are

$$C_A^{\pi/2} \circ e^{-i\varphi B} = e^{i\varphi B}, \quad C_A^{\pi/4} \circ e^{-i\varphi B} = e^{\varphi AB}.$$

Using this, we first generate $e^{-i\varphi X_1 X_2} = U_{12}(\varphi/2, \varphi^z) C_{X_1}^{\pi/2} \circ U_{12}(\varphi/2, \varphi^z)$, which takes six elementary steps [where an elementary step is defined as one of the operations $U_{ij}(\varphi^{\perp}, \varphi^{z}), R_{j\beta}$]. Second, as we show below, our gate-teleportation procedure can prepare $R_{j\beta}^{\dagger}$ just as efficiently as $R_{j\beta}$ [also note that $R_{j\beta}^{\dagger} = -(R_{j\beta})^3$], so that with two additional steps we have $e^{-i\varphi Y_1 X_2} = C_{Z_1}^{-\pi/4} \circ e^{-i\varphi X_1 X_2}$. Finally, with a total of $8 + 6 + 8 = 22$ elementary steps we have $e^{-i\varphi Z_1} = C_{Y_1 X_2}^{\pi/4} \circ e^{-i\varphi X_1 X_2}$, where φ is arbitrary. Similarly, we can generate $e^{-i\varphi Y_1}$ in 22 steps using $C_{X_1}^{\pi/4}$ instead of $C_{Z_1}^{-\pi/4}$. Using a standard Euler angle construction we can generate arbitrary single-qubit operations by composing $e^{-i\varphi Z_1}$ and $e^{-i\varphi Y_1}$ [1]. ■

It is important to note that optimization of the number of steps given in the proof above may be possible. We now show that the single-qubit gates $R_{j\beta}$ can be implemented using cooling, weak spin measurements, and evolution under exchange Hamiltonians of the Heisenberg, XY, or XXZ type. Our method is inspired by the gate-teleportation idea [6–10]. We proceed in two cycles. In cycle (i), consider a spin (our "data qubit") in an unknown state $|\psi\rangle = a|0\rangle + b|1\rangle$, and two additional ("ancilla") spins, as shown in Fig. 1. Our task is to apply the one-qubit operation R_β to the data qubit. As in gate teleporation, we require an entangled pair of ancilla spins. However, it turns out that rather than one of the Bell states we need an entangled state that has a phase of i between its components. To obtain this state, we first turn on the exchange interaction $H_{23}^{\rm ex}$ between the ancilla spins such that $J^\perp > 0$. The eigenvalues (eigenstates) are $\{-2J^\perp - J^z, 2J^\perp - J^z, J^z, J^z\}$ ($|S\rangle$, $|T_0\rangle$, $|00\rangle$, $|11\rangle$), where $|S\rangle = 1/\sqrt{2}(|01\rangle - |10\rangle)$, $|T_0\rangle = 1/\sqrt{2}(|01\rangle + |10\rangle)$ are the singlet and one of the triplet states. Provided $J^\perp > -J^z$ [which is the case for all QC proposals of interest, in which either $\mathrm{sgn}(J^\perp) = \mathrm{sgn}(J^z)$, or $J^z = 0$] and we cool the system significantly below $-2J^\perp - J^z$, the resulting ground state is $|S\rangle$. We then perform a single-spin measurement of the observable σ_j^z on one or both of the ancillas, which will yield either $|01\rangle$ or $|10\rangle$. For definiteness, assume that the outcome was $|01\rangle$. We then immediately apply an exchange pulse to the ancilla spins [Fig. 1(a)]: $U(\pi/8, \varphi_0^z)|10\rangle = e^{i\varphi^z}/\sqrt{2}(|01\rangle - i|10\rangle)$ [as follows from Eq. (2)]. The total state of the three spins then reads (neglecting an overall phase $e^{i\varphi^z}$):

$$
\begin{aligned}
|\psi\rangle_1 U_{23}(\pi/8, \varphi_0^z)|10\rangle_{23} \\
&= (1/\sqrt{2})(a|001\rangle - ib|110\rangle) + \tfrac{1}{2} r |T_0\rangle_{12} R_{3z}^\dagger |\psi\rangle_3 \\
&\quad - \tfrac{1}{2} r^* |S\rangle_{12} R_{3z} |\psi\rangle_3,
\end{aligned}
\tag{3}
$$

where $r = \exp(-i\pi/4)$ and the subscripts denote the spin index.

