Entanglement evolution of a spin-chain bath coupled to a quantum spin

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For an electron spin in coupling with an interacting spin chain via hyperfine-type interaction, we investigate the dynamical evolutions of the pairwise entanglement of the spin chain, and a correlation function joined the electron spin with a pair of chain spins in correspondence to the electron-spin coherence evolution. Both quantities manifest a periodic and a decaying evolution. The entanglement of the spin bath is significant in distinguishing the zero-coherence status exhibited in periodic and decoherence evolutions of the electron spin. The periodical concurrence evolution of the spin bath characterizes the whole system in a coherence-preserving phase, particularly for the case that the associated periodic coherence evolution is predominated by zero value in the infinite chain-length limit, which was often regarded as the realization of decoherence.

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I. INTRODUCTION

Quantum coherence in terms of state superposition is one of the most important features of quantum mechanics. Usually the unavoidable environment would render the quantum object decohered. In most cases the environment can be efficiently modeled by an infinite number of noninteracting oscillators, i.e., the boson bath as in the Caldeira-Legget model.[1] The decohering environment can also be a spin bath. An interesting example is the so-called Coleman-Hepp model,[2] proposed in the study of quantum measurement. In this model, the apparatus is a one-dimensional chain with infinite number of noninteracting spins, which is used to measure and to collapse the superposed spin states of a relativistic electron.

The decoherence of an electron spin S induced by an environment E can be viewed as a quantum-measurement process. The electron spin in a state $|\phi^S\rangle = c_1|+\rangle + c_2|-\rangle$ is to be measured by E as the apparatus. The quantum dynamics of the universe, i.e., the closed system formed by S and E, is expected to result in the von Neumann's wave-packet collapse postulate[3,4]

$$\rho^S(t) = \mathrm{Tr}_E[\rho(t)] \to |c_1|^2|+\rangle\langle+| + |c_2|^2|-\rangle\langle-|,$$

where $\rho^S(t)$ is the reduced density matrix of S, and $\rho(t)$ is the evolving density matrix of the universe. The decoherence of S is actually the realization of wave-packet collapse in sense of von Neumann's postulate.

Since the universe is composed of two mutually interacting parts, S and E, the bath-traced electron-spin coherence should be complemented by a proper description of the environment status. The relation between the entanglement of the bath and the coherence of the system has stimulated extensive interest.[5,6] However, the dynamical evolution of the coherence status of the environment in correspondence to the coherence evolution (CE) of S has not been systematically explored. In this paper, we investigate the entanglement evolution of a spin-chain bath in driving the decoherence of a coupled electron spin. In fact, zero-coherence status can be realized in two kinds of qualitatively different electron-spin CEs, which exhibit different responses to the spin-echo effect.[7] It is then desirable to explore the underlying difference between these two coherence states and its implications by studying the status of the environment.

In this paper, we introduce the pairwise concurrence which measures the entanglement of the spin chain, and a joint-correlation function which correlates the system spin with a pair of bath spins. We find that the evolutions of both quantities exhibit critically different behaviors in correspondence to the different CEs of S. The apparent zero-coherence status that appeared in the two kinds of CEs prevails in the joint-correlation evolution but can be discriminated by the concurrence of the spin chain. In particular, corresponding to the periodic coherence evolution predominated by zero values, the bath chain maintains a stable entangled state with periodic concurrence evolution while the zero-coherence status appearing in the decoherence evolution corresponds to the bath chain with disentangled spin pairs. Moreover, the periodicity of bath concurrence persists even when the coherence exhibits a nondecaying irregularly oscillating evolution. The periodic concurrence evolution of the spin bath with nonzero amplitude characterizes the whole system in a coherence-preserving phase.

II. MODEL DESCRIPTION

The Hamiltonian for the universe,

$$H = H_S + H_E + H_I, \tag{1}$$

is composed of the system part $H_S = \epsilon_+|+\rangle\langle+| + \epsilon_-|-\rangle\langle-|$, the environment part $H_E = B/2\sum_{j=1}^{N-1}(I_j^+ I_{j+1}^- + I_j^- I_{j+1}^+)$, and the interaction part $H_I = \sum_{j=1}^{N} A_j S^z I_j^z$, which is the longitudinal hyperfine (HF)-type interaction[8] between S and E. Here j is the site index of the spin chain, $I_j^\pm = I_j^x \pm i I_j^y$, and $\mathbf{I}_j$ is the corresponding spin operator with $I = \frac{1}{2}$. $\epsilon_\pm$ are the Zeeman energies of the electron spin under applied magnetic field, B is the coupling constant of the nearest-neighbor chain spins, and A_j is the HF-type interaction strength between S and E, which varies from site to site on the spin chain.

