Dynamical Decoupling and Quantum Error Correction Codes

Gerardo A. Paz-Silva and Daniel Lidar

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GAPS and DAL paper in preparation

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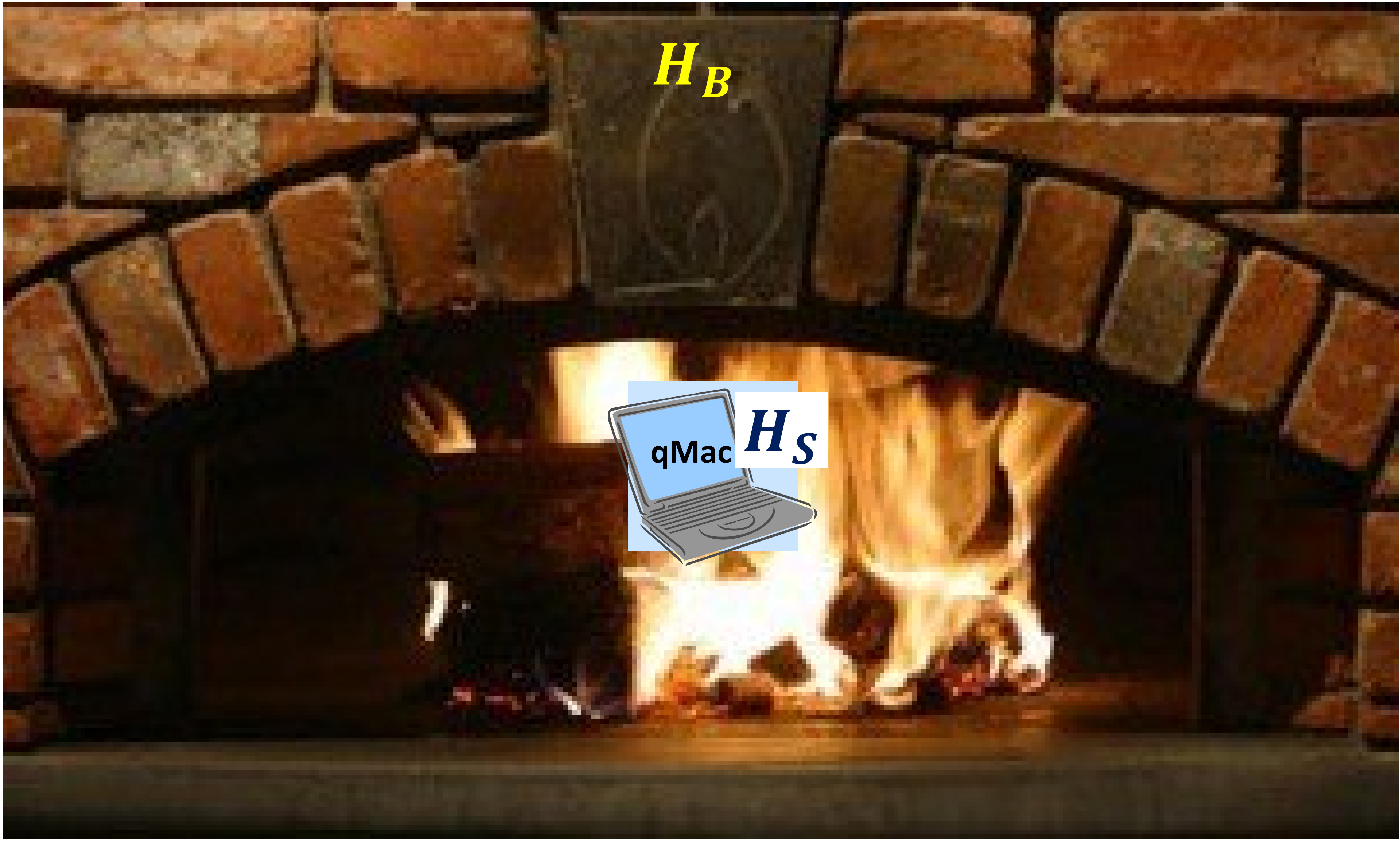