At this point Alice makes a weak measurement of her spins [Fig. 1(b)]. Let $\vec{S}_{ij} = \frac{1}{2}(\vec{\sigma}_i + \vec{\sigma}_j)$ be the total spin of qubits i,j; Alice measures $\vec{S}_{12}^2$ with eigenvalues $S(S+1)$. It follows that if the measurement yields 0, then the state has collapsed to $|S\rangle_{12} R_{3z} |\psi\rangle_3$. In this case, which occurs with probability 1/4, Bob has $R_{3z}|\psi\rangle_3$, and we are done [Fig. 1(c), bottom]. If, on the other hand, Alice finds $S = 1$, then the normalized postmeasurement state is

$$
(1/\sqrt{3})[r|T_0\rangle_{12} R_{3z}^\dagger |\psi\rangle_3 + a\sqrt{2}(|001\rangle - ib|110\rangle)]. \tag{4}
$$

Similar to the gate-teleportation protocol [9,10], Alice and Bob now need to engage in a series of correction steps. In the next step, Alice measures $S_z^2 = \frac{1}{4}(\sigma_1^z + \sigma_2^z)^2 = \frac{1}{2}(I + \sigma_1^z \sigma_2^z)$ [Fig. 1(c), top]. If Alice finds $S_z^2 = 0$ then with probability 1/3 the state collapses to $|T_0\rangle_{12} R_{3z}^\dagger |\psi\rangle_3$ and Bob ends up with the opposite of the desired operation, namely, $R_z^\dagger|\psi\rangle$ [Fig. 1(d), bottom]. We describe the required corrective action below, in Cycle (ii). If Alice finds $S_z^2 = 1$, then the state is $a|001\rangle - ib|110\rangle = 1/\sqrt{2}(r^* R_{1z}^\dagger |\psi\rangle_1 |S\rangle_{23} + r R_{1z} |\psi\rangle_1 |T_0\rangle_{23})$. Bob now measures $\vec{S}_{23}^2$. If he finds $S = 0$ then the state has collapsed to $R_{1z}^\dagger |\psi\rangle_1 |S\rangle_{23}$, while if $S = 1$ then the outcome is $R_{1z}|\psi\rangle_1 |T_0\rangle_{23}$, equiprobably. In the latter case Alice ends up with the desired operation [Fig. 1(e)].

In a similar manner one can generate R_x or $R_x^\dagger$ acting on an arbitrary qubit state $|\psi\rangle$. Let $|\pm\rangle$ denote the ± 1 eigenstates of the Pauli operator σ^x. As in the R_z case above, first prepare a singlet state $|S\rangle = 1/\sqrt{2}(|-+\rangle - |+-\rangle)$ on the ancilla spins 2,3 by cooling. Then perform a single-spin measurement of the observable σ_j^x on each ancilla, which will yield either $|+-\rangle$ or $|-+\rangle$. For definiteness assume that the outcome was $|+-\rangle_{23}$. Observing that in the $\{|+-\rangle, |-+\rangle\}$ subspace, $H_{ij}^{\rm ex} = -J_{ij}^\perp I + (J_{ij}^\perp + J_{ij}^z)\tilde{X}$, where $\tilde{X}: |+-\rangle \leftrightarrow |-+\rangle$, it follows that $U(\pi/4 - \varphi_0^z, \varphi_0^z)|+-\rangle = e^{-i\varphi^\perp}/\sqrt{2}(|+-\rangle - i|-+\rangle)$, so that we have a means of generating an entangled initial state. The unknown state $|\psi\rangle_1$ of the data qubit can be expressed as $|\psi\rangle = a_x|+\rangle + b_x|-\rangle$, where $a_x = (a+b)/\sqrt{2}$ and $b_x = (a-b)/\sqrt{2}$. Then (neglecting the overall phase $e^{-i\varphi^\perp}$),

