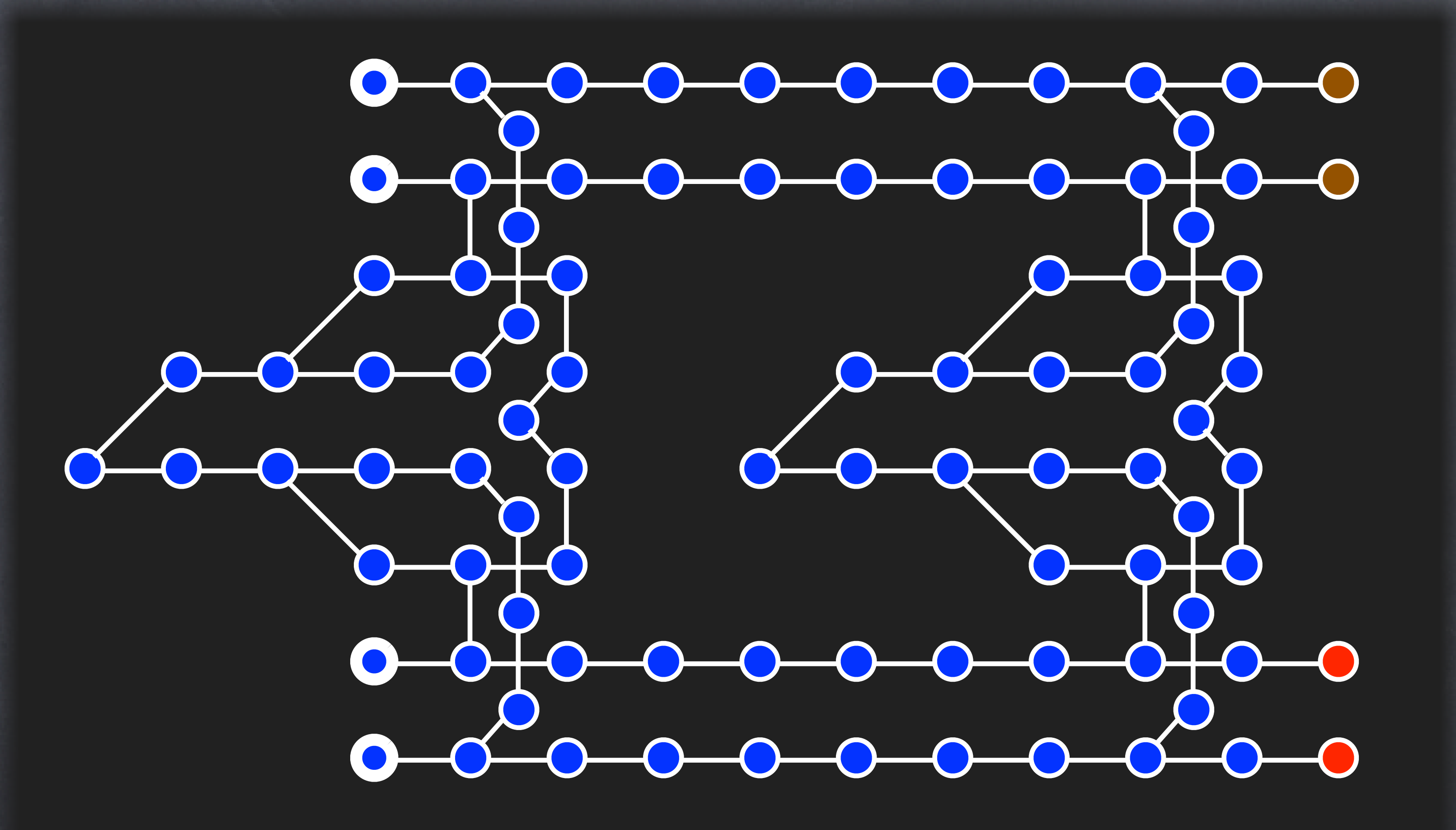
Quantum Error Correcting Codes For Locavores



Dave Bacon* and Jonathan Shi
University of Washington

*= decohered to Google

Thanks Daniel!

