Fault-tolerant quantum computation via adiabatic holonomies

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Second International Conference on Quantum Error Correction, 2011

Outline

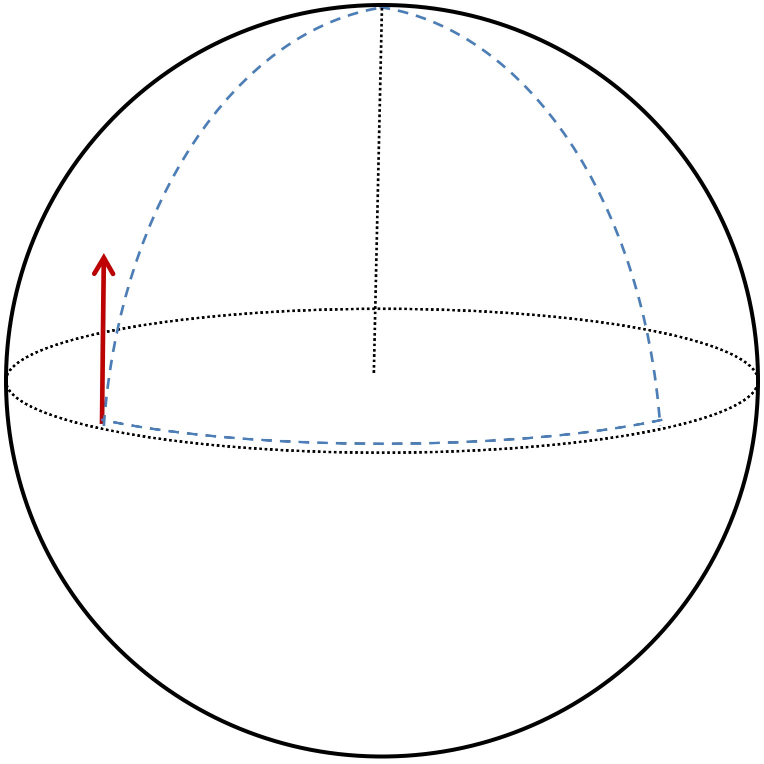
- Holonomic quantum computation (HQC)
- HQC in subsystems
- Two approaches to fault-tolerant HQC

  1) no extra qubits, but Hamiltonians depend on the error-correcting code

  2) extra (noisy) qubits needed, but Hamiltonians independent of the code
- FTHQC with 2-qubit Hamiltonians
- Related schemes
- Conclusion and outlook

Geometric phases and HQC

Parallel transport of a vector on a curved surface (example):



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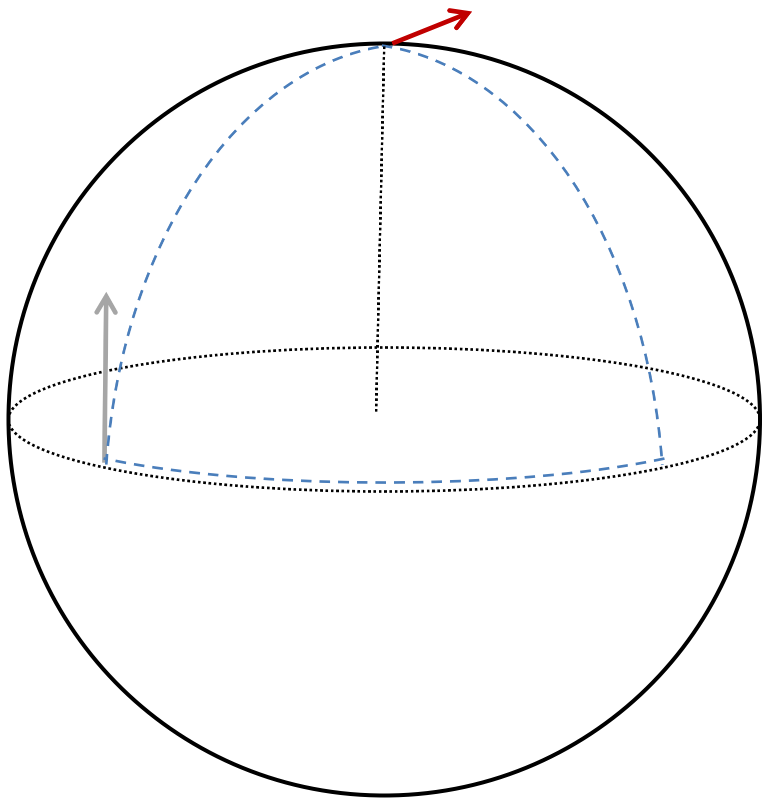