We notice that the z component of the total bath spin $I^z = \Sigma_j I_j^z$ is a constant of motion of the whole system, $[I^z, H] = 0$. It is then physically reasonable to confine the bath states to the $I^z = 0$ section of the full Hilbert space. As a result, the contribution of the longitudinal HF-type interaction depends only on the differences of the local interaction strength, i.e., it keeps unchanged with the substitution $A_j \rightarrow A_j + \text{common const.}$ for all sites.[7] Moreover, the z component of the electron spin S^z is also a good quantum number and the total Hamiltonian is diagonal in the S^z representation as

$$H = |+\rangle\langle+|H_+ + |-\rangle\langle-|H_-, \tag{2}$$

with electron-spin-conditioned bath Hamiltonian

$$H_\pm = \epsilon_\pm + H_E \pm \sum_j \frac{A_j}{2} I_j^z. \tag{3}$$

We consider two kinds of HF-type interactions: (i) the linear form $A_j = (N-j)\Delta A + A_N$ with ΔA being a constant, and the resulted CE depending only on the ratio of the slope of the HF-type interaction and the intrabath interaction strength, $\xi = \frac{\Delta A}{B}$.[7] (ii) The cosine form $A_j = \mathcal{A} \cos^2 \frac{(j-1)\pi}{2N}$ with magnitude $\mathcal{A}$ as constant, which resembles the HF coupling in a quantum dot with A_j proportional to the norm of the electron wave function.[9] The former is a minimal model clarifying the underlying mechanism for decoherence while the latter is more realistic.

At time $t=0$, S and E are in pure states and are disentangled from each other as $\rho_0 = \rho_0^S \otimes \rho_0^E = (|\phi^S\rangle\langle\phi^S|) \otimes (|\Phi^E\rangle\langle\Phi^E|)$, where $|\phi^S\rangle = c_1|+\rangle + c_2|-\rangle$ and $|\Phi^E\rangle$ are the initial states of S and E, respectively. Without loss of generality, we take the initial bath state randomly selected and fully polarized as $|\Phi^E\rangle = |\uparrow\uparrow\downarrow\downarrow\cdots\rangle_{\{j_\alpha\}}$, where $\{j_\alpha\}$ denotes the initial bath spin configuration with M spin-up states on sites $j_1, j_2, \cdots, j_\alpha, \cdots, j_M$ and spin-down states on other sites. At time t, the density matrix of the universe is $\rho(t) = U(t)\rho_0 U^\dagger(t)$ with $U(t)$ as the evolution operator of the universe. In the reduced density matrix of the system

$$\rho^S(t) = |c_1|^2|+\rangle\langle+| + |c_2|^2|-\rangle\langle-| + c_1 c_2^*|+\rangle\langle-|\rho_{+,-}^S(t) + \text{H.c.}, \tag{4}$$

the off-diagonal element,

$$\rho_{+,-}^S(t) = \text{Tr}[\rho(t)S^-] = \langle\psi_-(t)|\psi_+(t)\rangle, \tag{5}$$

measures the coherence of S, where $|\psi_\pm(t)\rangle = e^{-iH_\pm t}|\Phi^E\rangle$ is the wave function of the spin chain conditioned on the electron-spin states.

Under the Jordan-Wigner transformation,[10] $I_j^+ = c_j^\dagger \exp(i\pi\Sigma_{l=1}^{j-1} c_l^\dagger c_l)$, I_j^- its Hermitian conjugate, and $I_j^z = c_j^\dagger c_j - \frac{1}{2}$, with $c_j^\dagger$ and c_j as the creation and annihilation spinless fermionic operators on site j, the initial bath state reads $|\Phi^E\rangle = \Pi_{j_\alpha} c_{j_\alpha}^\dagger |0\rangle$, the Hamiltonian $H_\pm$, and the coherence $\rho_{+,-}^S(t)$ take the forms

$$H_\pm = \tilde{\epsilon}_\pm + \sum_{j,j'=1}^N c_j^\dagger h_{j,j'}^\pm c_{j'},$$