Concatenating Decoherence-Free Subspaces with Quantum Error Correcting Codes

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(Received 28 September 1998)

An operator sum representation is derived for a decoherence-free subspace (DFS) and used to (i) show that DFS's are the class of quantum error correcting codes (QECC's) with fixed, *unitary* recovery operators and (ii) find explicit representations for the Kraus operators of collective decoherence. We demonstrate how this can be used to construct a concatenated DFS-QECC code which protects against collective decoherence perturbed by independent decoherence. The code yields an error threshold which depends only on the perturbing independent decoherence rate. [S0031-9007(99)09301-1]

PACS numbers: 03.67.Lx, 03.65.Bz, 03.65.Fd, 89.70.+c

Decoherence-free subspaces (DFS's) have recently emerged [1–6] as an alternative way to protect fragile quantum states against decoherence, alongside "conventional" quantum error correcting codes (QECC's) [7,8] and the new "dynamical decoupling" schemes [9]. This is of particular importance in quantum computation, where the promise of a speedup compared to classical computers hinges crucially on the possibility to maintain quantum coherence throughout the computation [10]. So far, DFS's and QECC's have been considered as distinct methods, often characterized as "passive" and "active," respectively. However, as we will show here, in fact, DFS's can be considered as a special class of QECC's, characterized as

tonian is $\mathbf{H} = \mathbf{H}_S \otimes \mathbf{I}_B + \mathbf{I}_S \otimes \mathbf{H}_B + \mathbf{H}_I$, where $\mathbf{H}_S$, $\mathbf{H}_B$, and $\mathbf{H}_I$ are, respectively, the system, bath, and interaction Hamiltonians, and $\mathbf{I}$ is the identity operator. Assuming initial decoupling between system and bath, the evolution of the closed system is given by $\rho_{SB}(t) = \mathbf{U}[\rho_S(0) \otimes \rho_B(0)]\mathbf{U}^\dagger$. Quite generally, the interaction Hamiltonian can be written as $\mathbf{H}_I = \sum_\alpha \mathbf{F}_\alpha \otimes \mathbf{B}_\alpha$, where $\mathbf{F}_\alpha$ and $\mathbf{B}_\alpha$ are, respectively, system and bath operators. Suppose that there exists a degenerate subset $\{|\tilde{k}\rangle\}$ of eigenvectors of the $\mathbf{F}_\alpha$'s such that

$$\mathbf{F}_\alpha|\tilde{k}\rangle = a_\alpha|\tilde{k}\rangle \quad \forall\ \alpha, |\tilde{k}\rangle. \qquad (1)$$

But Why?

20 . "Concatenating Decoherence-Free Subspaces with Quantum Error Correcting Codes", Phys. Rev. Lett. **82**, 4556 (1999), by D.A. Lidar, D. Bacon, and K.B. Whaley [pdf]

19 . "Calculating the Thermal Rate Constant with Exponential Speedup on a Quantum Computer", Phys. Rev. E 59, 2429 (1999), by D.A. Lidar and H. Wang [pdf]

18 . "Pattern Formation and a Clustering Transition in Power-Law Sequential Adsorption", Phys. Rev. E **59**, R4713 (1999), O. Biham, O. Malcai, D.A. Lidar (Hamburger), and D. Avnir [pdf]

17 . "How to Teleport Superpositions of Chiral Amplitudes", Phys. Rev. Lett. **81**, 5928 (1998), C.S. Maierle, D.A. Lidar, and Robert A. Harris [pdf]

16 . "Decoherence-Free Subspaces for Quantum Computation", Phys. Rev. Lett. **81**, 2594 (1998), by D.A. Lidar, I.L. Chuang, and K.B. Whaley [pdf]

15 . "Fractality in Nature (Reply to a letter)", Science **279**, 1611 (1998), by O. Biham, O. Malcai, D.A. Lidar, and D. Avnir [pdf]

14 . "Is Nature Fractal? (Reply to letters)", Science **279**, 783 (1998), by O. Biham, O. Malcai, D.A. Lidar, and D. Avnir [pdf]

13 . "Is the Geometry of Nature Fractal?", Science **279**, 39 (1998), by D. Avnir, O. Biham, D.A. Lidar, and O. Malcai [pdf]

12 . "Inversion of Randomly Corrugated Surface Structure from Atom Scattering Data", Inverse Problems **14**, 1299 (1998) [pdf]

11 . "Atom Scattering from Disordered Surfaces in the Sudden Approximation: Double Collisions Effects and Quantum Liquids", Surf. Sci. **411**, 231 (1998), by D.A. Lidar (Hamburger) [pdf]

