Efficient Universal Leakage Elimination for Physical and Encoded Qubits

L.-A. Wu, M. S. Byrd, and D. A. Lidar
Chemical Physics Theory Group, University of Toronto, 80 St. George Street, Toronto, Ontario M5S 3H6, Canada
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Decoherence-induced leakage errors can couple a physical or encoded qubit to other levels, thus potentially damaging the qubit. They can therefore be very detrimental in quantum information processing and require special attention. Here we present a general method for removing such errors by using simple decoupling and recoupling pulse sequences. The proposed gates are experimentally accessible in a variety of promising quantum-computing proposals.

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The unit of quantum information is the qubit: an idealized two-level system consisting of a pair of orthonormal quantum states. However, this idealization neglects other levels which are typically present and can mix with those defining the qubit. Such mixing, the prevention of which is the subject of this work, is known as "leakage." Leakage may be the result of the application of logical operations, or induced by system-bath coupling. In the former case, a rather general solution was proposed in [1]. Here we are interested in decoherence-induced leakage. This is part of a more general problem: quantum computation (QC) depends on reliable components and a high degree of isolation from a noisy environment. When these conditions are satisfied, it is known that it is possible to stabilize a quantum computer using an encoding of a "logical qubit" into several physical qubits. Methods which profitably exploit such an encoding are, e.g., (closed-loop) quantum error correcting codes (QECC) [2,3] and (open-loop) decoherence-free subspaces or subsystems (DFS) [4–6]. The logical qubits of these codes can also undergo leakage errors, which are particularly serious: by mixing states from within the code and outside the code space, leakage completely invalidates the encoding. A simple procedure to detect and correct leakage, which can be incorporated into a fault-tolerant QECC circuit, was given in [2]. This scheme is, however, not necessarily compatible with all encodings [7]. Here we present a *universal*, open-loop solution to leakage elimination, which makes use of fast and strong "bang-bang" (BB) pulses [8,9]. We first give a general scheme for protecting qubits (whether encoded or physical) from leakage errors using an efficient pulse-sequence. Then we illustrate the general result with examples taken from a variety of promising QC proposals. Particularly important is the fact that our scheme is experimentally feasible in these examples, in the sense that we only make use of the naturally available interactions.

Universal leakage-elimination operator.—Here we give a general, existential argument for eliminating all leakage errors on encoded or physical qubits. We generalize previous results on leakage elimination via symmetrization [10] to a multi-qubit setting. Suppose that n two-level systems (e.g., electron spins in quantum dots [11]) are used to encode one logical qubit, or that an N-level Hilbert space $\mathcal{H}_N$ supports a two-dimensional physical qubit subspace (e.g., hyperfine energy levels of an ion). Let us arrange the basis vectors $\{|j\rangle\}_{j=0}^{N-1}$ of $\mathcal{H}_N$ so that $|0\rangle$ and $|1\rangle$ represent the (physical or encoded) qubit states ($N = 2^n$ for the encoded case). In this ordered basis we can classify all system operators as follows:

$$E = \begin{pmatrix} B & 0 \\ 0 & 0 \end{pmatrix} \qquad E^{\perp} = \begin{pmatrix} 0 & 0 \\ 0 & C \end{pmatrix} \qquad L = \begin{pmatrix} 0 & D \\ F & 0 \end{pmatrix}, \tag{1}$$

where B and C are 2×2 and $(N-2) \times (N-2)$ blocks, respectively, and D, F are $2 \times (N-2)$, $(N-2) \times 2$ blocks. Operators of type E represent logical operations, i.e., they act entirely within the qubit subspace. $E^{\perp}$ operators, on the other hand, have no effect on the qubit as they act entirely outside the qubit subspace. Finally, L represents the leakage operators. The total system-bath Hamiltonian can be written as $H_{SB} = H_E + H_{E^{\perp}} + H_L$, where H_E ($H_{E^{\perp}}, H_L$) is a linear combination of elements of the set E ($E^{\perp}, L$), tensored with bath operators. Now consider

$$R_L = e^{i\phi} \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \tag{2}$$

where the blocks have the same dimensions as in Eq. (1) and ϕ is an overall phase. This operator satisfies $\{R_L, L\} = 0$, while $[R_L, E] = [R_L, E^{\perp}] = 0$. Using a BB parity-kick sequence [8] it follows that R_L is a *leakage-elimination operator* (LEO):

$$\lim_{m\to\infty} (e^{-iH_{SB}t/m} R_L^{\dagger} e^{-iH_{SB}t/m} R_L)^m = e^{-iH_E t} e^{-iH_{E^{\perp}} t}. \tag{3}$$