$$
\begin{aligned}
|\psi\rangle_1 U_{23}(\pi/4 - \varphi_0^z, \varphi_0^z)|+-\rangle_{23} \\
&= \tfrac{1}{2} r^* |S\rangle_{12} R_{3x} |\psi\rangle_3 + \tfrac{1}{2} r |T_0^x\rangle_{12} R_{3x}^\dagger |\psi\rangle_3 \\
&\quad + (1/\sqrt{2})(a_x|++-\rangle - ib_x|--+\rangle),
\end{aligned}
$$

where $|T_0^x\rangle = 1/\sqrt{2}(|+-\rangle + |-+\rangle)$ is a triplet state, a zero eigenstate of the observable $\sigma_1^x + \sigma_2^x$. The gate-teleportation procedure is now repeated to yield R_x or $R_x^\dagger$. First, Alice measures the total spin $\vec{S}_{12}^2$. If she find $S = 0$ (with probability 1/4) Bob has spin 3 in the desired state $R_{3x}|\psi\rangle_3$. If she finds $S = 1$ then she proceeds to measure the total length of the x component $S_x^2 = \frac{1}{4}(\sigma_1^x + \sigma_2^x)^2$, yielding, provided she finds $S_x^2 = 0$, the state $|T_0^x\rangle_{12} R_{3x}^\dagger |\psi\rangle_3$ with probability 1/3. If, on the other hand, she finds $S_x^2 = 1$, i.e., the state is $a_x|++-\rangle - ib_x|--+\rangle$, then by letting Bob measure $\vec{S}_{23}^2$, the states $R_{1x}^\dagger |\psi\rangle_1 |S\rangle_{23}$ or $R_{1x}|\psi\rangle_1 |T_0^x\rangle_{23}$ are obtained, with equal probabilities.

Figure 1 summarizes the protocol we have described thus far. The overall effect is to transform the input state $|\psi\rangle$ to either the output state $R_\beta|\psi\rangle$ or $R_\beta^\dagger|\psi\rangle$, equiprobably.

We have now arrived at cycle (ii), in which we must fix the erred state $R_{j\beta}^\dagger|\psi\rangle_j$ ($j = 1$ or 3). To do so we essentially repeat the procedure shown in Fig. 1. We explicitly discuss one example; all other cases are similar. Suppose we obtain the erred state $R_{1z}^\dagger|\psi\rangle_1|S\rangle_{23}$ [Fig. 1(e)]. It can be rewritten as

$$
\begin{aligned}
rR_{1z}^\dagger|\psi\rangle_1|S\rangle_{23} &= -(i/\sqrt{2})(a|001\rangle - ib|110\rangle) \\
&\quad - \tfrac{1}{2} r |S\rangle_{12} R_{3z}^\dagger |\psi\rangle_3 + \tfrac{1}{2} r^* |T_0\rangle_{12} R_{3z} |\psi\rangle_3,
\end{aligned}
$$

which up to unimportant phases is identical to Eq. (3), except that the position of $R_{3z}^\dagger$ and R_{3z} has flipped. Correspondingly, flipping the decision pathway in Fig. 1 will therefore lead to the correct action $R_\beta|\psi\rangle$ with probability 1/2, while the overall probability of obtaining the faulty outcome $R_\beta^\dagger|\psi\rangle$ after the second cycle of measurements is 1/4. Clearly, after n measurement cycles as shown in Fig. 1, the probability for the correct outcome is $1-2^{-n}$. The expected number of measurements per cycle is $1\frac{1}{4}+3\frac{3}{4}\frac{2}{3}\frac{1}{2}=1$, and the expected number of measurement cycles needed is $\Sigma_{n=1}^{\infty}n2^{-n}=2$.