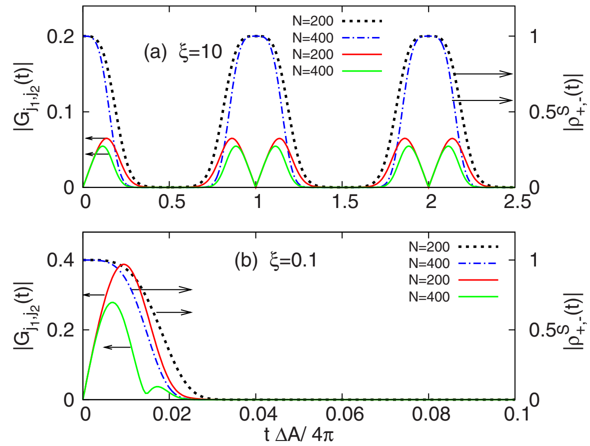

FIG. 1. (Color online) Joint-correlation evolution $|G_{j_1,j_2}(t)|$ for (a) $\xi=10$ and (b) $\xi=0.1$ with $N=200,400$, and $j_1=N/2$, $j_2=N/2+1$. Corresponding CE $|\rho_{+,-}^S(t)|$ are plotted for comparison. In (a), $|G_{j_1,j_2}(t)|$ is periodic with period π. The zero-value regimes coincide with those for CE. In (b), after a peak close to $t=0$, $|G_{j_1,j_2}(t)|$ dies away completely.

$$\rho_{+,-}^S(t) = e^{i\varepsilon(\tilde{\epsilon}_- - \tilde{\epsilon}_+)t} \det[\chi(t)_{j_\beta,j_{\beta'}}], \tag{6}$$

where $h_{j,j'}^\pm = \pm\frac{A_j}{2}\delta_{j,j'} + \frac{B}{2}(\delta_{j,j'+1} + \delta_{j,j'-1})$, $\tilde{\epsilon}_\pm = \epsilon_\pm \mp \frac{1}{4}\Sigma_j A_j$, and $\chi(t)_{j_\beta,j_{\beta'}} = [e^{ih^- t} e^{-ih^+ t}]_{j_\beta,j_{\beta'}}$ with $j_\beta, j_{\beta'} \in \{j_\alpha\}$.

III. JOINT-CORRELATION FUNCTION

We consider first the joint-correlation function $G_{j_1,j_2}(t) = \text{Tr}[\rho(t)S^+ I_{j_1}^+ I_{j_2}^-] = \langle\psi_+(t)|I_{j_1}^+ I_{j_2}^-|\psi_-(t)\rangle$, which embodies both the information of the system S via S^+ and that of the bath spins via $I_{j_1}^+ I_{j_2}^-$. We choose j_1, j_2 as a pair of nearest-neighbor sites with opposite initial spins. Time-dependent density-matrix renormalization-group method[11] is employed for the calculation of $G_{j_1,j_2}(t)$.

For linear HF-type interaction, we normalize the time variable t by $4/\Delta A$. Our calculation shows that the joint-correlation evolution exhibits the same two kinds of qualitatively different evolutions as the corresponding CEs of S,[7] a periodic evolution and a decaying evolution, see Fig. 1. For $\xi=10$, $G_{j_1,j_2}(t)$ is typically periodic with period π while for $\xi=0.1$ it decays monotonically after a narrow peak close to $t=0$. As the chain size increases, the zero-value intervals in the periodic evolution keep extending and the peak width shrinking while the peak in the decaying evolution moves closer to $t=0$.

It is interesting to notice that the zero-value time regimes for $G_{j_1,j_2}(t)$ coincide with those in CE of S not only in the decaying evolution but also in the periodic evolution. This is excellently in consistence with Coleman-Hepp's argument for the quantum-measurement theory.[2] In the zero-coherence regime, the two pointer states $|\psi_\pm(t)\rangle$ "remain orthogonal after any operation involving only finitely many lattice points" in the limit of infinite chain length. We have here $I_{j_1}^+$ and $I_{j_2}^-$ as the local operators.

IV. PAIRWISE CONCURRENCE OF THE SPIN CHAIN

We next investigate the nearest-neighbor pairwise entanglement of the environment in correspondence to the CE of S. The dynamical entanglement of the spin chain is driven by not only the intrabath spin-spin interaction but also the inhomogeneous interaction between S and E. For $I=\frac{1}{2}$ spins, the *concurrence* $\mathcal{C}$ gives a proper measure of the amount of the pairwise entanglement,[12] which varies from $\mathcal{C}=0$ for a separable state to $\mathcal{C}=1$ for a maximally entangled state.