In practice one takes $m = 1$ and makes $t \ll 1/\omega_c$, where ω_c is the bath high-frequency cutoff (e.g., the Debye frequency for a bath of harmonic oscillators) [8]. Equation (3) then holds to order t^2, and implies that one intersperses periods of free evolution for time t with

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$R_L, R_L^\dagger$ pulses which are so strong that H_{SB} is negligible during these BB pulses. This combination of fast and strong pulses is why the procedure is termed bang-bang [8]. The term $e^{-iH_{E^\perp}t}$ in Eq. (3) has no effect on the qubit subspace. The term $e^{-iH_E t}$ may result in logical errors, which will have to be treated by other methods, e.g., concatenation with a QECC [2,3,12], or additional BB pulses [13]. Note that since R_L commutes with the logical operations, they can be performed at the same time, i.e., *our leakage-elimination procedure is fully compatible with universal QC*. We now give a procedure for generating LEOs from a controllable system Hamiltonian H_S acting for a time τ, i.e., $R_L = \exp(-iH_S\tau)$. From Eq. (2) it follows that H_S must act as a projection operator P onto the qubit subspace. Furthermore, τ must be chosen so that R_L acts as $-I$ in the qubit subspace. A general choice is

$$R_L^{\mathrm{E}(1)} = \exp(\pm i\pi\hat{n}\cdot\vec{\sigma}P), \tag{4}$$

where $\vec{\sigma}$ denotes the vector of Pauli matrices, which we refer to as logical X, Y, Z operations, and $\hat{n}$ is a real unit vector. This is a valid LEO since $\exp(-i\pi\hat{n}\cdot\vec{\sigma})$ expresses a 2π rotation about the axis $\hat{n}$ on the qubit Bloch sphere, upon which the qubit state acquires a minus sign. A useful example is $\tau = \pi$ and $H_S = |0\rangle\langle 0| + |1\rangle\langle 1|$, which is a projector onto the qubit subspace and acts as identity there. This example generalizes immediately to d-dimensional qu*d*its [10]:

$$R_L^{\mathrm{id}(d)} = \exp\left(\pm i\pi\sum_{k=0}^{d-1}|k\rangle\langle k|\right). \tag{5}$$

Now let us consider leakage prevention on a code subspace of K logical qubits, each supported by n physical qubits. In analogy to $R_L^{\mathrm{E}(1)}$ we can construct a general LEO as follows. Let S_i be a single-qubit logical (unitary) operation on the ith (encoded or physical) qubit, and let P_i be a projection onto the code subspace of that qubit. Then

$$R_L^{\mathrm{E}(K)} = \exp(\pm i\pi\otimes_{i=1}^K S_iP_i) \tag{6}$$

is a valid LEO.

Proof: We can always rotate the Bloch sphere of a qubit so that each S_i is independently transformed into Z_i: $S_i = U_iZ_iU_i^\dagger$ (where U_i is an appropriate single-qubit unitary); $\otimes_{i=1}^K Z_i$ is a diagonal matrix of ± 1, so $\exp(-i\pi\otimes_{i=1}^K Z_i) = -I$. Thus $R_L^{\mathrm{E}(K)} = \exp(-i\pi\otimes_{i=1}^K U_iZ_iU_i^\dagger P_i) = (\otimes_{i=1}^K U_i)\exp(-i\pi\otimes_{i=1}^K Z_iP_i)(\otimes_{i=1}^K U_i^\dagger) = -I$ on the code subspace, and (because of the P_i) $= I$ on the orthogonal complement. QED.

In all examples considered below, we are able to construct single-qubit logical operators which are automatically projectors on that qubit's subspace. We refer to such operators as "canonical." We are now ready to apply these considerations to a number of promising QC proposals.

