Comprehensive Encoding and Decoupling Solution to Problems of Decoherence and Design in Solid-State Quantum Computing

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(Received 6 February 2002; published 3 July 2002)

Proposals for scalable quantum computing devices suffer not only from decoherence due to the interaction with their environment, but also from severe engineering constraints. Here we introduce a practical solution to these major concerns, addressing solid-state proposals in particular. Decoherence is first reduced by encoding a logical qubit into two qubits, then completely eliminated by an efficient set of decoupling pulse sequences. The same encoding removes the need for single-qubit operations, which pose a difficult design constraint. We further show how the dominant decoherence processes can be identified empirically, in order to optimize the decoupling pulses.

DOI: 10.1103/PhysRevLett.89.047901 PACS numbers: 03.67.Lx, 03.65.Yz, 05.40.Ca

Essentially all promising solid-state quantum computing (QC) proposals are based on direct or effective exchange interactions between qubits, with a Hamiltonian:

$$H_{\rm ex} = \sum_{i<j} J_{ij}^x X_i X_j + J_{ij}^y Y_i Y_j + J_{ij}^z Z_i Z_j . \qquad (1)$$

(X_i represents the Pauli matrix σ_x acting on the ith qubit, etc.) Examples are quantum dots [1,2], donor atoms in silicon [3], quantum Hall systems [4], and electrons on helium [5]. These implementations combine scalability with controllability of qubit interactions via tunable exchange couplings J_{ij}^α and single qubit operations. Two major problems arise in these proposals. Problem I, shared by all QCs, concerns the coupling to the environment (lattice, impurities, and other degrees of freedom). This leads to decoherence, which introduces computational errors that must be prevented [6–11], corrected [12,13], or suppressed [14–17]. Problem II is somewhat unique to solid-state QC architectures and concerns single-qubit versus two-qubit operations. For reasons detailed, e.g., in [18–20], single-qubit operations often involve significantly more demanding constraints. A large body of literature proposes solutions to the decoherence problem, e.g., [6–17]. A number of recent papers have proposed solutions to the problems imposed by single- and two-qubit operations, e.g., [18–20]. *Here, we propose a comprehensive solution to both problems by making use of a simple encoding and efficient pulse sequences.* The encoding uses a decoherence-free subspace (DFS) [7–11], whereas the pulse sequences combine strong and fast "bang-bang" (BB) pulses [14–17] with selective recoupling [19]. The BB pulse sequence we present eliminates all leakage errors (i.e., errors that violate the DFS encoding) *with a single pair of pulses per cycle.* We illustrate our results with a discussion of quantum dots and then generalize them to include quantum error correcting codes (QECC) [12,13].

Encoding.—We use a well-known code [6] of blocks of two qubits encoding a single logical qubit:

$$|0_L\rangle_i \equiv |0\rangle_{2i-1} \otimes |1\rangle_{2i}, \qquad |1_L\rangle \equiv |1\rangle_{2i-1} \otimes |0\rangle_{2i} . \quad (2)$$

Here $i = 1, \ldots, N/2$ indexes logical qubits, and N is the number of physical qubits. We can define logical operations (denoted by a bar) which act on the encoded qubits as do the unencoded operations on physical qubits. For example, $\overline{X}$: $|0_L\rangle \leftrightarrow |1_L\rangle$. Then, the single-encoded-qubit logical operations, $\overline{X}_i = (X_{2i-1}X_{2i} + Y_{2i-1}Y_{2i})/2$ and $\overline{Z}_i = (Z_{2i-1} - Z_{2i})/2$, generate all encoded-qubit SU(2) transformations, through time evolution. With the two-encoded-qubits operation $\overline{Z}_i\overline{Z}_{i+1} = Z_{2i}Z_{2i+1}$ coupling qubits in two neighboring blocks, they form a *universal set of Hamiltonians* [19]. Universality means that by selectively turning on/off $\{\overline{X}_i, \overline{Z}_i, \overline{Z}_i\overline{Z}_{i+1}\}$ it is possible to generate a dense subgroup of the unitary group $U(2^{N/2})$ of all logical transformations. Let us assume that the single-qubit spectrum is nondegenerate (e.g., due to Zeeman splitting), but $\overline{Z}_i$ are not necessarily controllable. It is sufficient to control *only* $\overline{X}_i$ to achieve (encoded) universality in the Heisenberg ($J_{ij}^x = J_{ij}^y = J_{ij}^z$), *XXZ* ($J_{ij}^x = J_{ij}^y \neq J_{ij}^z$), and *XY* ($J_{ij}^x = J_{ij}^y$, $J_{ij}^z = 0$) instances of the general exchange Hamiltonian, Eq. (1) [19]. The "encoded recoupling" method that accomplishes this eliminates the need for single-qubit control in exchange-based QC architectures, thus solving problem II.