10 . "Structure Determination of Disordered Metallic Sub-Monolayers by Helium Scattering: A Theoretical and Experimental Study", Surf. Sci. **410**, L721 (1998), by A.T. Yinnon, D.A. Lidar (Hamburger), R.B. Gerber, P. Zeppenfeld, M. Krzyzowski, and G. Comsa [pdf]

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20 . "Concatena ... Rev. Lett. **82**, 4556 (1999

19 . "Calculatin ... ", Phys. Rev. E 59, 2

18 . "Pattern ... Rev. E **59**, R4713 (1999

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16 . "Decoherence- ... 998), ... A. Lidar, I.L. Chuang,

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14 . "Is Nature ... D.A. Lidar, and D. Avnir [pdf

13 . "Is the Geo ... A. Lidar, and O. Malcai [pdf]

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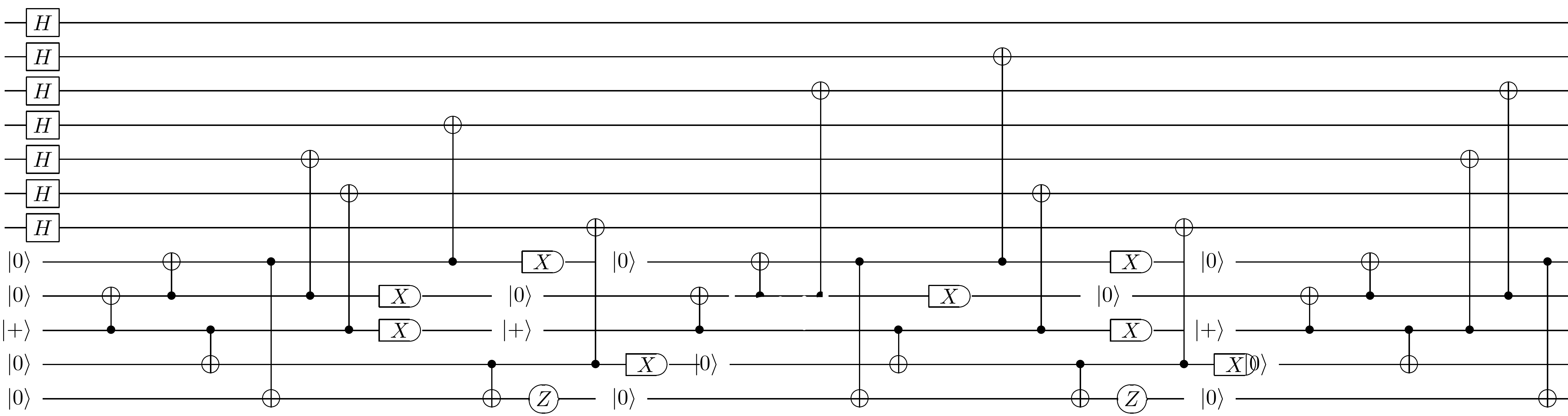
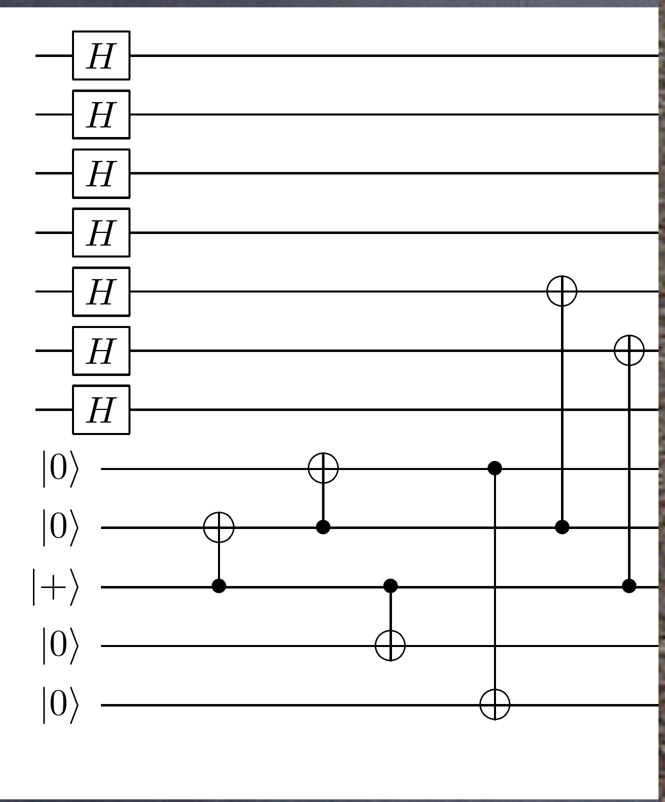
Quantum Error Correction