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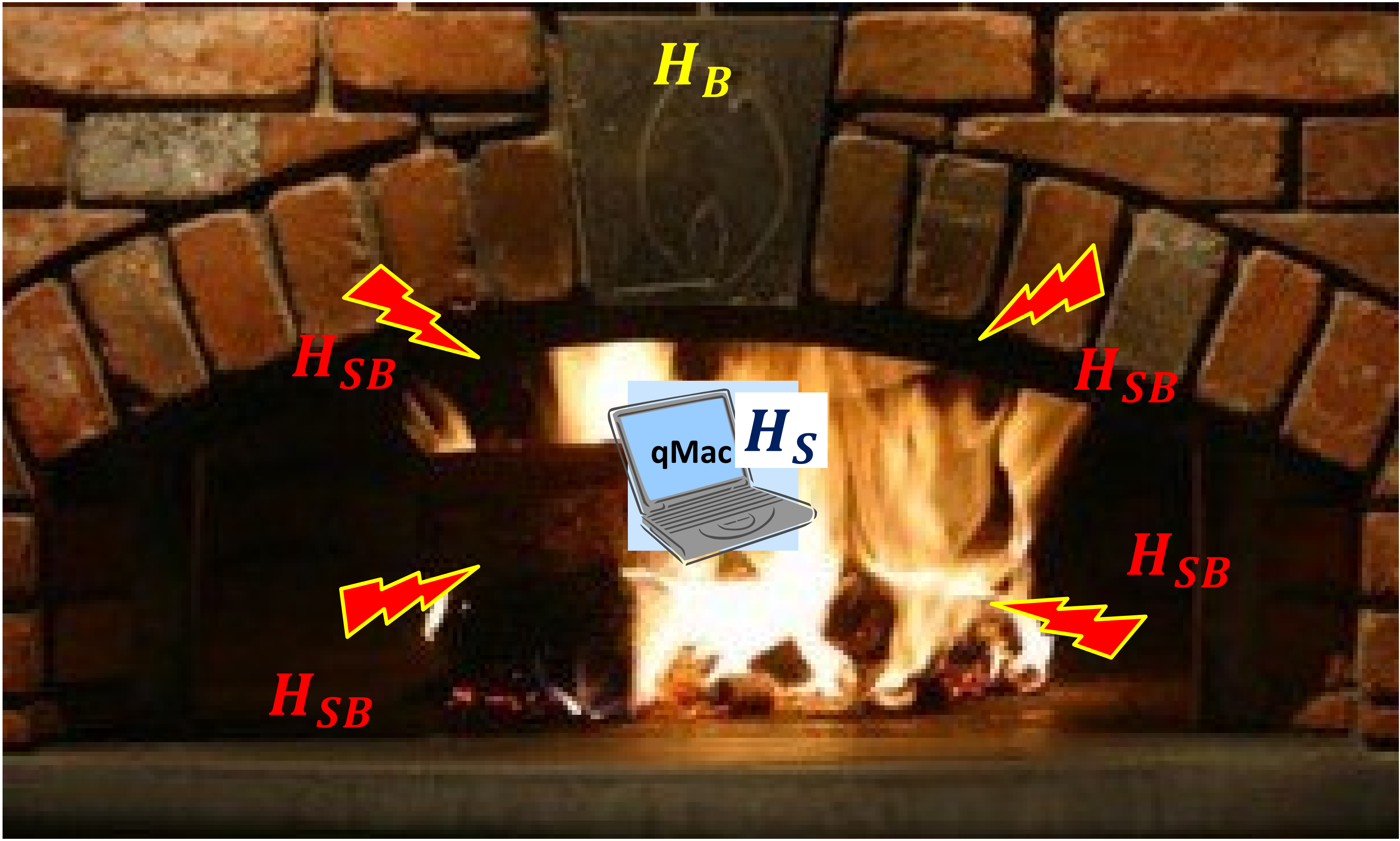

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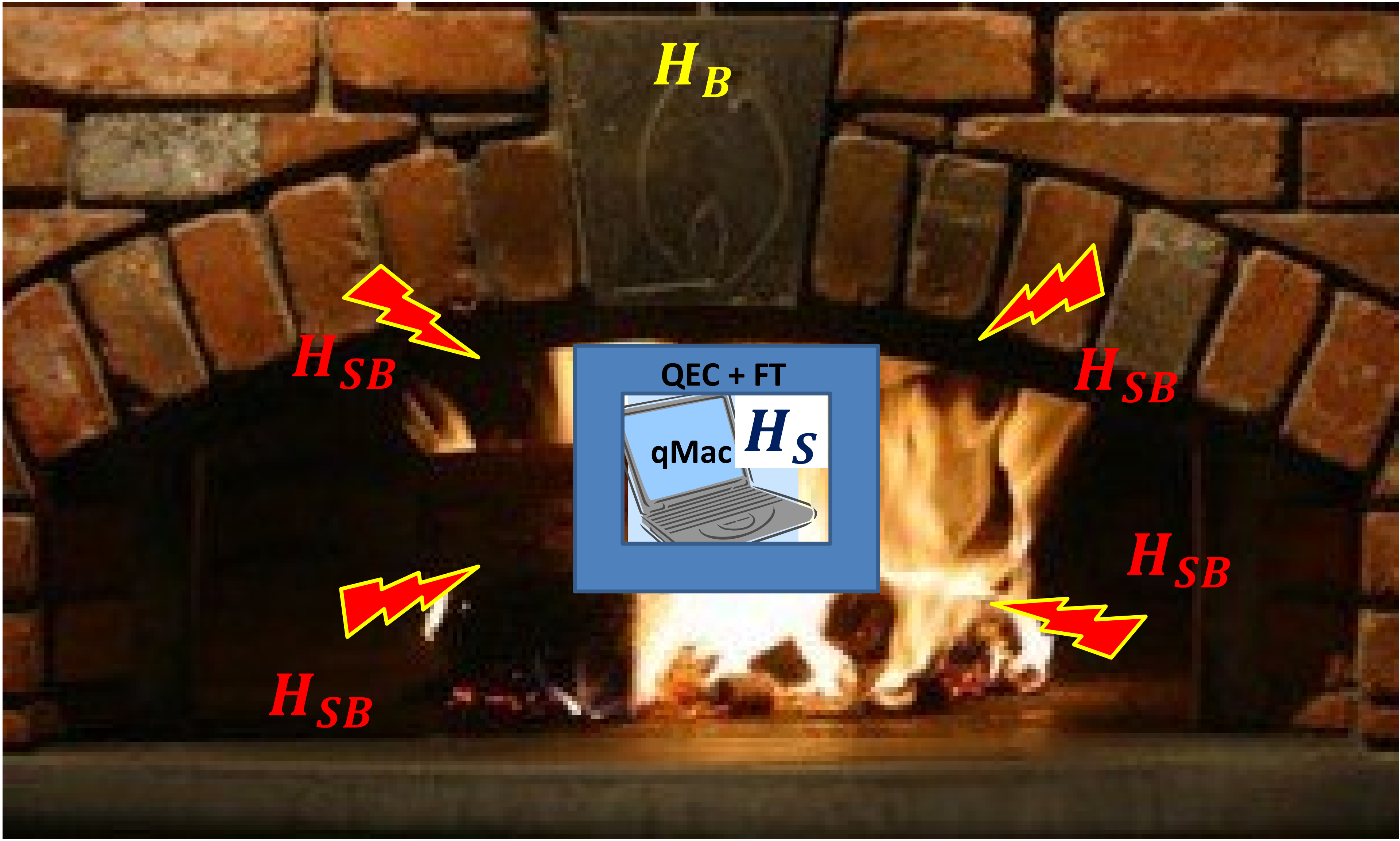