The second advantage of the encoding (2) is that it is a DFS with regard to collective dephasing [6,8,10]: Suppose the system interacts with a bath through the Hamiltonian $H_I = S_z \otimes B_z$, where $S_z = \sum_i Z_i$. For logical qubit states $|\psi_L\rangle = a|0_L\rangle + b|1_L\rangle$ we have $S_z|\psi_L\rangle = 0$, so H_I does not affect the code. This immunity to the system-bath interaction, generally associated with a symmetry, is the reason that the "code" $|\psi_L\rangle$ is a DFS. Collective errors are expected to be particularly relevant for solid-state systems at low temperatures and dephasing is one of the main problems in the corresponding class of QC devices. However, in reality there are other sources of decoherence and errors. Our goal is to show how the aforementioned methods can be extended to treat these as well. We do this by introducing BB pulses as a second layer of protection, except that, using encoded recoupling [19], we apply BB to *encoded* qubits. The DFS encoding together with BB operations will serve to counter decoherence, thus solving problem I, while encoded recoupling will allow for universal QC. We note that an interesting alternative proposal for dealing with problems I and II is to combine the DFS method with energetic suppression of decoherence [21]. This perturbative result requires an encoding into at least four spins.

Bang-bang operations.—BB controls are strong and fast pulses, repeatedly applied to average out the environment-induced noise [14]. The simplest example is the "parity kick" [14,15]. Suppose that an error E acts on the system and that a pulse U (a unitary operator) anticommutes with E and therefore inverts the sign of this error:

$$\{E, U\} = 0, \Rightarrow U^{\dagger} E U = -E. \quad (3)$$

Repeatedly implementing the cycle {apply U, evolve freely under E (for time Δt), apply U^{-1}, evolve freely} averages out the errors ("symmetrizes" [16,17]), thus *decoupling* system and bath. The time for a complete cycle, T_c, must be shorter than τ_c, the inverse of the bath spectral density high-frequency cutoff:

$$\Delta t \leq T_c \ll \tau_c. \quad (4)$$

If the time scales are close, one can still achieve noise reduction [14,15]. Knowledge of τ_c is clearly useful for the success of the procedure and will be discussed below for quantum dots. We also outline an alternative empirical method for the determination and evaluation of the BB operations. Let us now show how BB can be applied to the encoded qubits, enabling the comprehensive solution to the problems posed above.

Applying bang-bang operations on a decoherence-free subspace.—As noted above, the logical qubits of Eq. (2) are immune to collective dephasing errors $Z_{2i-1} + Z_{2i}$. Let us consider other errors. A basis for all possible errors is the 2^4 different tensor products of Pauli matrices (including the identity I) acting on two qubits. Four types of operations affecting a DFS can be identified [10]: (i) two operations to which the DFS is invariant—$(I, Z_1 + Z_2)$; (ii) three that interchange states outside the DFS; both (i) and (ii) have no effect on the DFS; (iii) three logical operations—$[\overline{X}, \overline{Y}, \overline{Z}]$ which can cause *logical* errors; (iv) eight operations which mix DFS states with states out of the DFS [Eq. (5)]. These cause leakage from, and to, the DFS. Sets (iii) and (iv) damage the encoding. Both cause decoherence by entangling the encoded information with the bath. Let us apply this classification to our code. A basis for leakage errors (iv) on the first logical qubit is represented by the following set:

$$\{X_1, X_2, Y_1, Y_2, X_1Z_2, Z_1X_2, Y_1Z_2, Z_1Y_2\}. \quad (5)$$

These can be seen to take the encoded states of Eq. (2) out of the DFS (and vice versa) since they involve single bit flips, or bit and phase flips on individual physical qubits. We now come to our key observation.

Theorem 1: Let $U_{\overline{X}_i}(\phi) \equiv \exp(-i\phi\overline{X}_i)$. Then cycles of a *single* pair of BB pulses $U_{\overline{X}_i}(\pi)$ and $U_{\overline{X}_i}(\pi)^{\dagger}$, where

$$U_{\overline{X}_i}(\pi) = e^{-i\pi(X_{2i-1}X_{2i}+Y_{2i-1}Y_{2i})/2} = -Z_{2i-1}Z_{2i}, \quad (6)$$

eliminate all type (iv) leakage errors on the ith logical qubit.