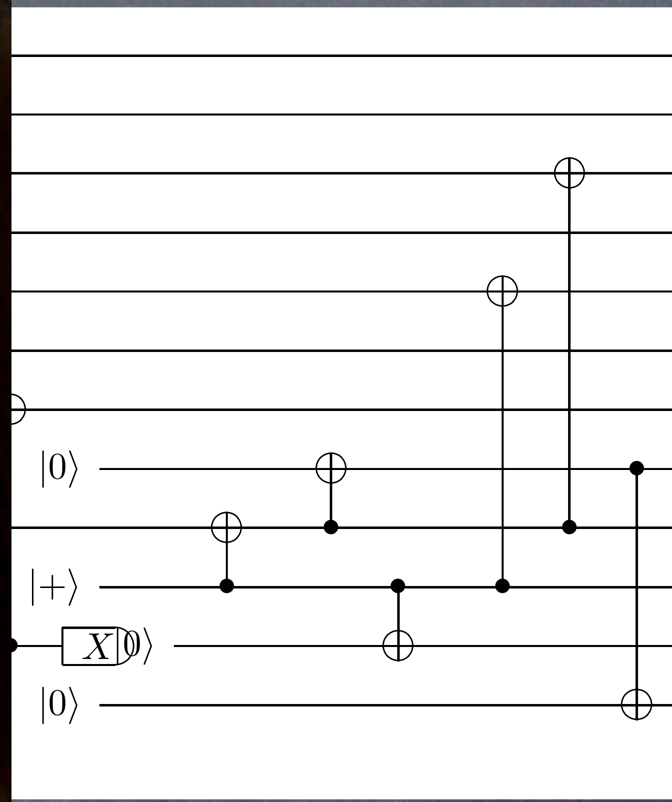


Time →

Complicated, time dependent recipe for protecting quantum information

Quar ection





Compl be for
pr on

Stabilizer Codes

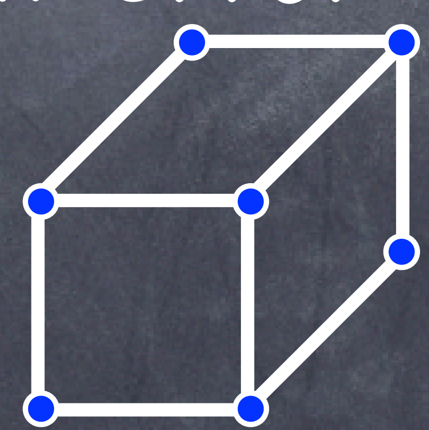
[[7,1,3]] Steane code

S_1				Z	Z	Z	Z
S_2		Z	Z			Z	Z
S_3	Z		Z		Z		Z
S_4				X	X	X	X