Example 1.—As a simple first example, consider physical qubits, such as electrons on liquid helium [14], or an electron-spin qubit in quantum dots [11,15], or a nuclear- or electron-spin qubit in donor atoms in silicon [16]. In those cases, a potential well at each site traps one fermion. Usually, the ground and first excited state are taken as a qubit for a given site: $|k\rangle = c_k^\dagger|\mathrm{vac}\rangle$, where $c_k^\dagger$ is a fermionic creation operator for level $k = 0, 1$. Let $n_k = c_k^\dagger c_k$ be the fermion number operator. The logical operations for this qubit are $E = \{X = c_0^\dagger c_1 + c_0^\dagger c_1, Y = i(c_1^\dagger c_0 - c_0^\dagger c_1), Z = n_0 - n_1\}$ whose elements satisfy $su(2)$ commutation relations. In this case, a general linear Hamiltonian which includes hopping terms, $H_{SB} = \sum_{k,l=0}^{N-1} a_{kl}c_k^\dagger c_l$, where a_{kl} includes parameters and bath operators, and k, l denote all electron states, can leak the qubit states $k = 0, 1$ into any of the other states. Using parity kicks, we can eliminate this leakage in terms of the LEO: $R_L^{\mathrm{id}(1)} = \exp[\pm i\pi(n_0 + n_1)]$. This LEO is implemented by controlling on-site energies.

Let us now generalize this to K qubits. The states of the ith qubit are $|k\rangle_i = c_k^\dagger(i)|\mathrm{vac}\rangle$. States outside of the code subspace contain at least one creation operator $c_k^\dagger(i)$ with $k \geq 2$. A logical Z operator on the ith qubit is $Z_i = n_0(i) - n_1(i)$, which is canonical. It follows from Eq. (6) that an LEO is

$$(R_L^{\mathrm{E}(K)})_{\mathrm{ferm}} = \exp[\pm i\pi(Z_1Z_2\cdots Z_K)]. \tag{7}$$

The term $\otimes_{i=1}^K Z_i$ involves a many-body interaction which is not naturally available. However, it can be constructed from available interactions as follows: Let us assume that the interaction between neighboring sites i, j contains a controllable Z_iZ_j term (in reality such control may have to be obtained indirectly, e.g., by controlling an $X_iX_j + Y_iY_j$ term, as shown in [17], and as discussed in more detail below). We note the following useful "conjugation by $\pi/4$" formula:

$$T_A \circ e^{i\theta B} \equiv e^{i(\pi/4)A}e^{i\theta B}e^{-i(\pi/4)A} = e^{i\theta(iAB)}, \tag{8}$$

which holds if $\{A, B\} = 0$ and $A^2 = B^2 = I$. Using this we can efficiently generate long-range interactions by alternately switching interactions A, B on/off. E.g.,

$$T_{Y_2} \circ [T_{Z_2Z_3} \circ (T_{X_2} \circ e^{i\theta Z_1Z_2})] = e^{i\theta Z_1Z_2Z_3}, \tag{9}$$

which can, in turn, be used to generate $e^{i\theta Z_1Z_2Z_3Z_4}$, etc. Using this recursive process, the implementation of the LEO $(R_L^{\mathrm{E}(K)})_{\mathrm{ferm}}$ takes $O(K)$ steps. Figure 1 shows a circuit for the 4-qubit case.

We conclude Example 1 with an estimate of its feasibility in the case of electrons on helium. There the major source of decoherence is the ripplon bath, with $\omega_c = 0.1$ GHz, while all operations needed to implement $(R_L^{\mathrm{E}(K)})_{\mathrm{ferm}}$ can be performed at several GHz [14]. The BB time-scale condition is thus satisfied.

Example 2.—Similarly to the fermionic case, we can also treat bosonic systems, such as the linear optical QC

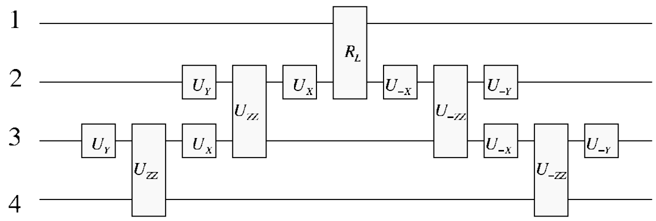

FIG. 1. Quantum circuit of the LEO for 4 physical (encoded) qubits in Example 1 (3). $U_A = e^{i\frac{\pi}{4}A}$, where A stands for X_i, Y_i, Z_i, Z_iZ_j ($\overline{X}_i, \overline{Y}_i, \overline{Z}_i, \overline{Z}_i\overline{Z}_j$) for Example 1 (3).