The pairwise concurrence for chain sites j_1 and j_2 can be calculated from the corresponding reduced density matrix $\rho_{j_1 j_2}=\mathrm{Tr}_S[\mathrm{Tr}_{j\backslash_1 j\backslash_2}\rho(t)]$, where $\mathrm{Tr}_{j\backslash_1 j\backslash_2}$ denotes tracing over the spin chain except sites j_1 and j_2. Noting that $I^z=\Sigma_{j=1}^{N} I_j^z$ is a good quantum number, $\rho_{j_1 j_2}$ takes the form

$$\rho_{j_1 j_2}=\begin{pmatrix} u_1 & 0 & 0 & 0 \\ 0 & w_1 & z & 0 \\ 0 & z^* & w_2 & 0 \\ 0 & 0 & 0 & u_2 \end{pmatrix} \tag{7}$$

in the standard basis $|\uparrow_{j_1}\uparrow_{j_2}\rangle$, $|\uparrow_{j_1}\downarrow_{j_2}\rangle$, $|\downarrow_{j_1}\uparrow_{j_2}\rangle$, and $|\downarrow_{j_1}\downarrow_{j_2}\rangle$, and the corresponding concurrence is[13]

$$\mathcal{C}(t)=2\max[0,|z|-\sqrt{u_1 u_2}]. \tag{8}$$

The entities of matrix (7) are two-point correlation functions. In the fermionic representation their exact expressions can be derived as $u_1=|c_1|^2\det[\gamma^{(+)}]+|c_2|^2\det[\gamma^{(-)}]$, $u_2=1-\mathrm{Tr}[|c_1|^2\gamma^{(+)}+|c_2|^2\gamma^{(-)}]+u_1$, and $z=|c_1|^2\gamma^{(+)}_{j_1,j_2}+|c_2|^2\gamma^{(-)}_{j_1,j_2}$, where $\gamma^{(\pm)}_{j,j'}=\Sigma_{j_\beta\in\{j_\alpha\}}[e^{-ih^{\pm}t}]_{j,j_\beta}[e^{ih^{\pm}t}]_{j_\beta,j'}$ with $j,j'\in\{j_1,j_2\}$.

For linear HF-type interaction, for the case of ΔA much larger than B, the transverse correlation $z=\langle I^+_{j_1}I^-_{j_2}\rangle$ dominates the longitudinal parts $u_{1,2}=\langle(1/2\pm I^z_{j_1})(1/2\pm I^z_{j_2})\rangle$. The concurrence takes value $2|z|$ and is also periodic with the same period as that of CE of S, see Fig. 2(a). It takes zero values only at $t=n\pi$, $n=0,1,2,\ldots$, and keeps nonzero within each period. For small ξ, the longitudinal correlation overwhelms its transverse counterpart, the pairwise bath concurrence exhibits an extremely narrow peak close to $t=0$ and then dies away completely, see Fig. 2(b).

We notice that, in the periodic CE, the zero-coherence interval in each period extends with the increase in the chain length, see the dashed lines in Fig. 2(a). In the limit $N\to\infty$, it predominates the whole evolution and the periodical coherence revival shrinks into instantaneous pulses with zero width. This was often understood as a complete decoherence.[4,6] However, the bath concurrence $\mathcal{C}$ is maintained as nonzero in these zero-coherence time intervals, and is chain-length independent, as shown by the solid lines in Fig. 2(a). In other words, even when the periodic coherence evolution is predominated by zero value, the bath still keeps a stable entangled state. Taking into consideration the periodically reviving coherence evolution, this persistently entangled bath state suggests that the whole universe is in a coherence-preserving phase.