Proof: $U_{\overline{X}_1}(\pi)$ anticommutes with all errors in Eq. (5) so it satisfies the parity-kick condition (3). Q.E.D.

This *single pair of pulses* aspect is extremely important given the time constraints of Eq. (4).

In order to implement $U_{\overline{X}_1}(\pi)$ one must switch on $J(X_1X_2 + Y_1Y_2)$ for a time $t = \pi/2J$. This (XY) Hamiltonian is available in a number of QC proposals (quantum dots/atoms in cavities [2,22] and quantum Hall systems [4]). Systems governed by Heisenberg or XXZ Hamiltonians can be made to simulate the XY type using encoded recoupling [19]. The Heisenberg case applies to the spin-coupled quantum dots [1] and donor-spin proposals [3]; the XXZ case applies to the electrons on helium proposal [5], and the XY and Heisenberg if symmetry breaking mechanisms are present [19]. Spin-orbit coupling induces corrections to Eq. (1), which can be overcome by a number of methods [20,23].

Eliminating all leakage errors on a DFS encoding a logical qubit by cycles of a single pair of BB pulses is a drastic alternative to the previous proposal of concatenation of a DFS and QECC [10]. The advantage is diminished somewhat if logical errors (iii) are present, which the DFS-QECC method can correct at no extra cost [10]. To eliminate such errors here, we need another pulse, $U_{\overline{X}_i}(\pi/2) = -i\overline{X}_i$, which anticommutes with both $\overline{Y}_i$ and $\overline{Z}_i$ (this also implies that this pulse can be used to *create* the conditions of collective dephasing [24]). Hence *all but one error* ($\overline{X}_i$ itself) *can be eliminated using only the single BB control Hamiltonian* $\overline{X}_i$. To eliminate $\overline{X}_i$ itself without destroying the previous step of eliminating $\overline{Y}_i$ and $\overline{Z}_i$, we must introduce two more BB controls, e.g., $\overline{Z}_i$ and $\overline{Y}_i$. The encoded recoupling method [19] can be used to switch on/off $\overline{Z}_i$ *solely by controlling* $\overline{X}_i$, and $e^{-i\theta\overline{Y}_i} = e^{-i(\pi/4)\overline{Z}_i} e^{-i\theta\overline{X}_i} e^{i(\pi/4)\overline{Z}_i}$. This procedure already involves several pulses and may not meet the strict BB time constraints. We note that cycles of three parity-kick pulses (+ the identity operation) can suppress all single-qubit errors *without* encoding [16]. In contrast, the advantages of our method, which is compatible with encoded recoupling, are (i) the elimination of *leakage* errors on a logical qubit with a *single* parity kick sequence depending on a controllable XY Hamiltonian and (ii) the elimination of all other errors using control of the *same* Hamiltonian. Leakage elimination on M logical qubits can also be treated in a number of pulses that scale as $O(M)$ [25].

Estimation of bath cutoff frequency in quantum dots.—We turn to an assessment of the feasibility of our encoded BB method. We concentrate on the spin-based GaAs quantum dots QC proposals [1,2]. For a review of the main spin relaxation and dephasing mechanisms, see [26]. The dominant low temperature mechanisms are related to spin-orbit coupling. However, no detailed understanding of the various decoherence mechanisms exists. It is noteworthy that our approach to error suppression does not rely on a detailed microscopic understanding of these mechanisms.

Spin-bath and spin-boson models, which are rather general models of low energy effective Hamiltonians, are adaptable to a wide range of problems, including ours. The spin-boson model describes dephasing due to coupling to delocalized modes (lattice vibrations), while the spin-bath model captures the coupling to localized modes, such as nuclear and paramagnetic spins, and defects [27]. In both models it can be shown that the characteristic decay time of coherence, $T_2 = f(\tau_c, T)$ (T is the temperature), and the function f can be analytically determined in various cases [6,14,27–29]. Note that exponential decay is rigorously valid only in the Markovian limit: e.g., in the spin-boson model at $T = 0$ with Ohmic damping, coherence decays polynomially as $1/[1 + (t/\tau_c)^2]$ [29], in which case one can identify $T_2 = \tau_c$. In fact, since τ_c is the primary time scale describing the bath, it is not unreasonable to quite generally identify $T_2 = c(T)\tau_c$, where c is a model-dependent function. This is supported by a variety of instances of the spin-boson and spin-bath models, differing by the specific form of the bath spectral density. Furthermore, at low temperature $c(T) \approx 1$. Given $T_2 \sim 100$ ns [30], we thus conservatively estimate $\tau_c \sim 1-100$ ns for spin-coupled GaAs quantum dots. The gate operation time in these systems is of the order of 50 ps [26] and cannot be made much shorter because of induced spin-orbit excitations [31]. Thus a range of 20–2000 BB parity-kick pulses seems attainable. The first order correction to the ideal limit of infinitely fast and strong BB operations is $O((T_c/\tau_c)^2)$ [14], which, for parity kicks, in our case translates to a correction of $O(10^{-2})$–$O(10^{-6})$.