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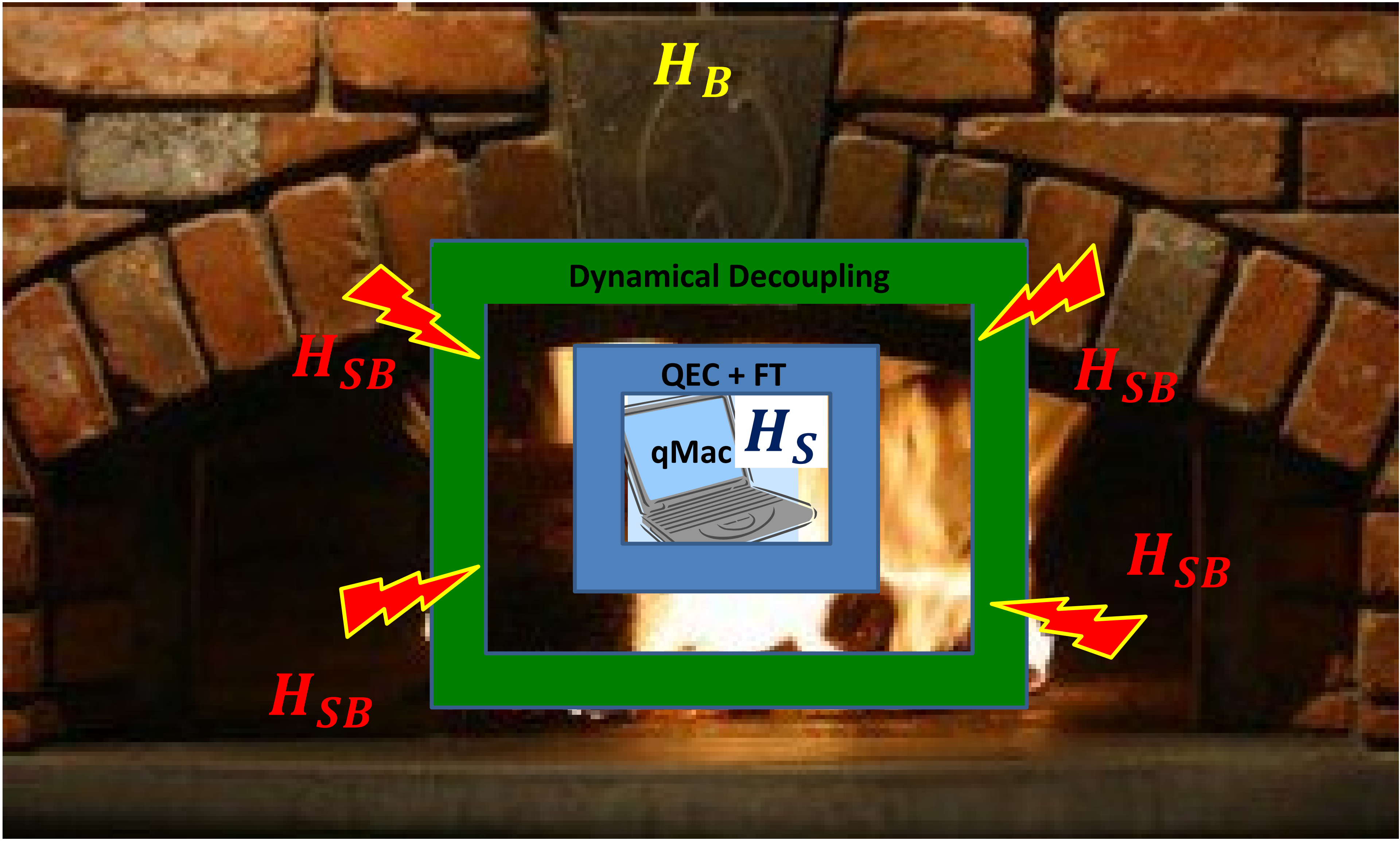