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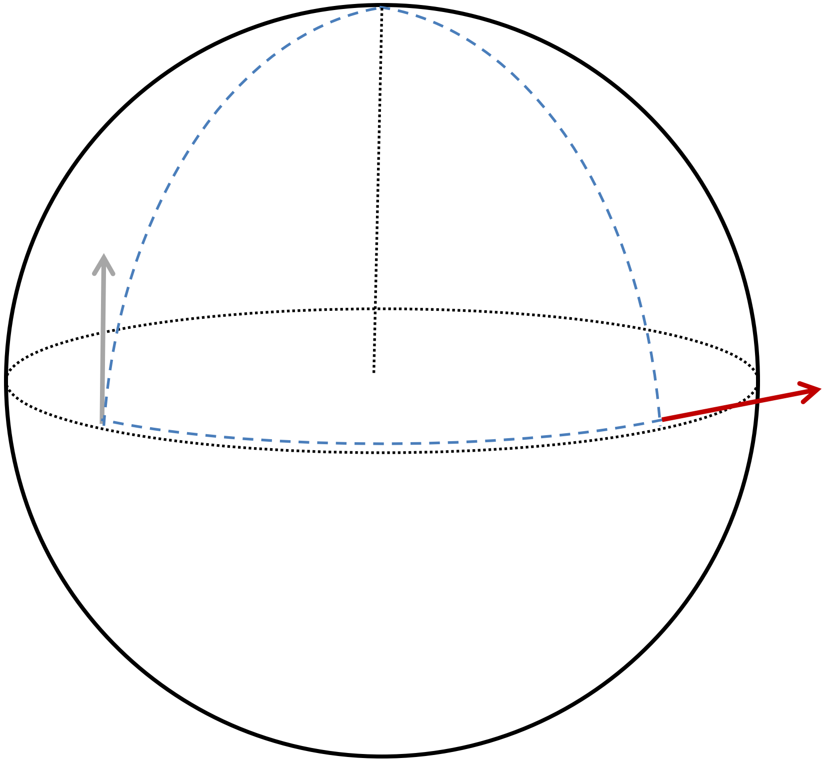

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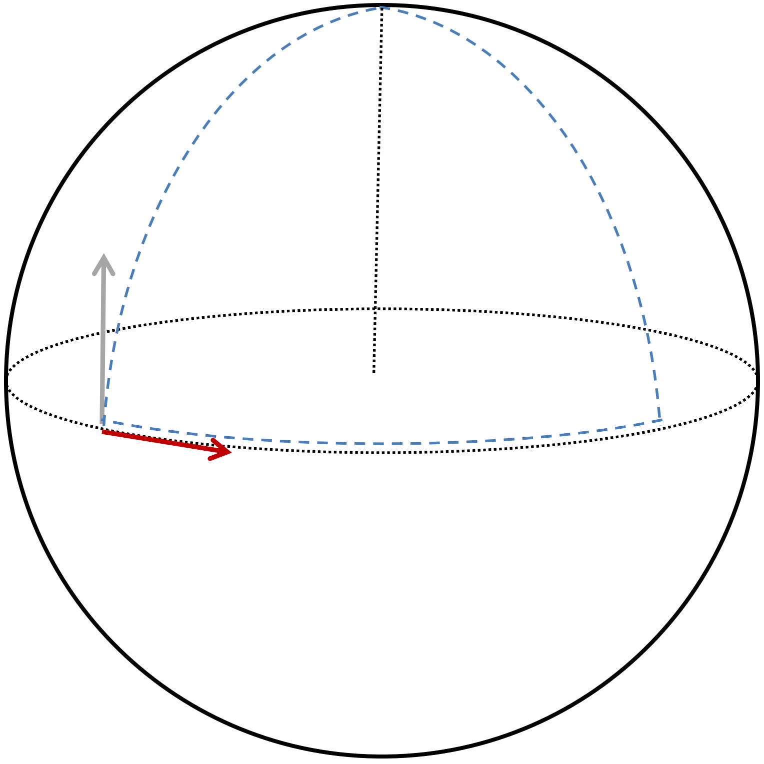