Empirical bang bang.—As an alternative to model-based approaches of determining the bath cutoff and the BBs, we propose "empirical bang bang." This requires neither a microscopic understanding nor a detailed experimental analysis of each of the decoherence processes in the system. It requires only quantum process tomography measurements (QPT) [32] to determine the *types* of errors. One may then empirically determine the *required* corrective pulses and the effectiveness of the *experimentally available* set [33].

Empirical BB is based on the evolution of an open quantum system, described by a density matrix ρ, that satisfies

$$\rho(t) = \sum_{\alpha,\beta} \chi_{\alpha\beta}(t) K_\alpha \rho(0) K_\beta^\dagger, \tag{7}$$

where the matrix $\chi_{\alpha\beta}(t)$ is Hermitian and $\{K_\alpha\}$ is a system operator basis [28,32]. The χ matrix is the output of QPT [32], i.e., it is *measurable*. For BB operations, a short-time expansion of Eq. (7) is relevant. In this case, choosing a Hermitian operator basis $\{K_\alpha\}$ ($K_0 = I$), to first order in τ (where $\tau \ll \tau_c$): $\rho(\tau) = i[S(\tau), \rho(0)]$. Here $S(\tau) = \sum_{\alpha\geq 1} \mathrm{Im}[\chi_{\alpha 0}^{(1)}(\tau)]K_\alpha$, and $\chi_{\alpha 0}^{(1)}(\tau) = \tau[d(\chi_{\alpha 0}^{(1)})/dt]_{t=0}$ [28]. $S(\tau)$ plays the role of a Hamiltonian. Under the action of a group $\mathcal{G} = \{U_k\}_{k=1}^N$ of unitary BB controls the operator basis transforms as $K_\alpha \to \frac{1}{N}\sum_k U_k^\dagger K_\alpha U_k$. This implies a transformation of χ under the adjoint representation of $\mathcal{G}$, defined by $\sum_\beta R_{\alpha\beta}^{(k)} K_\beta = U_k^\dagger K_\alpha U_k$ [e.g., $R \in SO(3)$ for $U \in SU(2)$, [33]]. Specifically, using the abbreviation $\overline{\chi}_\alpha \equiv \mathrm{Im}[\chi_{\alpha 0}^{(1)}(\tau)]$, we have under BB that $\sum_{\alpha\geq 1} \overline{\chi}_\alpha K_\alpha \to \sum_{\beta\geq 1} \tilde{\chi}_\beta K_\beta$, where

$$\tilde{\chi}_\beta = \frac{1}{N}\sum_k \sum_{\alpha\geq 1} \overline{\chi}_\alpha R_{\alpha\beta}^{(k)}. \tag{8}$$

The coefficients $\tilde{\chi}_\beta$ are the expansion coefficients of a "desired" Hamiltonian. For example, for *storage* we would require BB to eliminate all errors due to $S(\tau)$, so that all $\tilde{\chi}_\beta$ vanish. For *computation* we would have nonvanishing $\tilde{\chi}_\beta$ describing the Hamiltonian we wish to implement [33]. *The key idea of empirical BB is to use the experimentally determined* $\overline{\chi}_\alpha$, *together with a specified set of* $\tilde{\chi}_\beta$ *(corresponding to a desired evolution), to solve Eq. (8) for the rotation matrices* $R_{\alpha\beta}^{(k)}$. These determine a set of BB operations. Thus, using empirical BB, *one may determine the required BB operations directly from experimental data*. Repeatedly performing such a procedure (measure χ, apply BB) determines the optimal BB process, given the available controls and accounting for constraints, through a learning loop [34]. In this manner only the experimentally relevant errors are addressed, thus potentially reducing the set of BB operations. In addition, a reduction in decoherence, as manifested in χ, is a direct indication that condition (4) is satisfied. The use of empirical BB therefore allows for a direct test of the feasibility of our encoded parity-kick scheme.