proposal [18]. In this case, a qubit is encoded into two modes. Two qubits are then $|0\rangle_1 = b_1^\dagger|\text{vac}\rangle$, $|1\rangle_1 = b_2^\dagger|\text{vac}\rangle$, and $|0\rangle_2 = b_3^\dagger|\text{vac}\rangle$, $|1\rangle_2 = b_4^\dagger|\text{vac}\rangle$, where $b_i^\dagger$ are bosonic creation operators. Encoded two-qubit states are $|00\rangle = b_1^\dagger b_3^\dagger|\text{vac}\rangle$, $|01\rangle = b_1^\dagger b_4^\dagger|\text{vac}\rangle$, $|10\rangle = b_2^\dagger b_3^\dagger|\text{vac}\rangle$, and $|11\rangle = b_2^\dagger b_4^\dagger|\text{vac}\rangle$. But the linear optical Hamiltonian $H = \sum_{k,l=1}^4 a_{kl} b_k^\dagger b_l$, contains beam-splitter terms like $b_1^\dagger b_3$ and $b_2^\dagger b_3$, which can cause leakage into states such as $b_1^\dagger b_2^\dagger|\text{vac}\rangle$ or $b_1^{\dagger 2}|\text{vac}\rangle$. By using the LEO $R_L^{\text{id}(1)} = \exp[\pm i\pi(b_1^\dagger b_1 + b_2^\dagger b_2)]$, we can eliminate the leakage terms (see also [10]). This LEO can be implemented simply using a linear optical phaseshifter.

Example 3.—A substantial number of promising solid-state QC proposals, e.g., [11,14,15,16,19], are governed by effective isotropic and anisotropic exchange interactions, which quite generally, can be written as

$$H_{\text{ex}} = \sum_{i<j} J_{ij}^x X_i X_j + J_{ij}^y Y_i Y_j + J_{ij}^z Z_i Z_j. \tag{10}$$

The encoding $|0\rangle_L = |01\rangle$, $|1\rangle_L = |10\rangle$ (using 2 physical qubits per logical qubit) is highly compatible with H_{ex}, in the sense that universal QC can be performed by controlling the *single* parameter J_{ij}^x in the Heisenberg ($J_{ij}^x = J_{ij}^y = J_{ij}^z$), XXZ ($J_{ij}^x = \pm J_{ij}^y \neq J_{ij}^z$), and XY ($J_{ij}^x = J_{ij}^y$, $J_{ij}^z = 0$) instances of H_{ex}, provided there is a Zeeman splitting that distinguishes single-qubit Z_i terms. This is done using the "encoded selective recoupling" method [17]. Furthermore, the $\{|01\rangle, |10\rangle\}$ encoding is a DFS for collective dephasing (where the bath couples only to system $Z_{2i-1} + Z_{2i}$ operators) [4,12,20]. A set of logical operations on this code is $E_{2-\text{DFS}} = \{\overline{X}_1 = (X_1X_2 + Y_1Y_2)/2,\ \overline{Y}_1 = (X_2Y_1 - Y_2X_1)/2,\ \overline{Z}_1 = (Z_1 - Z_2)/2\}$. Only the $\overline{X}_1$ term is assumed to be directly controllable (by manipulation of J_{12}^x), while the $\overline{Z}_1$ term can be turned on/off using recoupling [17]. Then $e^{-i\theta\overline{Y}_1} = T_{\overline{X}_1} \circ e^{-i\theta\overline{Z}_1}$. The leakage errors are due to system-bath interactions where the system terms include any of X_i, Y_j, X_iZ_j, and Y_iZ_j, since as is easily seen, such terms do not preserve the $\{|01\rangle, |10\rangle\}$ code subspace. As pointed out first in [13], the LEO can be expressed as $(R_L^{\text{E}(1)})_{2-\text{DFS}} = \exp(i\pi\overline{X}_1) = Z_1Z_2$, which means that it is implementable using just the controllable J_{12}^x parameter in the instances of H_{ex} mentioned above. This form for $(R_L^{\text{E}(1)})_{2-\text{DFS}}$ is an instance of Eq. (4), with $\hat{n} = \hat{x}$. Note that, in agreement with our general comments above, $(R_L^{\text{E}(1)})_{2-\text{DFS}}$ commutes with every element of $E_{2-\text{DFS}}$, meaning that logical operations can be performed on the encoded subspace *while eliminating leakage.*