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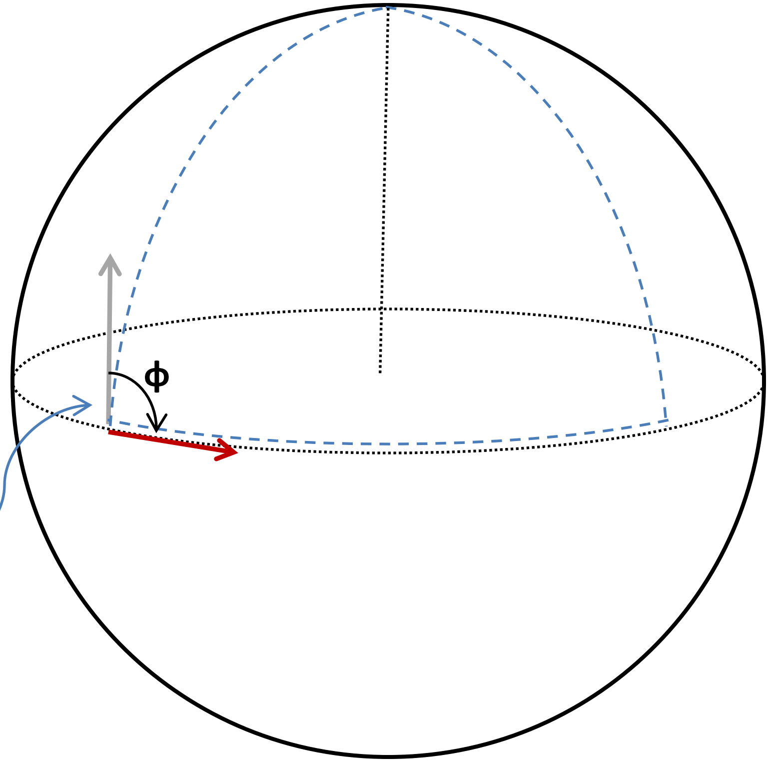

holonomy

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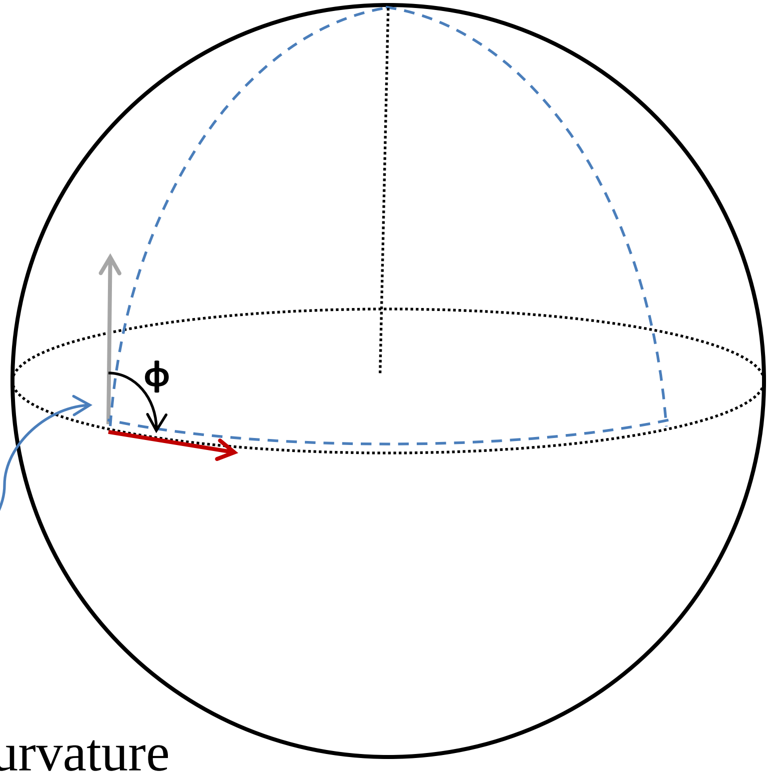

holonomy

(equals the flux of the curvature field through the enclosed area)

Holonomic quantum computation

Adiabatic theorem (Kato, 1950): Consider a time-dependent Hamiltonian $H(t/T)$ changing along a curve $H(s), s \in [0,1]$. Let $\epsilon(s)$ be an eigenvalue with constant degenearacy, whose eigenspace $\mathcal{H}_\epsilon(s)$ has a twice-differentiable projector $\Pi_\epsilon(s)$.

