Approximate simulation of quantum channels

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Why approximate QEC?

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- The KL conditions could be far from necessary

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Crépeau, Gottesman, Smith (2005)

- However, one can correct $\sim n/2$ errors for n qubit, for large n

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Knill, Laflamme, Viola (2000)

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Tyson (2010)

- Bound similar to ours, different methods

Channel simulation

- Error correction: find $\mathcal{C}$, $\mathcal{R}$ such that

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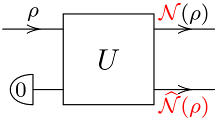
- Equivalence relation

$$\mathcal{N} \sim \mathcal{M} \quad \text{if} \quad \mathcal{N} \succ \mathcal{M} \text{ and } \mathcal{M} \succ \mathcal{N}$$

Complementary class

- $\widehat{\mathcal{N}}$ complementary to $\mathcal{N}$



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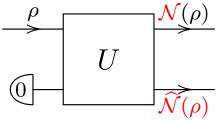

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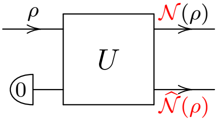

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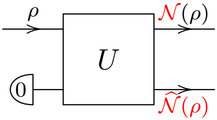

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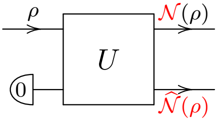

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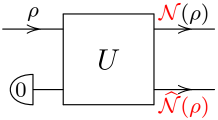

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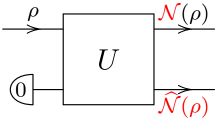

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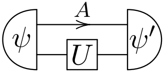
- E.g. we obtain the generalized KL conditions for subsystems codes when

$$\mathcal{A} = \{A \otimes \mathbf{1} : A\}$$

Approximate versions (fixed-state fidelity)

- Fidelity between states $\rho = \mathrm{Tr}_B |\psi\rangle\langle\psi|$ and $\rho' = \mathrm{Tr}_B |\psi'\rangle\langle\psi'|$

$$f(\rho, \rho') = \max_U \Bigg|$$  $$\Bigg|$$

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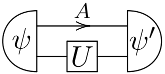

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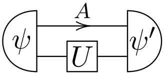

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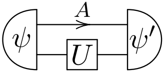

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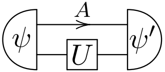

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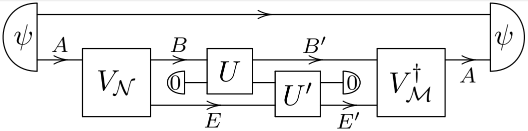

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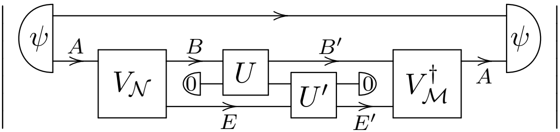
Proof

$$\max_{U} \min_{\psi} \max_{U'}$$



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- When $\mathcal{M} = \mathcal{P}_{\mathcal{A}}$: approx. subsystem QEC.

Near-optimal decoding channel

If we found σ such that

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and for any CP map $\mathcal{S}$,

$$\overline{\mathcal{S}(\rho)} = \mathcal{S}(\mathcal{S}^{\dagger}(\mathbf{1})^{-\frac{1}{2}}\, \rho\, \mathcal{S}^{\dagger}(\mathbf{1})^{-\frac{1}{2}}).$$

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- General perturbative channel:

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Theorem

$\mathcal{N}$ correctable to order t if and only if

$$PL_i^\dagger L_j P \propto P \quad \text{and} \quad PL_i P \propto P.$$

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- Answers Leung, Nielsen, Chuang, Yamamoto, 1997

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$\mathcal{N}$ correctable to order ϵ^2 if and only if $\forall i \neq 0$,

$$PE_i^\dagger E_j P \propto P \quad \text{and} \quad PE_i P \propto P$$

Perturbative QEC

- Weak interaction system-environment: $\epsilon = \lambda t \ll 1$.

$$H = \lambda \sum_i J_i \otimes K_i$$

- Noise operators

$$N_i = \delta_{0i}\mathbf{1} + \epsilon E_i + \mathcal{O}(\epsilon^2)$$

- where

$$E_i = \imath \sum_j J_j \langle i | K_j | 0 \rangle$$

Theorem

$\mathcal{N}$ correctable to order ϵ^2 if and only if $\forall i \neq 0$,

$$P E_i^\dagger E_j P \propto P \quad \text{and} \quad P E_i P \propto P$$

- Weaker than Knill, Laflamme, Viola 2000

General theory: 1103.0649

Perturbative QEC: 1102.3809