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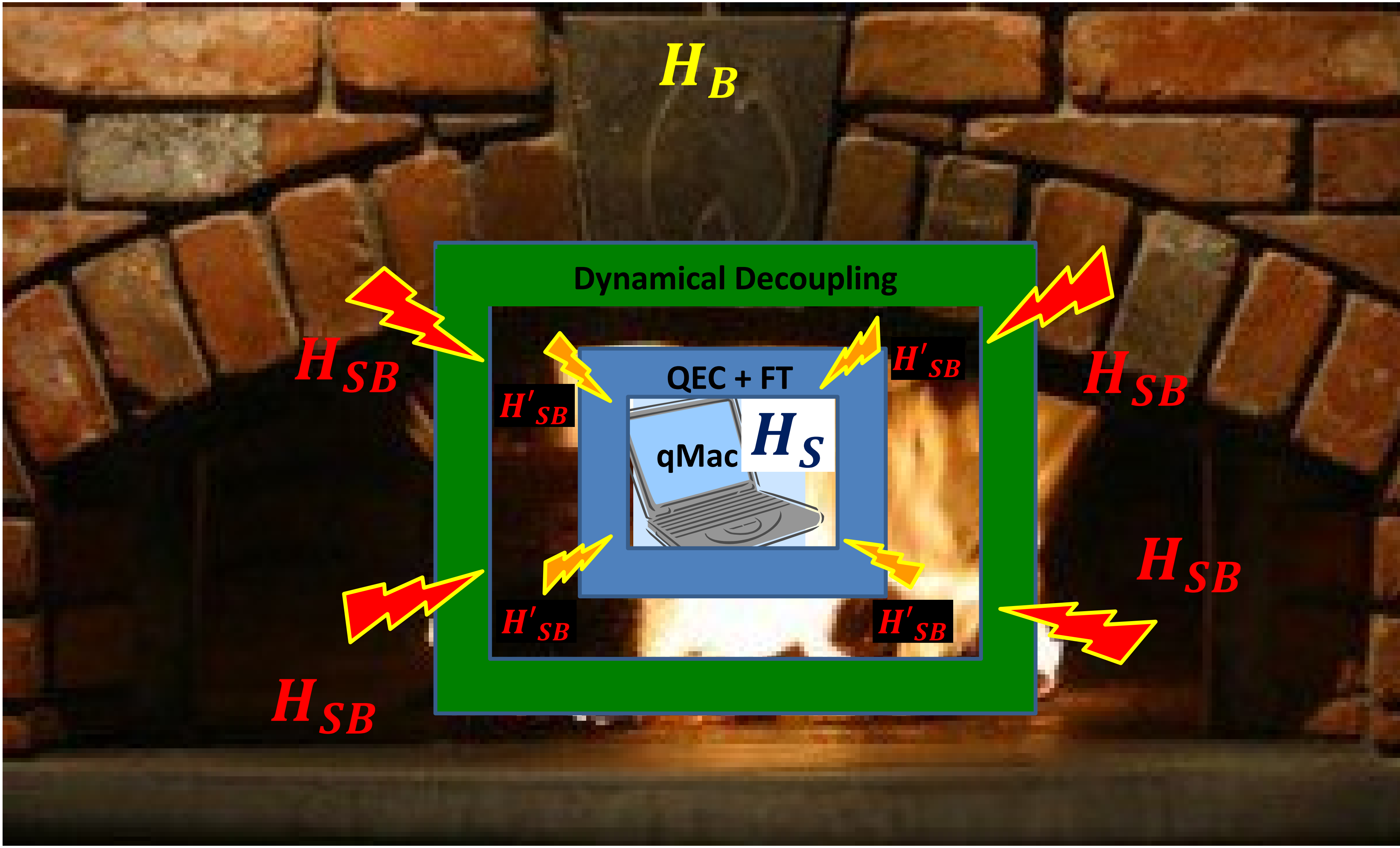
H_B
Dynamical Decoupling
QEC + FT
qMac
H_S
H'_{SB}
H'_{SB}
H'_{SB}
H'_{SB}
H_{SB}
H_{SB}
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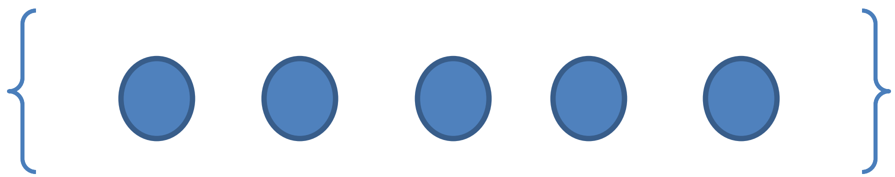
$$H = I \otimes H_B + H_{SB} \rightarrow U(\tau_{min}) = e^{-i(H\,\tau_{min})}$$

$$\overline{\eta_0} = \|H_{SB}\|\,\tau_{min}$$



[[n,k,d]] QEC code

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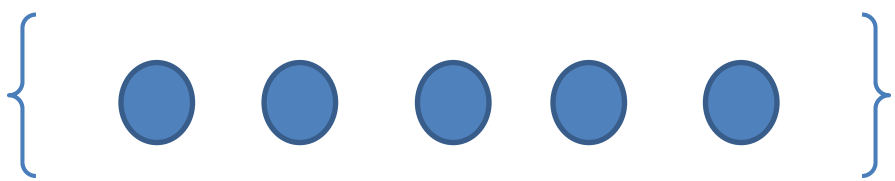

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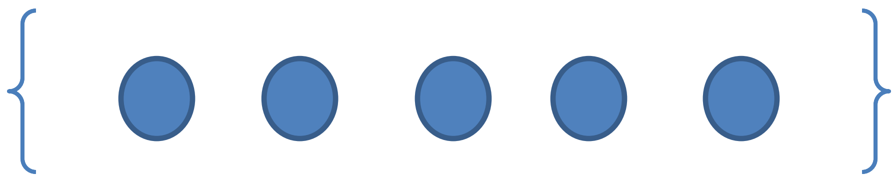