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- Specifically, if $\{|i,s\rangle\}$ is an arbitrary (differentiable) basis of $\mathcal{H}_\epsilon(s)$, the evolution of any initial state is given by [up to an error $O(T^{-1})$]

$$|\psi(t)\rangle = e^{-i\int_0^t \epsilon(\tau/T)d\tau}\Gamma(t/T)|\psi(0)\rangle \quad ,$$

where $\Gamma(s) = \lim_{\delta s \to 0} \Pi_\epsilon(s)\Pi_\epsilon(s-\delta s)...\Pi_\epsilon(\delta s)\Pi_\epsilon(0) = \sum_{ij} U_{ij}(s)|i,s\rangle\langle j,0|$.

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Appeal: robustness due to adiabaticity and geometric nature of gates

Holonomic quantum computation

However,

- Any system interacts with its environment.

  - HQC in DFSs (Wu, Zanardi, Lidar, 2005), but no symmetry is exact.

- Robustness does not mean flawlessness (control errors are inevitable).

- Scalability of any computational method requires fault tolerance.

Need for active error correction!

Prospects: combine the inherent resilience of all-geometric control with the software protection of QEC

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$\longleftarrow$ **The structure of preserved information!**

Blume-Kohout, Ng, Poulin, Viola, PRL 100, 030501 (2008) .
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Example (QEC code): $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B,$ *Code space:* $\mathcal{H}^A \otimes \mathrm{Span}\{|0\rangle^B\}$

logical information

error syndromes

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HQC in subsystems

Theorem: Consider a decomposition of the Hilbert space $\mathcal{H} = \bigoplus_i \mathcal{H}_i^A \otimes \mathcal{H}_i^B$.
Choose a starting Hamiltonian

$$H(0) = \sum_i I_i^A \otimes H_i^B \ ,$$

where H_i^B and H_j^B have different eigenvalues for $i \neq j$. (In the case of $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$ require H^B to have at least two different eigenvalues .)

By varying this Hamiltonian adiabatically along suitable loops, we can generate

$$U = \sum_i W_i^A \otimes V_i^B \ ,$$

where $\{W_i^A\}$ is any desired set of purely geometric (holonomic) operations.

Oreshkov, PRL 103, 090502 (2009) .

HQC in subsystems

Consequence: HQC without initialization

When $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$, we can implement an arbitrary purely geometric transformation on $\mathcal{H}^A$ without having to initialize the system in any subspace.

→ Given any system, we can apply to it an arbitrary geometric transformation by appending to it a *noisy* qubit (which absorbs all unwanted dynamical effects).

May be useful where initialization is difficult to perform.

Oreshkov, PRL 103, 090502 (2009) .

Fault-tolerant quantum computation

For a (stabilizer) QEC code, $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$.

logical information

syndrome, and
gauge degrees of
freedom (operator QEC)

Logical observables are generally highly nonlocal !

→ to perform computation in $\mathcal{H}^A$ (and error correction in $\mathcal{H}^B$) using local interactions, the logical states will have to go outside of $\mathcal{H}^A$ during the implementation, i.e., the logical subsystem has to be moved around:

$$\mathcal{H} = \mathcal{H}^A(t) \otimes \mathcal{H}^B(t) = U(t)\mathcal{H}^A \otimes \mathcal{H}^B \,.$$

generated by local interactions

Can it be done so that the logical information is not exposed to further danger?

Fault-tolerant computation

Shor, DiVincenzo, Knill, Laflamme, Zurek, Aharonov, Ben-Or, Kitaev, Gottesman…. (1996 - 1997)

Definition (fault tolerance): a QEC circuit is fault-tolerant if an error occurring *during* its implementation renders the result correctable.