Generalizations.—Let us now discuss generalizations which will shed further light and suggest additional applications. Let $\overline{\mathcal{G}}$ denote the group of logical operations (e.g., for SU(2), generated by $\{\overline{X}, \overline{Y}, \overline{Z}\}$) , acting as *gates* on encoded qubits. In analogy to standard BB theory [14–17] we define "symmetrization of a Hamiltonian H with respect to $\overline{\mathcal{G}}$" as: $H \mapsto (1/|\overline{\mathcal{G}}|)\sum_{U\in\mathcal{G}} U^\dagger H U$. Then:

Theorem 2: Symmetrization with respect to $\overline{\mathcal{G}}$ suffices to completely decouple the dynamics of the encoded subspace from the bath.

Proof: Symmetrization takes any system-bath Hamiltonian and projects it onto the centralizer of the group generated by $\overline{\mathcal{G}}$ (i.e., the set of elements that commute with all elements of this group). By irreducibility of the representation of $\overline{\mathcal{G}}$, it follows, from Shur's Lemma, that the BB-modified system-bath Hamiltonian is proportional to identity on the code space; i.e., the code space dynamics will be decoupled. Q.E.D.

This shows that *any encoding* may be combined with BB operations. In particular, it motivates us to consider combining encoded BB with QECCs. Such codes can often be described by a stabilizer $S = \{S_i\}$: a group that has all codewords as eigenstates with eigenvalue 1 [12]. The errors $\mathcal{E} = \{E_j\}$ that a stabilizer code can detect are exactly the operators that *anticommute* with some element of S. This naturally leads to an encoded parity-kick scheme: *to suppress* $\mathcal{E}$*, apply the generators of* S *as a set of BB operations.* Consider as a simple, but important example, the case of protecting against all single-qubit errors. The smallest QECC uses five physical qubits per logical qubit [13]. Instead, we could start by encoding one logical qubit into three: $|0\rangle_L = |000\rangle$, $|1\rangle_L = |111\rangle$, in order to protect just against independent bit flip errors $\mathcal{E}_X = \{X_1, X_2, X_3\}$. This leaves us with the independent phase flip errors $\mathcal{E}_Z = \{Z_1, Z_2, Z_3\}$. These can be suppressed using BB on the encoded qubits. The stabilizer for the three-qubit code for phase flips is generated by $S_X = \{X_1X_2, X_2X_3\}$, which anticommutes with $\mathcal{E}_Z$. Thus, frequent application of X_1X_2 and X_2X_3 (which can be implemented using simultaneous application of single-body Hamiltonians X_i and X_j) as parity kicks will suppress the $\mathcal{E}_Z$ errors. The advantage of this compared to the five-qubit code is in the conservation of qubit resources. This comes at the expense of additional gate operations that must be included in the QECC circuitry, but this may well be a worthwhile tradeoff when qubits are scarce. Another possibility, which illustrates Theorem 2 directly, is to use the normalizer elements of the QECC [12]. These are logical operations that must commute with stabilizer elements in order to preserve the code space. Therefore they can be applied at any time during a QECC circuit (with the exception of during the recovery operations), in particular, as BB pulses. Let us choose a subset of logical operations, $\overline{g}_i \in \mathcal{G} \neq S$, such that the parity-kick condition is satisified: $\{\overline{g}_i, E_j\} = 0$. For example, for the three-qubit code for phase flips, $\overline{Z} = X_1X_2X_3$, and as required for parity kicks, $\{\overline{Z}, \mathcal{E}_Z\} = 0$. Finally, another interesting possibility is to combine encoded BB pulses with the method of thermally suppressed DFS-encoded qubits [21].

Conclusions.—We have proposed a solution to problems of decoherence and gate implementation in quantum computation (QC) proposals governed by exchange Hamiltonians. Our solution combines ideas from the theory of decoherence-free subspaces [7–11], bang-bang controls [14–17], and encoded recoupling [19]. By encoding physical qubits into logical qubits, a first level of protection against collective dephasing is obtained. Control of an exchange Hamiltonian suffices for universal QC on this code. Cycles of pairs of BB pulses, generated from the *same exchange Hamiltonian,* can be used to eliminate all *leakage* errors on a logical qubit and to further suppress *all* other errors using two more BBs. In fact, the same Hamiltonian can be used to *create* the conditions of collective dephasing, using BB pulses [24]. We estimate that 10−1000 parity-kick cycles can be implemented in GaAs spin-coupled quantum dots within the required bath-correlation time. We further proposed an empirical BB method to determine the set of BB operations and to check the efficacy of the implemented BB procedure. Combining the BB method with encoding is general (see Theorem 2) and can be extended to other encodings, e.g., stabilizer codes [12]. In conjunction with the elimination of the need for difficult-to-implement single-qubit operations by the encoded recoupling method [19], we believe that we offer a realistic and comprehensive solution to some of the major difficulties related to decoherence in, and the design of, quantum dot, and other exchange-based quantum computers.