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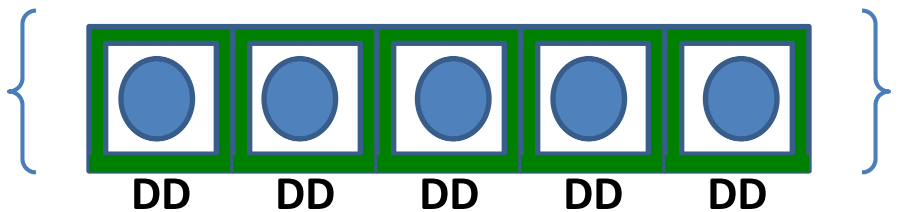

*Ng,Lidar,Preskill PRA 84, 012305(2011)*

- Enhanced fidelity of *physical gates* via appended DD sequences

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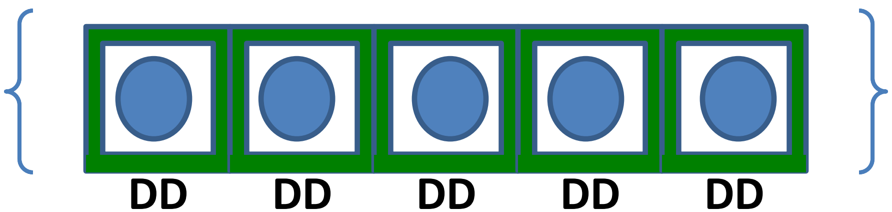

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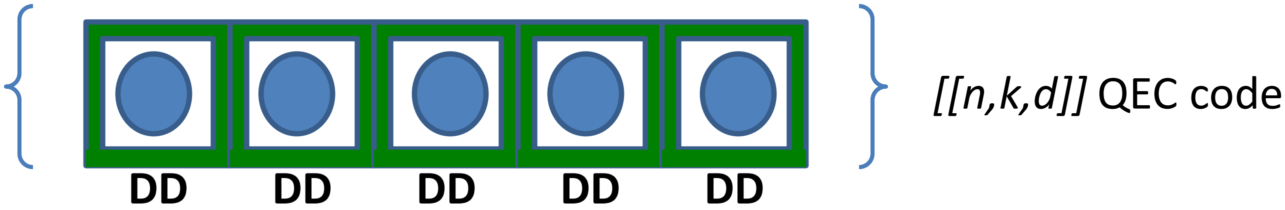

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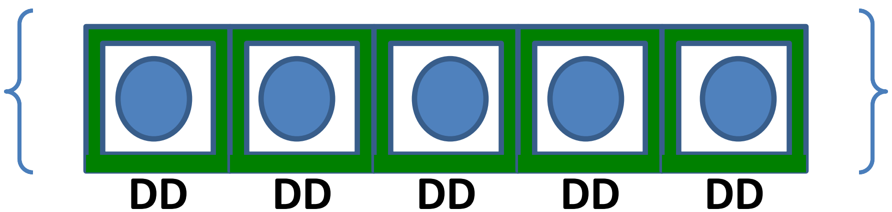

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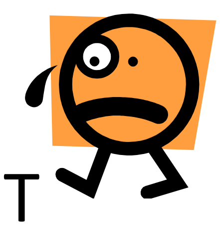

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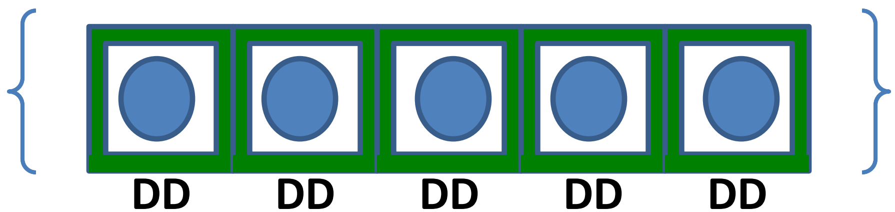

[[n,k,d]] QEC code

<u>n –qubit Pauli basis as decoupling group</u> → *No ‘local bath assumption’*

- Length of sequence exponential in *2n*
- Pulses look like errors to the code → limits possible integration with other schemes

Desiderata for DD +QEC:

I. No extra locality assumptions

II. Pulses in the code

III. Shorter sequences than full decoupling approach.

$$\overline{\eta_{DD}} < \overline{\eta_0}$$

The magic is in the decoupling group

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Too small → No arbitrary order decoupling
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- Mutually Orthogonal Operator (generator) Set = $\{\Omega_i\}_{i=1,\ldots,K}$

$$(\Omega_i)^2 = I$$

$$\Omega_i\,\Omega_j = (-1)^{f(i,j)}\Omega_j\,\Omega_i; \quad f(i,j) = \{0,1\}$$

$$\Omega_i\,\Omega_j \neq \Omega_k$$

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Concatenated Dynamical Decoupling **(CDD)**
[Khodjasteh and Lidar, Phys. Rev. Lett. 95, 180501 (2005)]

{ Pulses → **<MOOS>** $(2^K)^N$ **pulses** }

Nested Uhrig Dynamical Decoupling **(NUDD)**
[Wang and Liu, Phys. Rev. A 83, 022306 (2011)]

{ Pulses → **MOOS** $(N+1)^K$ pulses }

What we propose…

- Stabilizer generators = $\{S_i\}_{i=1,\ldots,Q}$

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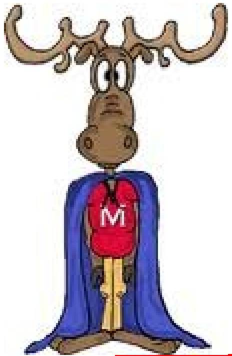


$$\text{MOOS} = \{S_i\}_{i=1,\ldots,Q} \cup \{ X_i^{(L)}, Z_i^{(L)}\}_{i=1,\ldots,k}$$

$$U_{DD}(T) = e^{-i(H_{\emptyset,eff} O(T) + H_{SB,eff}\, O(T^{N+1}))}$$

$$H_{\emptyset,eff} \propto \{S_i\}_{i=1,\ldots,Q}$$

Contains no physical or logical errors ! Only harmless terms !