Threshold theorem: If the probability for an error per elementary information carrier (e.g., qubit) per gate is below some value p, an arbitrarily long computation can be implemented reliably with a polylogarithmic computational overhead .

Fault-tolerant computation

Consider a stabilizer QEC code for the correction of single-qubit errors.

Building blocks of a dynamical fault-tolerant scheme:

- Transversal unitary operations:

  - single-qubit unitaries
  - transversal C-NOT

- Preparation and use of a 'cat' state $(|0...0\rangle + |1...1\rangle)/\sqrt{2}$:

  - preparation
  - verification (measurement of the parity of the state)
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- Single-qubit measurements in the computational basis.

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These procedures prescribe how to move the logical subsystem $\mathcal{H}^A(t)$ **!**

Fault-tolerant HQC
Approach 1: (no additional qubits)

Consider a [[n,1,r,3]] stabilizer code, $\mathcal{G} = \langle i, S_1, \ldots, S_s, X^{g_1}, Z^{g_1}, \ldots, X^{g_r}, Z^{g_r} \rangle$.

$$\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$$

logical qubits   syndrome, gauge, ancilla qubits

The starting Hamiltonian must have the form $H(0) = I^A \otimes H^B$.

$\Rightarrow$ $H(0)$ a linear combination of elements of $\mathcal{G}$.

During the evolution, $\mathcal{H} = \mathcal{H}^A(t) \otimes \mathcal{H}^B(t) = U(t)\mathcal{H}^A \otimes \mathcal{H}^B$ $\Rightarrow$

$H(t) = I^{A(t)} \otimes H^{B(t)}$ [a linear combination of elements of $\mathcal{G}(t)$] .

Operators in $\mathcal{G}$ couple qubits in the same block → transversality impossible!

Oreshkov, Brun, Lidar, PRL 102, 070502 (2009) ; PRA 80, 022325 (2009)

Fault-tolerant HQC
Approach 1: (no additional qubits)

Transversal unitaries are not the only ones that prevent propagation of errors:

A transversal unitary followed by a gauge transformation is also fault-tolerant!

The main idea:

1) Adiabatically drag the logical subsystem along a sequence of paths segments, such that during each segment the unitary that we generate in the full Hilbert space is transversal up to a gauge transformation (see next slide).

2) Follow a sequence of transversal operations just like in a dynamical FT scheme. The result is the desired operation followed by a gauge transformation.

3) When we complete each operation, the logical system has been taken around a loop whose associated holonomy is the desired logical gate.

1) & 2) → fault-tolerance; 3) → the computation is purely geometric

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How we do it…

Proposition: Take a starting Hamiltonian $H(0) = Z^1 \otimes \tilde{G}$, where $\tilde{G}$ is a Pauli operator on the rest of the qubits. Change the Hamiltonian adiabatically as

$$H(t) = H^1(t) \otimes \tilde{G} \quad , \qquad \mathrm{Tr} H^1(t) = 0 .$$

⇨ The resultant unitary is $\boxed{U(t) \approx U^1(t) \otimes \tilde{I}}$ (up to a gauge transformation).

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Example

Single-qubit X gate: $Z \otimes \tilde{G} \to Y \otimes \tilde{G} \to -Z \otimes \tilde{G}$

… In a similar way we can generate all necessary elementary operations.

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Properties of the scheme:

- **The threshold** (error per qubit per gate) **is the same** as for a dynamical scheme.

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  But the adiabatic approximation requires slow evolution: for error 10^{-4} ,
  holonomic gates may need to be 10 – 100 times slower than dynamical ones.
  → **environmental noise has to be weaker!**

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  HQC would be advantageous if it leads to an increase in control precision that outweighs the increase of environment errors due to the slowdown.

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- Universal FTHQC with this method **requires at least 3-local Hamiltonians**.
  This is achievable with suitable codes (e.g., Bacon-Shor code).

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  Can we reduce to 2-local Hamiltonians with perturbative techniques?