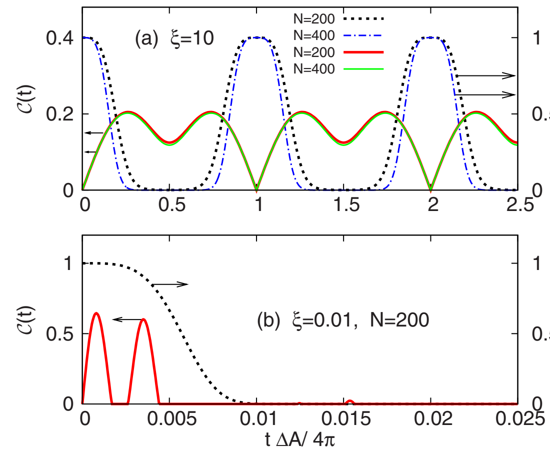

FIG. 2. (Color online) Solid lines are concurrence evolution $C(t)$ for (a) $\xi=10$ and (b) $\xi=0.01$ with $j_1=N/2$ and $j_2=N/2+1$. The electron-spin initial state is taken as $c_1=1/\sqrt{3}$, and $c_2=\sqrt{6}/3$. Dotted lines are correspondingly plotted for comparison. In (a), $\mathcal{C}(t)$ for $N=200$, and $N=400$ almost overlap with each other. The evolutions are periodic with period π and keep nonzero value except at $t=n\pi$, $n=0,1,2,\ldots$. In (b), $N=200$, after two narrow peaks close to $t=0$, $C(t)$ dies away completely.

V. HF-TYPE INTERACTION IN COSINE FORM

Now we consider the HF-type interaction in cosine form. The joint-correlation and bath concurrence again exhibits two types of evolutions, see Fig. 3. When the intrabath interaction B is large, these two quantities exhibit the same decaying behaviors as their counterparts in the case of linear HF-type interaction, respectively. For small B, unlike the nondecaying irregularly oscillating CE, interestingly, the

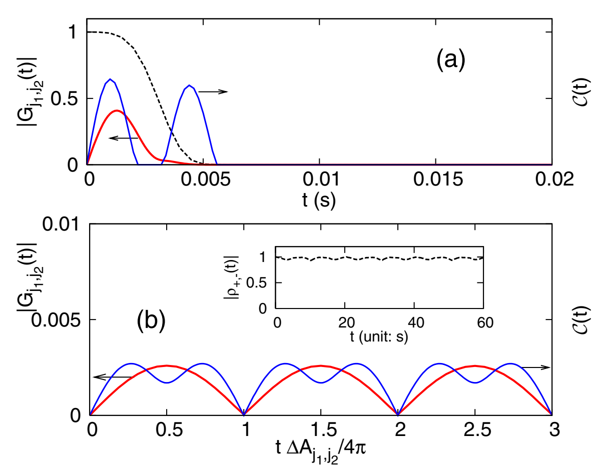

FIG. 3. (Color online) Joint-correlation and bath concurrence evolutions for $A_j=\mathcal{A}\cos^2\frac{(j-1)\pi}{2N}$, $N=200$, with $\mathcal{A}=10^4 s^{-1}$, $j_1=N/2$, and $j_2=N/2+1$. Dotted lines are corresponding CEs plotted for comparison. (a) For $B=10^3$ s^{-1}, both $|G_{j_1,j_2}(t)|$ and $\mathcal{C}(t)$ exhibit zero value after a narrow peak(s) close to $t=0$. The corresponding $|\rho^S_{+,-}(t)|$ decays monotonically. (b) For $B=0.1$ s^{-1}, $|G_{j_1,j_2}(t)|$ and $\mathcal{C}(t)$ both exhibit periodic evolution. The period is π with time normalized by local difference of the HF-type coupling strength $\Delta A_{j_1,j_2}$. The coherence (in inset) exhibits a nondecaying irregularly oscillating evolution.

evolutions of both $G_{j_1,j_2}(t)$ and $\mathcal{C}(t)$ are still periodic. If we normalize the time variable t by the local difference of the HF-type interaction strength at the two sites $\Delta A_{j_1,j_2}$, the resulting period π is exactly the same as in the periodic evolution with linear HF-type interaction. The scale introduced by the two bath spins picks out a periodic component hidden in the irregularly oscillating CE of S, and shows up in the joint-correlation and bath concurrence evolutions.