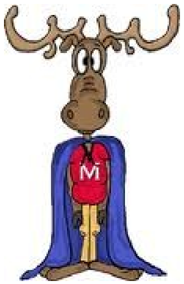
Even if H_{SB} is a logical error!

What do we gain ?

✓ No extra locality assumptions: The DD group is powerful enough.

CDD: → NO higher order Magnus term is UNDECOUPLABLE and HARMFUL
→ The next level of concatenation can deal with it

NUDD: (See proof in W.-J. Kuo, GAPS, G. Quiroz, D. Lidar in preparation)



1 - 0 H_{SB}

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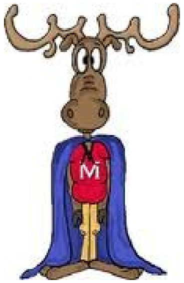

2 - 0 H_{SB}

✓ DD Pulses are bitwise / transversal in the code

Pulses do not look like errors to the code
→ Allows interaction with other protection schemes.

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What else do we gain ?

- Shorter sequences than full decoupling approach:

For stabilizer/subsystem codes: MOOS : *n-k-g* stabilizers + *2k* logical operators

$$CDD_{(<\Omega_i>,N)} \rightarrow 2^{(n+k-g)N} < 2^{2nN}$$
$$NUDD_{(\{\Omega_i,N\})} \rightarrow (N+1)^{n+k-g} < (N+1)^{2n}$$

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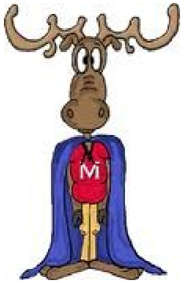
e.g. $[[n^2, 1, n]]$ Bacon Shor code:

Stabilizer generators: 2(*n-1)*
Logical generators: 2

	SXDD	Full decoupling
D(MOOS)	$2\,n$	$2\,n^2$
NUDD	$(N+1)^{2\,n}$	$(N+1)^{2\,n^2}$
CDD	$2^{2\,n\,N}$	$2^{2\,n^2 N}$



3 - 0 H_{SB}

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$\overline{\eta_{DD}} < \overline{\eta_0}$?

✓ Recall our (effective) noise rates:

$$\overline{\eta_0} = \|H_{SB}\, \tau_{min}\|$$

$$\overline{\eta_{DD}}(N) = \left\|H_{SB,eff}\; O(T^{N+1})\right\|$$

✓ Are an overestimation:

→ bounds obtained without using the QEC code structure. (work in progress)

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How to compute $\left\|H_{SB,eff}\; O(T^{N+1})\right\|$?? → NLP results (Eqs. 152-164)

Recursive relations for $\left\|H_{SB,eff}(q)\right\|$ and $\left\|H_{\emptyset,eff}(q)\right\|$ at every degree of concatenation q.

$$\left\|H_{SB,eff}(q)\right\| \;\leq\; R^{q(q+3)/2}\,(\bar{c}\,\|H_{SB}+H_B\|\tau_0)^{q-1}\|H_{SB}\|$$
$$T(q) = R^q\tau_{min}$$

where $R = 2^{D(MOOS)}$ and $\bar{c}\sim 1$

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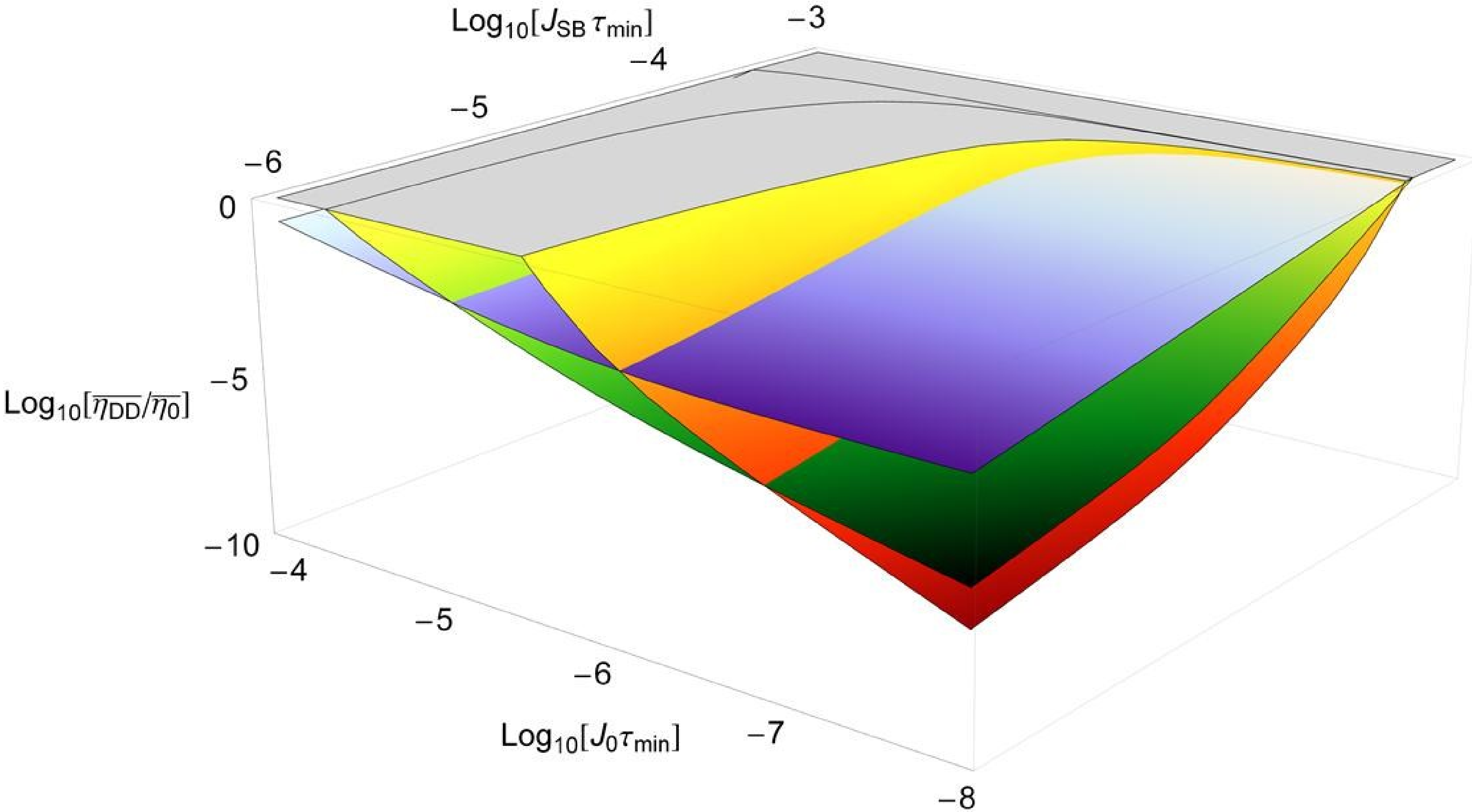