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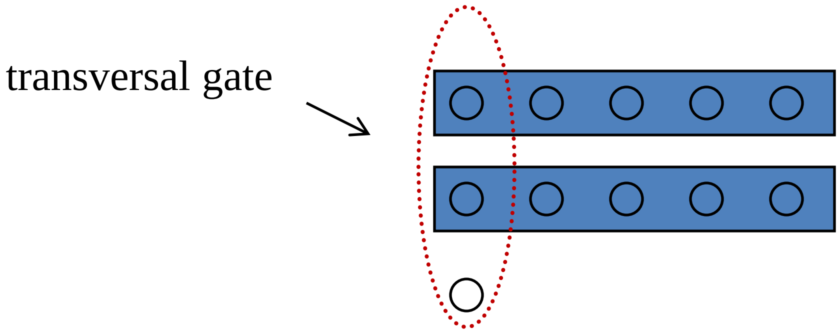
Fault-tolerant HQC
Approach 2: (extra gauge qubits)

An alternative (and conceptually simpler) scheme:

HQC is performed on the entire system's Hilbert space (by coupling each qubit or pair of qubits to an '*external*' gauge qubit).



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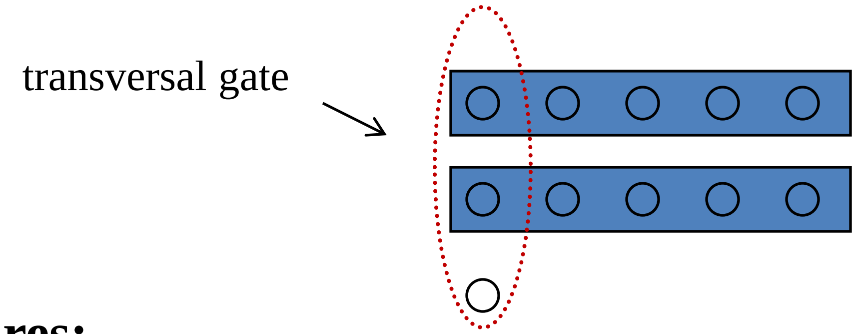

Key features:

- Qubits in the same block do not interact by construction.

- Extra qubits $\longrightarrow$ lower threshold (~1.5 times). [but no need to initialize them]

- The Hamiltonians are independent of the code.

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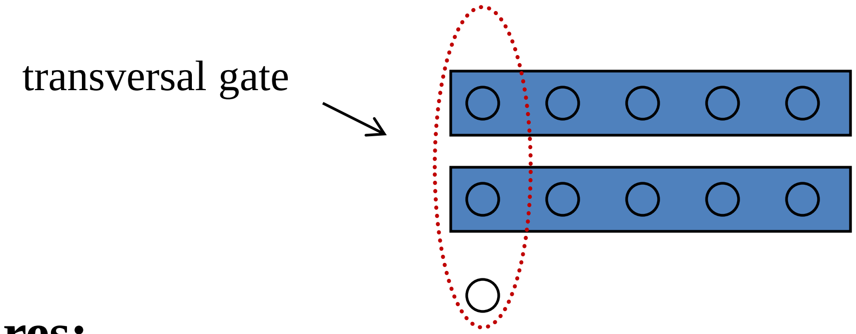

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- The Hamiltonians are independent of the code. [3-local Hamiltoni required]

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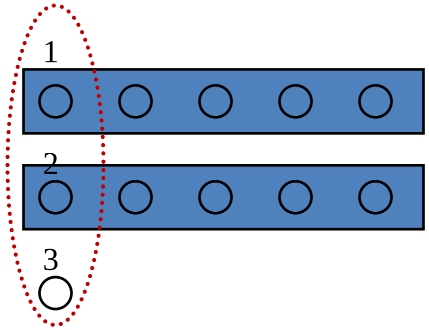
Fault-tolerant HQC with 2-qubit Hamiltonians

Perturbative gadgets:

Kempe, Kitaev, Regev, 2004; Oliveira, Terhal, 2005; Jordan, Farhi, 2008.