S_5		X	X			X	X
S_7	X		X		X		X

row = stabilizer
column = qubit

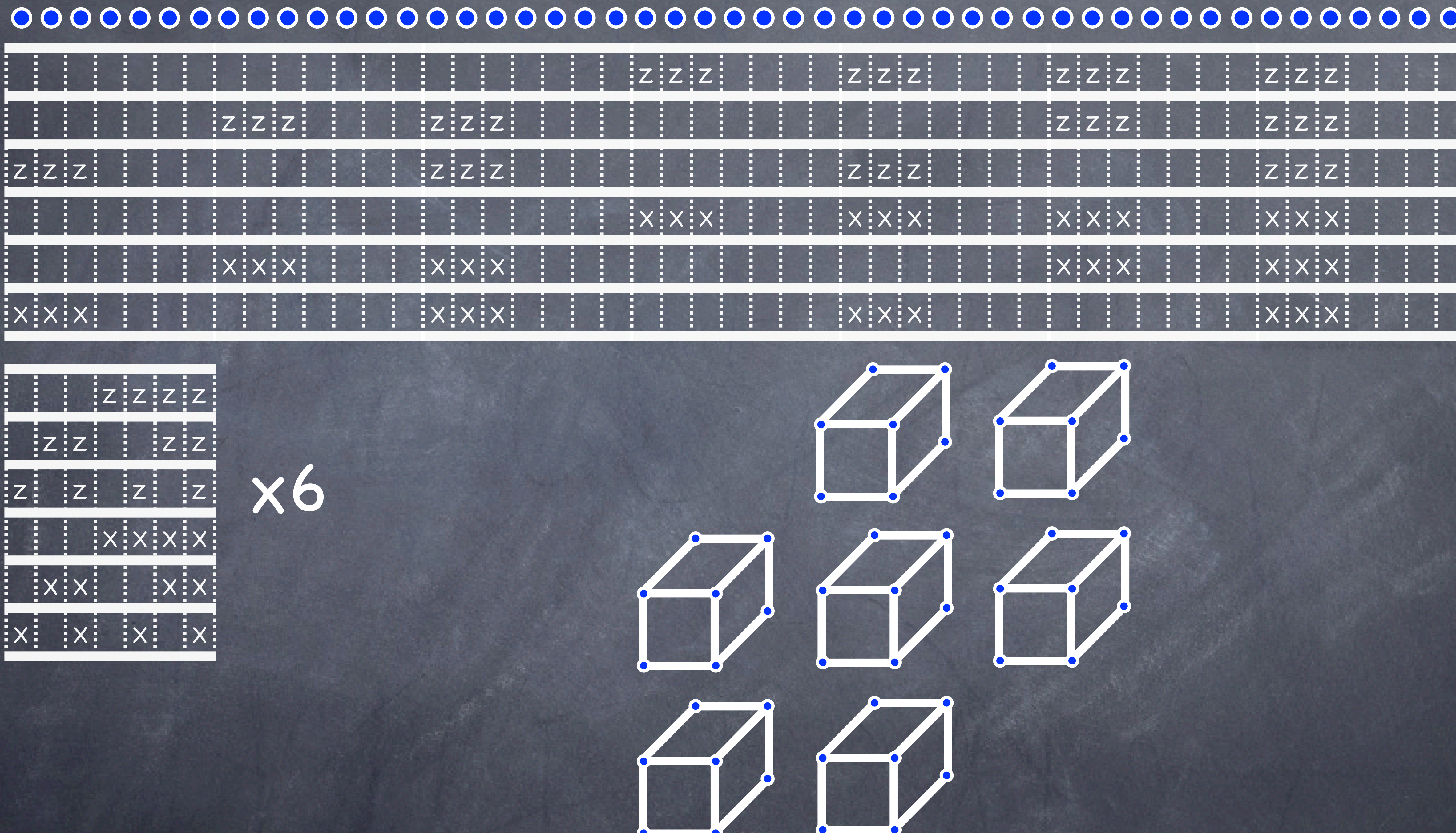
QEC Recipe

1. Measure 6 stabilizer operators
2. Conditional on measurement diagnose (and possibly fix) single qubit error



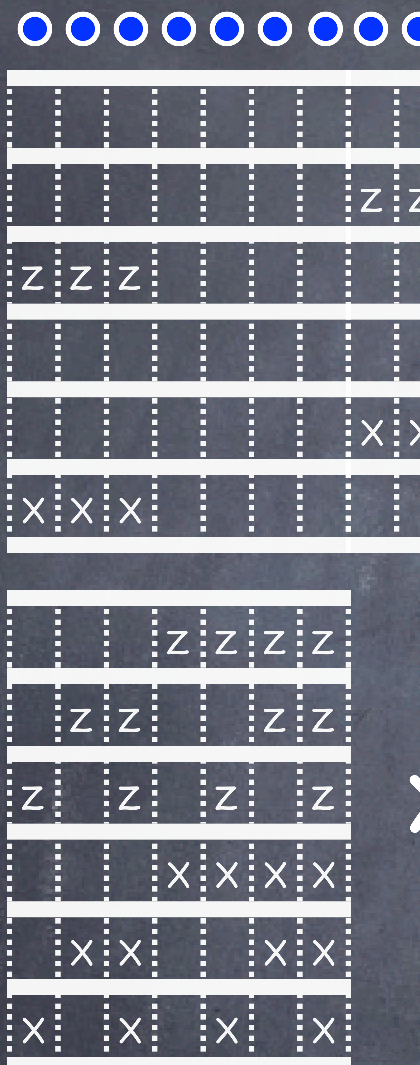
[[# qubits, # encoded qubits, distance]]