Next we now show how to efficiently eliminate leakage in this case on an arbitrary number of encoded qubits. The mth logical qubit is encoded as $|0_L\rangle_m = |0_{2m-1}1_{2m}\rangle$, $|1_L\rangle_m = |1_{2m-1}0_{2m}\rangle$. The logical Z operator is $\overline{Z}_m = (Z_{2m-1} - Z_{2m})/2$, and is canonical. It follows from Eq. (6) that a valid LEO is

$$(R_L^{\text{E}(K)})_{2-\text{DFS}} = \exp(\pm i\pi\overline{Z}_1\overline{Z}_2\cdots\overline{Z}_K). \tag{11}$$

The next question is how to efficiently construct such an operator. The term $\overline{Z}_1\overline{Z}_2 = \frac{1}{4}(Z_1Z_3 + Z_2Z_4 - Z_2Z_3 - Z_1Z_4)$ contains next and second-next nearest-neighbor interactions. Using "conjugation by $\pi/4$," they can all be generated using only nearest-neighbor interactions in terms of the relation

$$\begin{aligned} e^{i\theta Z_iZ_{i+2}} &= T_{X_{i+1}X_{i+2}} \circ (T_{Y_{i+1}Y_{i+2}} \circ e^{i\theta Z_iZ_{i+1}}) \\ &= T_{X_{i+1}X_{i+2}+Y_{i+1}Y_{i+2}} \circ e^{i\theta Z_iZ_{i+1}}, \end{aligned} \tag{12}$$

where, in accordance with [17], we have only assumed controllability of the XY interaction parameter $J_{i+1,i+2}^x$. At this point we can use the recursive construction of Eq. (9) again, by replacing X, Y, Z there by their encoded counterparts. Doing so takes $\overline{Z}_i\overline{Z}_{i+1}$ to $\overline{Z}_i\overline{Z}_{i+1}\overline{Z}_{i+2}$, etc., and will again *efficiently* construct the LEO $(R_L^{\text{E}(K)})_{2-\text{DFS}}$, i.e., using $O(K)$ steps. An example of this for four encoded qubits is shown in Fig. 1. It is interesting to contrast the linear scaling of this leakage-elimination procedure with general error elimination using BB pulses. As shown in [9], without additional symmetry assumptions restricting the order of coupling terms in the Hamiltonian Eq. (3), the BB procedure, if used to eliminate *all errors*, requires a number of pulses that is exponential in K.

Example 4.—Collective decoherence is a system-bath interaction that obeys full qubit permutation symmetry: $H_{SB}^{\text{Coll.Dec.}} = \sum_{\alpha=x,y,z}(\sum_i \sigma_i^\alpha) \otimes B^\alpha$, where σ_i^α are the Pauli matrices and B^α are bath operators [4,5]. This situation can be *created* from an arbitrary linear system-bath coupling $H_{SB}^{(1)} = \sum_i \vec{\sigma}_i \cdot \vec{B}_i$, where $\vec{\sigma}_i = (X_i, Y_i, Z_i)$ and $\vec{B}_i$ are bath operators, using a BB symmetrization pulse sequence that employs *only* the Heisenberg exchange interaction [21]. The shortest DFS (or "noiseless subsystem") encoding that protects a single logical qubit against collective decoherence uses 3 physical qubits [6]. In [20] it was shown that one can perform universal QC on this DFS, again using only the Heisenberg interaction. To explain the encoding, note that the Hilbert space of 3 spin 1/2's has total-spin $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2 + \vec{\sigma}_3)$ and splits into two $S = 1/2$ subspaces (denoted $\lambda = 0, 1$), and a $S = 3/2$ subspace. The $S = 1/2$ states can be labeled as $|\lambda, S_z\rangle$, and the DFS qubit is $|0_L\rangle = \alpha|0, 1/2\rangle + \beta|0, -1/2\rangle$, $|1_L\rangle = \alpha|1, 1/2\rangle + \beta|1, -1/2\rangle$, $|\alpha|^2 + |\beta|^2 = 1$, i.e., the encoding is into the degeneracy of the two $S = 1/2$ subspaces [6,20]. Collective errors can change the α, β coefficients, but have the same effect on the $|0_L\rangle$, $|1_L\rangle$ states, which is why this encoding is a DFS. If, however, we also consider bilinear system-bath coupling $H_{SB}^{(2)} = \sum_{i<j}\sum_{\alpha,\beta=\{x,y,z\}} g_{ij}^{\alpha\beta}\sigma_i^{\alpha}\sigma_j^{\beta}\otimes B_{ij}^{\alpha\beta}$, then the symmetrization procedure of [21], that prepares collective decoherence conditions, will not work. In this case we must consider the possibility of leakage. The bilinear term $g_{ij}^{\alpha\beta}\sigma_i^{\alpha}\sigma_j^{\beta}$ can be decomposed into (i) a scalar $g\vec{\sigma}_i \cdot \vec{\sigma}_j$, which has the effect of logical errors E; (ii) two operators $\vec{\beta}\cdot(\vec{\sigma}_i \times \vec{\sigma}_j)$ and $(\vec{\sigma}_i \cdot \vec{\gamma})(\vec{\sigma}_j \cdot \vec{\gamma})$, which can couple between $S = 1/2$ states, and can couple them to $S = 3/2$ states. Note that this also applies to *imperfect symmetrization* at the level of a *linear* system-bath Hamiltonian $H_{SB}^{(1)}$. Thus we see that the $S = 3/2$ subspace acts as a source for leakage, and that there is also the possibility of (noncollective) errors [both from (ii)] which do not have the same effect on the $|0_L\rangle$, $|1_L\rangle$ states. We defer an analysis of the latter "$S = 1/2 \rightarrow 1/2$" errors to a separate publication, but we note that they can be suppressed using techniques similar to those we discuss next.