Example: $H(t) = f(t) I^1 \otimes Y^2 \otimes Z^3 + g(t) Z^1 \otimes Z^2 \otimes Z^3$



Fault-tolerant HQC with 2-qubit Hamiltonians

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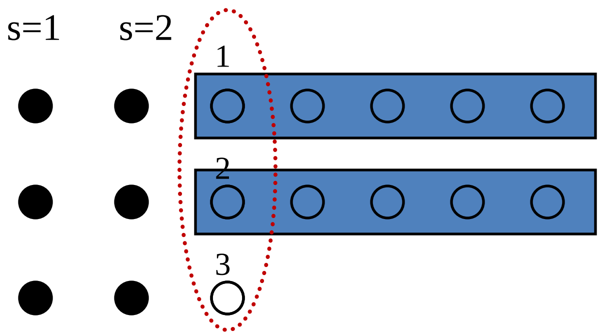

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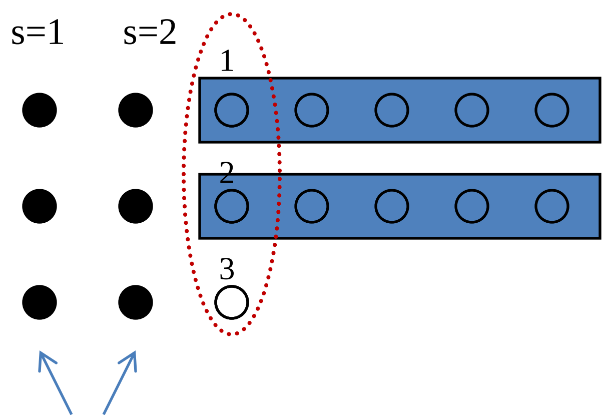

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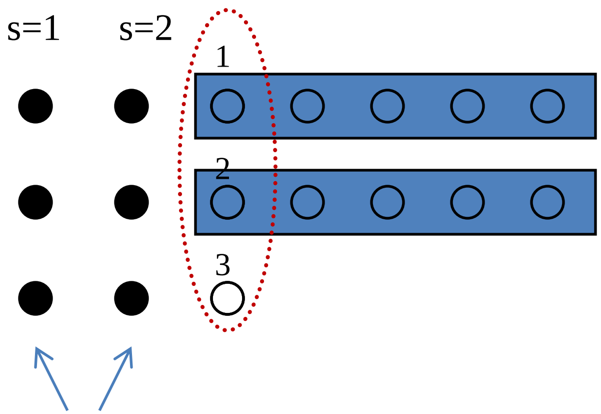

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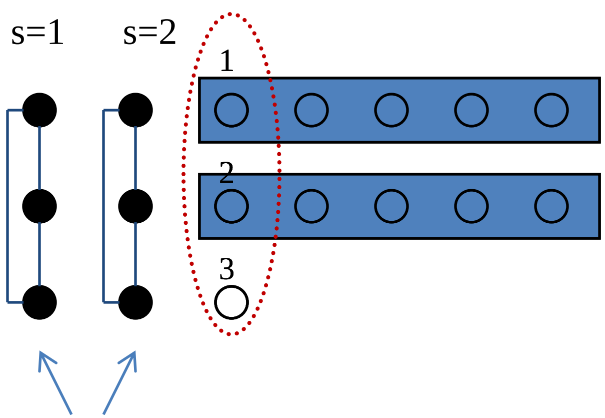

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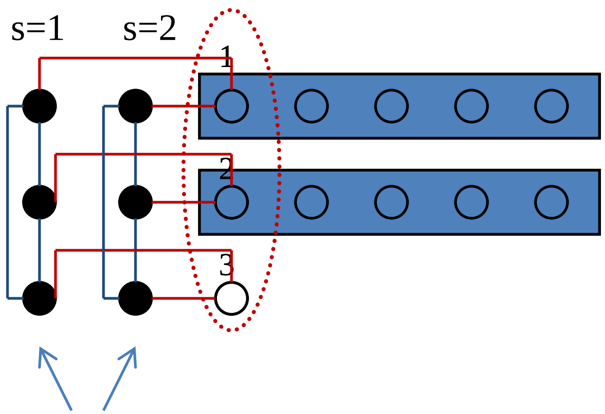

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$$H_s^{\mathrm{anc}} = \sum_{1 \le i < j \le 3} \frac{1}{2}(I - Z_{s,i} Z_{s,j})$$

$$V_1(t) = \sqrt[3]{f(t)} I^1 \otimes X_{1,1} + \sqrt[3]{f(t)} Y^2 \otimes X_{1,2} + \sqrt[3]{f(t)} Z^3 \otimes X_{1,3}$$

$$V_2(t) = \sqrt[3]{g(t)} Z^1 \otimes X_{2,1} + \sqrt[3]{g(t)} Z^2 \otimes X_{2,2} + \sqrt[3]{g(t)} Z^3 \otimes X_{2,3}$$