N=1
N=2
N=3

$\|I \otimes H_B\| = J_0$

$\|H_{SB}\| = J_{SB}$

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[[9,1,3]] – BS code: $\tau_{min} = 1\,;D(MOOS) = 4 + 2$

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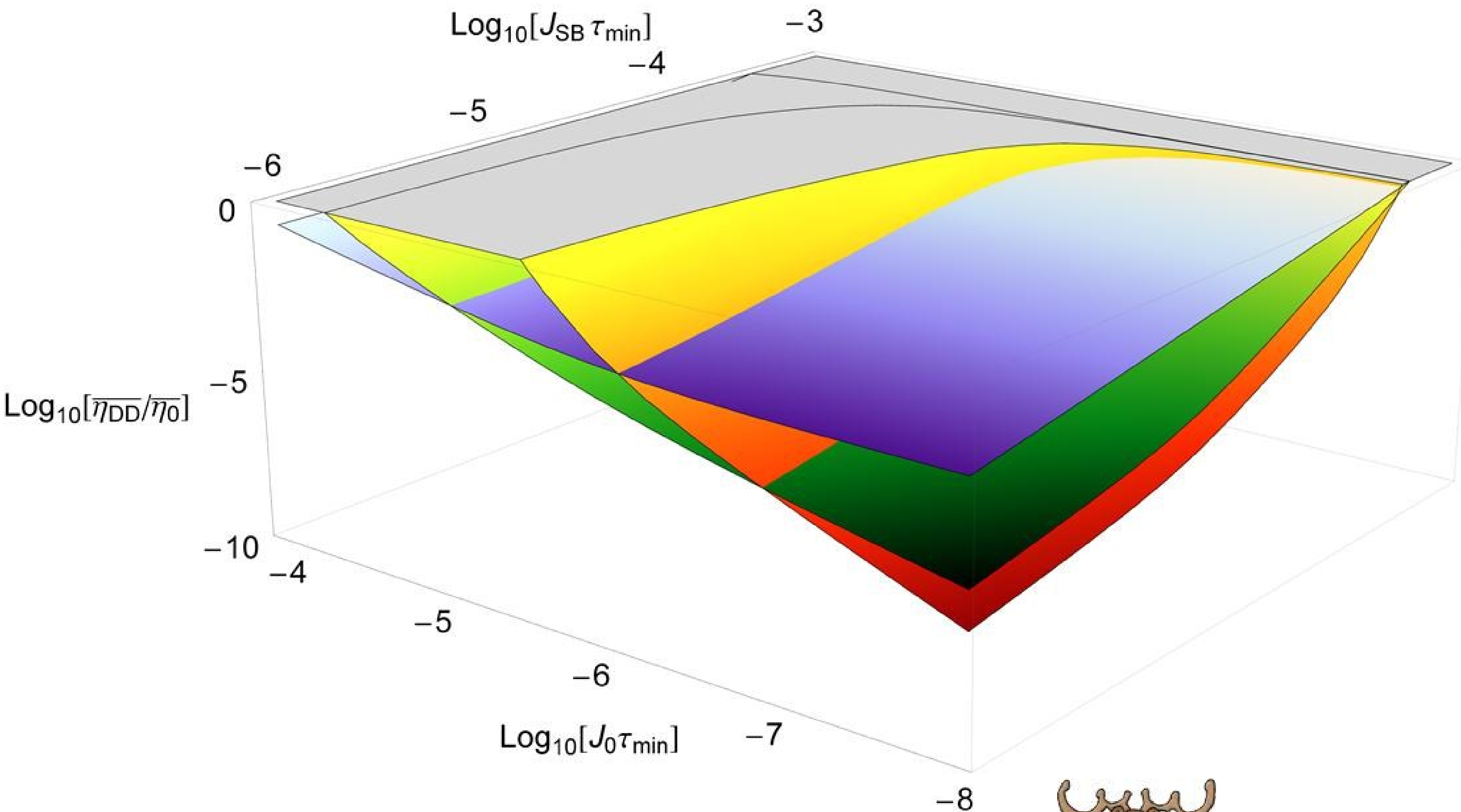