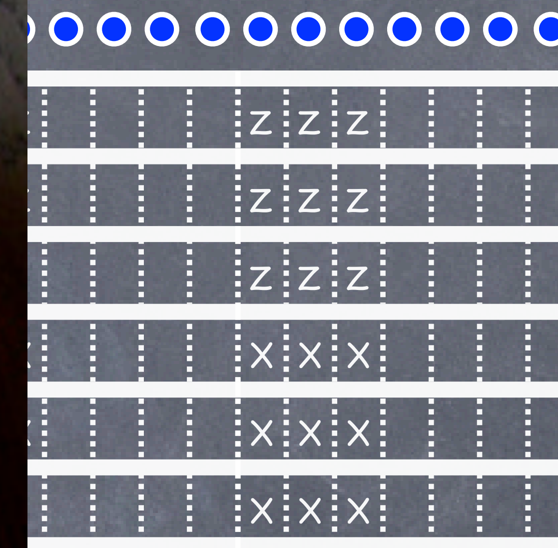
Concatenating Our Lives Away



Concatenation implies measurement of spatially distant, high weight, Pauli operators

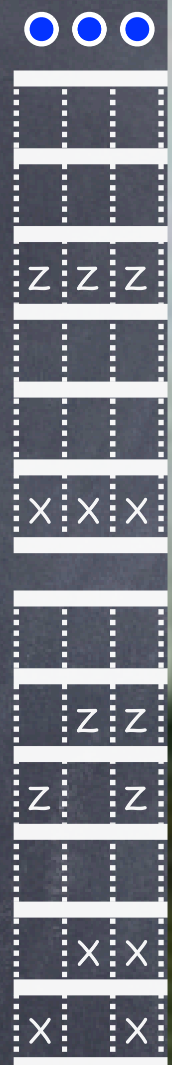
Conca… es Away





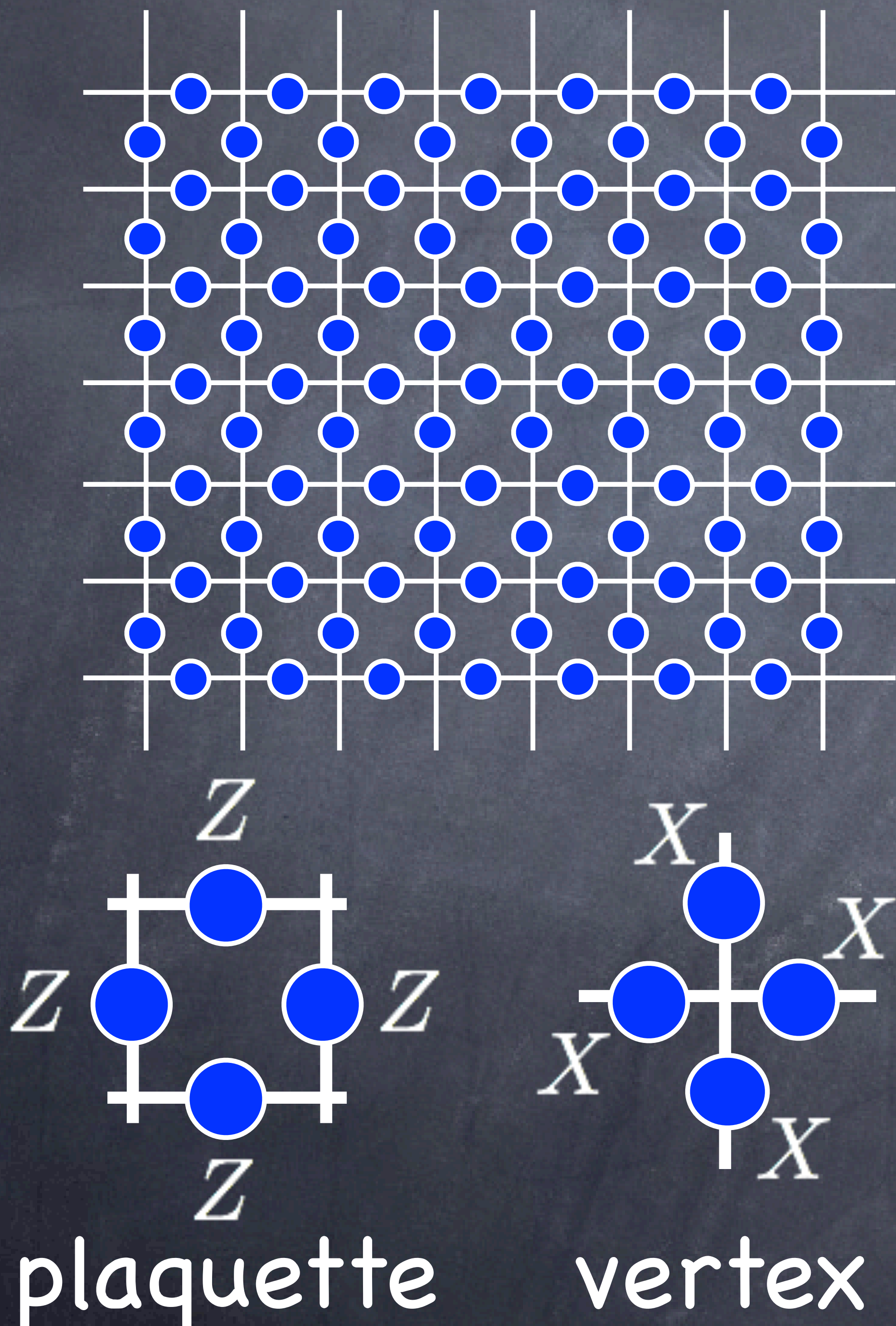
Concatena… spatially
distant, hig…

Cor ay



Conc
dista

Rays of Hope

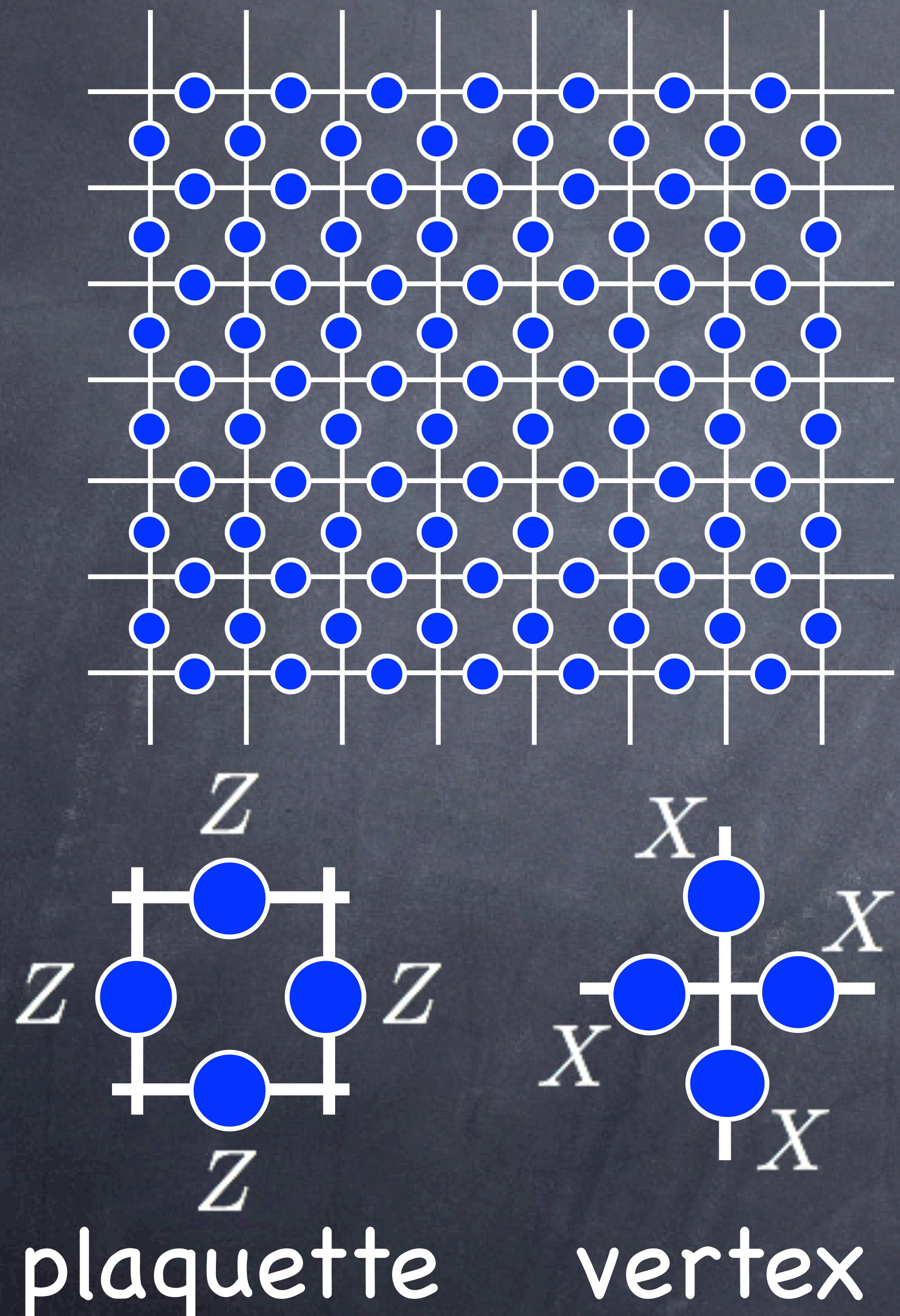


Surface code QEC

1. Measure plaquette and vertex operators

2. Classically post-process measurement results to diagnose error

Rays of Hope



Surface code QEC

1. Measure plaquette and vertex operators

2. Classically post-process measurement results to diagnose error

s-Local and s-Neighboring

weight of Pauli operator = # of operators $\neq I$