VI. DECOUPLING PHENOMENA

The decoherence effect is due to the temporally fluctuating random magnetic field exerted on the electron spin, originated from the fluctuations of the surrounding bath spin pairs. If the intrabath interaction increases, the decoherence will be enhanced owing to the increased bath spin fluctuations. However, as can be seen from Eq. (5) and its relevant formula, when the S-E interaction becomes negligible with respect to the intrabath interaction, the electron spin evolves independently, in decoupling from the bath, and its coherence takes constant value $\lim_{\xi\to 0}\rho^S_{+,-}(t)=1$. In correspondence to the above intuition, our calculation shows that both $\rho^S_{+,-}(t)$ and $G_{j_1,j_2}(t)$ manifest a continuous transition from the decaying evolution to the decoupling evolution, see Fig. 4. The bath concurrence, on the other hand, is in association only with the quantum fluctuations of the bath spins although the latter is driven by the full Hamiltonian of the universe. The decaying evolution of the bath concurrence therefore exhibits a scaling behavior in its dependence on the intrabath coupling B as $\mathcal{C}(t)\sim\mathcal{C}(Bt)$ while the scaling behavior for $G_{j_1,j_2}(t)$ only appears in the decoupling regime. The decoupling phenomenon occurs for HF-type interaction in both linear and cosine forms, and thus is a generic feature in the large intrabath interaction limit.

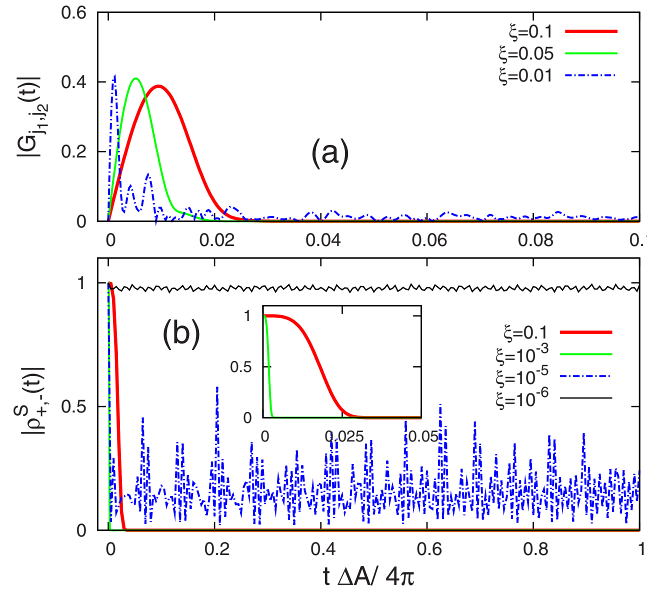

FIG. 4. (Color online) $N=200$, $j_1=N/2$, and $j_2=N/2+1$. (a) Transition behavior of joint-correlation from decaying to decoupling evolutions. For $\xi=0.1, 0.05$ lying in the parameter regime for decaying evolutions, $|G_{j_1,j_2}(t)|$ exhibits a single peak and then monotonically dies away. For $\xi=0.01$, the evolution starts oscillating after the single peak, which is a kind of evidence that the evolution is in the decoupling regime. (b) Decoupling behavior of CE. For $\xi=0.1$ and 10^{-3} in the decoherence regime, the coherence decays monotonically, see the inset for detail. For $\xi=10^{-5}$, oscillation appears during the decay. The deviation from the monotonic decay appears as a transition to the decoupling evolution, which occurs approximately at $\xi\sim 0.01/N\sim 10^{-5}$. For $\xi=10^{-6}$, as a typical decoupling behavior, $|\rho^S_{+,-}(t)|$ fluctuates around the limit $\lim_{\xi\to 0}|\rho^S_{+,-}(t)|=1$.

VII. CONCLUDING REMARKS

In summary, the bath entanglement plays a significant role in understanding the coherence status of the spin-bath system. In particular, the subtle difference between the zero-coherence status exhibited in two kinds of different electron-spin coherence evolutions can be discriminated by the entanglement status of the spin chain. The periodic coherence evolution predominated by zero-coherence and the non-decaying irregularly oscillating coherence evolution are both associated with a periodic evolution of the bath entanglement, which reveals the coherence-preserving nature of the whole system and is a kind of headspring of the spin-echo effect. The zero coherence appearing in the decoherence evolution corresponds to an environment with disentangled spin pairs. Any disturbance to the spin on one site would have no affect to its neighboring spins. The environment is in this sense silent and the whole system is in a true coherence collapse state. As a ramification, apparently, both the zero-coherence status in the two different parameter regimes meet the requirement of von Neumann's postulate. Yet they correspond to qualitatively different entanglement status of the environment. This brings up a question of whether a quantum measurement of the system should refer to the environment status. It is expected to stimulate further understanding of decoherence as well as quantum-measurement theories.

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