An open-loop leakage correction circuit for this DFS, that once more uses only the Heisenberg interaction, was given in [7]. There the DFS qubit was defined to be $|0_L\rangle = |0, 1/2\rangle$, $|1_L\rangle = |1, 1/2\rangle$ and transitions to any of the other six states were considered as leakage (this includes errors caused by collective decoherence, which are normally avoided by a DFS encoding). As stressed in [21], *the Heisenberg interaction can act as a generator of universal, fault-tolerant QC*. Here we add another element to this picture by showing that it can also provide an LEO. The importance of Heisenberg-only QC is in the relative ease of manipulating this interaction in a number of the most promising solid-state QC proposals [11,16]. Now, as shown in [20], $\overline{X} = (1/4\sqrt{3})(\vec{\sigma}_1 \cdot \vec{\sigma}_3 - \vec{\sigma}_2 \cdot \vec{\sigma}_3)$ acts as a logical X on the DFS qubit defined above, and annihilates the $S = 3/2$ states (i.e., it is canonical). Therefore, using Eq. (4), $(R_L^{\mathrm{E}(1)})_{3-\mathrm{DFS}} = \exp(\pm i\pi\overline{X})$ is a Heisenberg-only LEO for a single 3-qubit DFS which eliminates transitions to the $S = 3/2$ subspace. An LEO for the K-qubit case is then, from Eq. (6):

$$(R_L^{\mathrm{E}(K)})_{3-\mathrm{DFS}} = \exp[\pm i\pi(\overline{X}_1\overline{X}_2\cdots\overline{X}_K)]. \qquad (13)$$

To generate this LEO from available interactions we use a procedure similar to Eq. (9). First, note that $T_{\overline{Y}_2} \circ T_{\overline{Z}_1} \circ T_{\overline{Z}_1\overline{Z}_2} \circ e^{i\theta\overline{X}_1} = e^{i\theta\overline{X}_1\overline{X}_2}$. Efficient schemes for generating $\overline{Z}_i\overline{Z}_j$ were given in [22], while $\overline{Z}_i, \overline{Y}_i$ are directly obtainable from the Heisenberg interaction [20]. The recursive construction of $(R_L^{\mathrm{E}(K)})_{3-\mathrm{DFS}}$ then proceeds using $T_{\overline{Y}_3} \circ T_{\overline{Z}_2} \circ T_{\overline{Z}_2\overline{Z}_3} \circ e^{i\theta\overline{X}_1\overline{X}_2} = e^{i\theta\overline{X}_1\overline{X}_2\overline{X}_3}$, etc., which again is a procedure that scales as $O(K)$.

Conclusions.—Decoherence-induced leakage from the logical space of (physical or encoded) qubits is a severe source of errors for quantum computation. We have shown how to efficiently and universally eliminate such errors using sequences of strong and fast pulses. These pulses can be applied at the same time as logical operations, so that leakage elimination can be performed in conjunction with universal quantum computation. Applications to a variety of promising quantum-computing proposals were discussed, and leakage-elimination methods were presented that are directly applicable using only experimentally available interactions. In the case of electrons on helium we were able to confirm the feasibility of the proposed methods based on a time-scale analysis.

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