The present study was sponsored by the DARPA-QuIST program (managed by AFOSR under Agreement No. F49620-01-1-0468) and by PRO (to D. A. L.). We thank Dr. L.-A. Wu, Dr. K. Shiokawa, and Dr. S. Schneider for helpful discussions.

[1] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); J. Levy, Phys. Rev. A **64**, 052306 (2001).
[2] A. Imamoğlu *et al.,* Phys. Rev. Lett. **83**, 4204 (1999).
[3] B. E. Kane, Nature (London) **393**, 133 (1998); R. Vrijen *et al.,* Phys. Rev. A **62**, 012306 (2000).
[4] D. Mozyrsky *et al.,* Phys. Rev. Lett. **86**, 5112 (2001).
[5] M. I. Dykman *et al.,* Fortschr. Phys. **48**, 1095 (2000).
[6] G. M. Palma *et al.,* Proc. R. Soc. London A **452**, 567 (1996).
[7] P. Zanardi *et al.,* Phys. Rev. Lett. **79**, 3306 (1997).
[8] L.-M Duan and G.-C. Guo, Phys. Rev. A **57**, 737 (1998).
[9] D. A. Lidar *et al.,* Phys. Rev. Lett. **81**, 2594 (1998).
[10] D. A. Lidar *et al.,* Phys. Rev. Lett. **82**, 4556 (1999).
[11] D. Bacon *et al.,* Phys. Rev. Lett. **85**, 1758 (2000).
[12] D. Gottesman, Phys. Rev. A **54**, 1862 (1996).
[13] R. Laflamme *et al.,* Phys. Rev. Lett. **77**, 198 (1996).
[14] L. Viola and S. Lloyd, Phys. Rev. A **58**, 2733 (1998).
[15] D. Vitali and P. Tombesi, Phys. Rev. A **59**, 4178 (1999).
[16] P. Zanardi, Phys. Rev. A **60**, R729 (1999); L. Viola *et al.,* Phys. Rev. Lett. **82**, 2417 (1999).
[17] P. Zanardi, Phys. Lett. A **258**, 77 (1999).
[18] D. P. DiVincenzo *et al.,* Nature (London) **408**, 339 (2000); J. Levy, e-print quant-ph/0101057; J. Kempe *et al.,* Quant. Inf. Comp. **1**, 33 (2001).
[19] D. A. Lidar and L.-A. Wu, Phys. Rev. Lett. **88**, 017905 (2002).
[20] L.-A. Wu and D. A. Lidar, e-print quant-ph/0202135.
[21] D. Bacon *et al.,* Phys. Rev. Lett. **87**, 247902 (2001).
[22] S.-B. Zheng *et al.,* Phys. Rev. Lett. **85**, 2392 (2000).
[23] N. E. Bonesteel *et al.,* Phys. Rev. Lett. **87**, 207901 (2001); G. Burkard *et al.,* Phys. Rev. Lett. **88**, 047903 (2002).
[24] L.-A. Wu and D. A. Lidar, Phys. Rev. Lett. **88**, 207902 (2002); L. Viola, e-print quant-ph/0111167.
[25] L.-A. Wu *et al.,* e-print quant-ph/0202168.
[26] X. Hu *et al.,* e-print cond-mat/0108339.
[27] N. V. Prokof'ev and P. C. E. Stamp, Rep. Prog. Phys. **63**, 669 (2000).
[28] D. A. Lidar *et al.,* Chem. Phys. **268**, 35 (2001).
[29] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1993).
[30] J. M. Kikkawa *et al.,* Phys. Rev. Lett. **80**, 4313 (1998).
[31] G. Burkard *et al.,* Phys. Rev. B **59**, 2070 (1999).
[32] I. L. Chuang and M. A. Nielsen, J. Mod Opt. **44**, 2455 (1997).
[33] M. S. Byrd and D. A. Lidar, e-print quant-ph/0205156.
[34] C. Brif *et al.,* Phys. Rev. A **63**, 063404 (2001).