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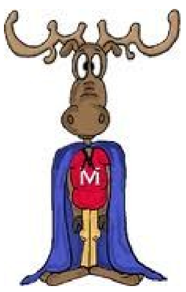

4 - 0 $\boldsymbol{H_{SB}}$

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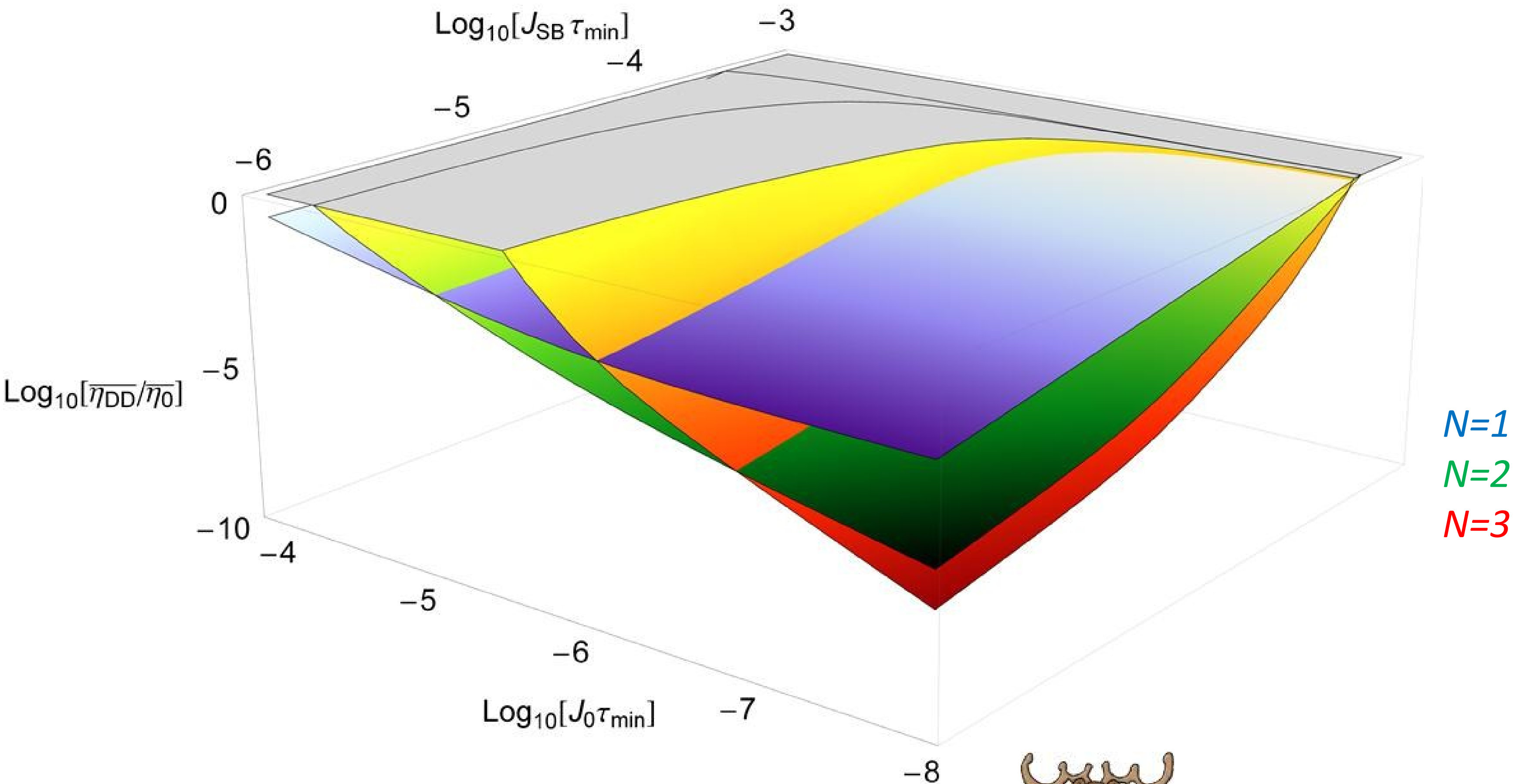

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Beyond $H_S = 0$

✓ DD-based methods for fidelity enhanced gates can be directly ported:

- Dynamically protected gates: works for both CDD and NUDD
  Append SXDD sequence to a gate.
  [NLP, PRA 84, 012305(2011)]

- (Concatenated) Dynamically corrected gates: based on CDD
  Eulerian cycle on the Caley graph of DD group

[Khodjasteh and Viola, PRL 102, 080501 (2009)]
[Khodjasteh, Lidar, Viola, PRL 104, 090501 (2010)]

5 - 1 H_{SB}

Conclusions

- We have shown how to integrate dynamical decoupling and quantum error correction codes in a ‘natural’ way.

  - ✓ No extra locality conditions
  - ✓ Pulses in the code.
  - ✓ Shorter sequences than full decoupling approach.
  - ✓ Improve effective error rates AND deal where Hamiltonians QEC fails.

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THANKS! QUESTIONS ?