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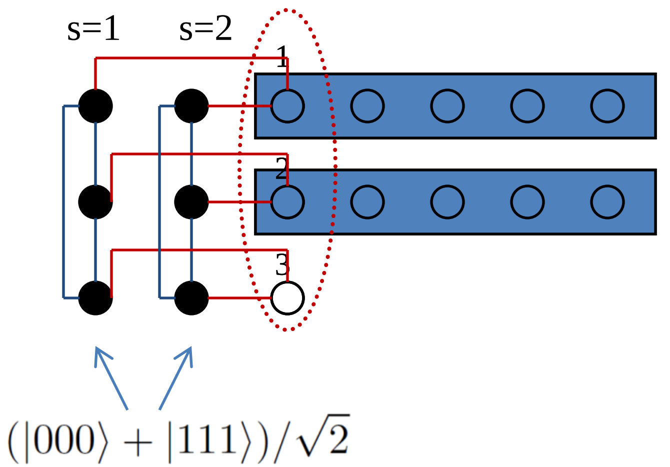

$$H^{\mathrm{gad}}(t) = \sum_{s=1}^{2} H_s^{\mathrm{anc}} + \lambda \sum_{s=1}^{2} V_s(t)$$

Effective Hamiltonian $\Rightarrow$

$$H_{\mathrm{eff}} = \frac{3\lambda^3}{2} H(t) \otimes P_{\mathrm{cat}} + O(\lambda^4)$$

Fault-tolerant HQC with 2-qubit Hamiltonians

Drawbacks of the gadgets:

- An error on any of the 9 qubits can result in an error on the 2 system qubits
  → the threshold is **9/2** times smaller.

- If for adiabatic precision $1-\delta$ with the Hamiltonian $H(t)$ we need time T ,
  with the effective Hamiltonian $\frac{3\lambda^3}{2}H(t)$ we need time $T' = \frac{2}{3\lambda^3}T$.
  But $O(\lambda^4 T') = O(\lambda T) = O(\delta)$ → $T' = O(\frac{T^4}{\delta^3})$.

  → We need $O(\frac{T^3}{\delta^3})$ times longer evolution! (very large slow-down)

Related schemes

- **Adiabatic gate teleportation:**

Bacon and Flammia,
PRL 103, 120504 (2009)

- Each gate is implemented by dragging the logical system only along a single line segment! (could be viewed as open-path HQC)

Compatible with fault-tolerant techniques and perturbative gadgets

- **Cluster state adiabatic computation:**

Bacon and Flammia,
PRA 82, 030303(R) (2010)

- One-way computing without measurements

- **HQC in ground states of spin chains protected by topological order:**

- Based on 2-local Hamiltonians without gadgets

Renes, Miyake, Brennen,
Bartlett, arXiv: 1103.5076

Related schemes

- **HQC by dissipation (Adiabatic Markovian Dynamics)**

$$\frac{d\rho}{dt} = -i[H(t),\rho] + \sum_i \left(L_i(t)\rho L_i(t)^\dagger - \frac{1}{2}L_i^\dagger(t)L_i(t)\rho - \frac{1}{2}\rho L_i^\dagger(t)L_i(t) \right)$$

Oreshkov and Calsamiglia,
PRL 105, 050503 (2010)

- A noiseless subsystem is dragged by a slowly varying Lindblad generator.

(Compatible with fault-tolerance Approach 2)

A new type of geometric phase: generalizes the Wilczek-Zee and Uhlmann holonomies.

Oreshkov and Sjoqvist,
in preparation

Conclusion and outlook

- HQC can be done in subsystems (rather than subspaces).

- Adiabaic geometric control is compatible with the techniques for fault-tolerant computation on stabilizer codes.

  → HQC is in principle scalable.

  → the software protection of QEC could be aided by the robustness of HQC.

- FTHQC is possible with 2-qubit Hamiltonians (but the gadgets are inefficienet).

- Is it possible to find a non-perturbative realization with 2-qubit interactions?

- Physical implementations?

- Can some of these ideas be useful for faul-tolerant adiabatic QC?