We note that in the case of the erred state $R_{jz}^{\dagger}|\psi\rangle_j$ ($j=1$ or 3), similarly to the case discussed in Sec. II there is an alternative that is potentially simpler than repeating the measurement scheme of Fig. 1. Provided the exchange Hamiltonian is of the XY type, or of the XXZ type with a tunable J^z exchange parameter, one can simply apply the correction operator $U_{j2}(\pi/2,0)=Z_jZ_2$ to $R_{jz}^{\dagger}|\psi\rangle_j$, yielding $R_{jz}|\psi\rangle_j$ as required. Finally, we note that Nielsen [9] has discussed the conditions for making a gate-teleportation procedure of the type we have proposed here, fault tolerant.

IV. SPIN MEASUREMENTS

Our proposal requires measurement of the following observables: σ^z, σ^x (single-spin projective measurements), $\vec{S}_{12}^2$ (distinguishing a singlet from a triplet state), $\sigma_1^{\alpha}\sigma_2^{\alpha}$ [distinguishing whether spins are parallel or antiparallel in the $|0\rangle,|1\rangle$ ($\alpha=z$) or $|0\rangle\pm|1\rangle$ ($\alpha=x$) basis]. Underlying our proposal is the assumption that these measurements are (or will be) easier to perform than the joint requirement of single-qubit gates and single-spin measurements. This assumption is partly motivated by the observation that measurements are necessary, and hence the technology to perform them will be perfected. Let us now briefly survey the subject of spin measurements (for a detailed discussion see, e.g., Refs. [2,3]). While direct measurement of spin is difficult because of the tiny magnetic moment (although possible, in principle, e.g., optically via the Faraday rotation [16]), spin measurement can be converted into charge detection via the Pauli exclusion principle. For example, a donor defect in Si can bind a second electron by 1 meV, provided the second electron has opposite spin to the first electron. Thus, spin measurement becomes electrical charge detection. This is the essential idea behind spin-resonance transistors. Vrijen *et al.* [4(a)] have proposed a field-effect transistor (FET) operating at low temperatures based on this idea. Alternatively, measuring the charge of single electrons is routine using single-electron transistors (SET), and has been proposed for measuring spin qubits [17]. The requirements for single-spin measurements have already been met using an rf-SET coupled to a Si:Te donor [3]. Another possibility, adapted to quantum dots, is to use a dot attached to current leads. This results in Coulomb blockade peaks and spin-polarized currents uniquely associated with the spin state on the dot [2(a)]. The same methods are easily adaptable to distinguishing a spin singlet from a triplet (measurement of $\vec{S}_{12}^2$), as discussed in Refs. [2,3]. The idea is to create an energy difference between the two states, and then to observe conductance. Measurement of the observable $\sigma_1^{\alpha}\sigma_2^{\alpha}$ is possible, e.g., given devices that measure σ_1^{α} and σ_2^{α} and have a nonlinear response: the devices must be coupled and tuned into a regime where the linear response coefficient vanishes, leaving only the second order contribution due to $\sigma_1^{\alpha}\sigma_2^{\alpha}$. A concrete example is provided by quadratic detection using magnetometers in the context of Josephson-junction qabits, as discussed in Ref. [18]. An alternative method is optical measurement: in the singlet state ($S=0$) there is no Faraday rotation, while in the triplet state ($S=1$) there is [2(a)]. Finally, we note that quantum error correction relies on measurements of multi-spin observables such as $\sigma_1^{\alpha}\sigma_2^{\alpha}$ [1], so the development of these techniques is as inevitable as that of single-spin measurements.

V. CONCLUSION

We have proposed a gate-teleportation method for universal quantum computation that is uniformly applicable to Heisenberg, XY, and XXZ-type exchange interaction-based QC proposals, and that replaces single-spin unitary gates by measurements of single- and two-spin observables. We hope that the flexibility offered by this approach will provide a useful alternative route towards the realization of universal quantum computers.

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