- ➡ $I \otimes X \otimes X \otimes Y \otimes Y \otimes I \otimes Z \otimes I$ has weight 5

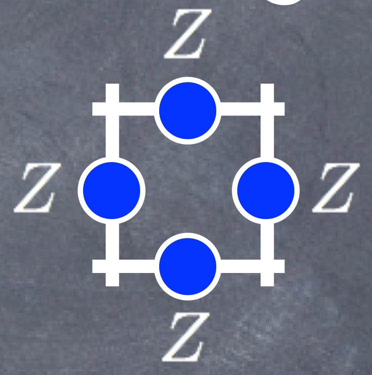
s-Local and s-Neighboring

weight of Pauli operator = # of operators ≠ I

➡ $I \otimes X \otimes X \otimes Y \otimes Y \otimes I \otimes Z \otimes I$ has weight 5

s-Local code: exists a set of stabilizer generators whose weight ≤s

➡ surface code is 4-local



s-Local and s-Neighboring

<u>weight</u> of Pauli operator = # of operators ≠ I

➡ $I \otimes X \otimes X \otimes Y \otimes Y \otimes I \otimes Z \otimes I$ has weight 5

<u>s-Local code</u>: exists a set of stabilizer generators whose weight ≤s

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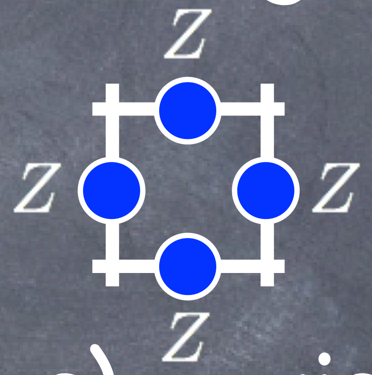


<u>s-Neighboring code (in D dimensions)</u>: exists a set of stabilizer generators where all ≠ I operators are within a distance s of each other on some lattice of qubits in D spatial dimensions

➡ surface code is 2-neighboring in 2D

Lament

Why can't all codes be s-neighboring in two or three spatial dimensions?

Main Result

For every [[n,k,d]] stabilizer code, there exists a [[N,k,r,d]] stabilizer subsystem code, $N=O(n^2)$, whose syndrome measurements can be made using s-neighboring measurements (s constant) in >1 spatial dimensions. (r = # gauge qubits)

Every stabilizer code can be made to act like the surface code!

Main Result

For every [[n,k,d]] stabilizer code, there exists a

[[N,k,r,d]] sta … , $N=O(n^2)$,

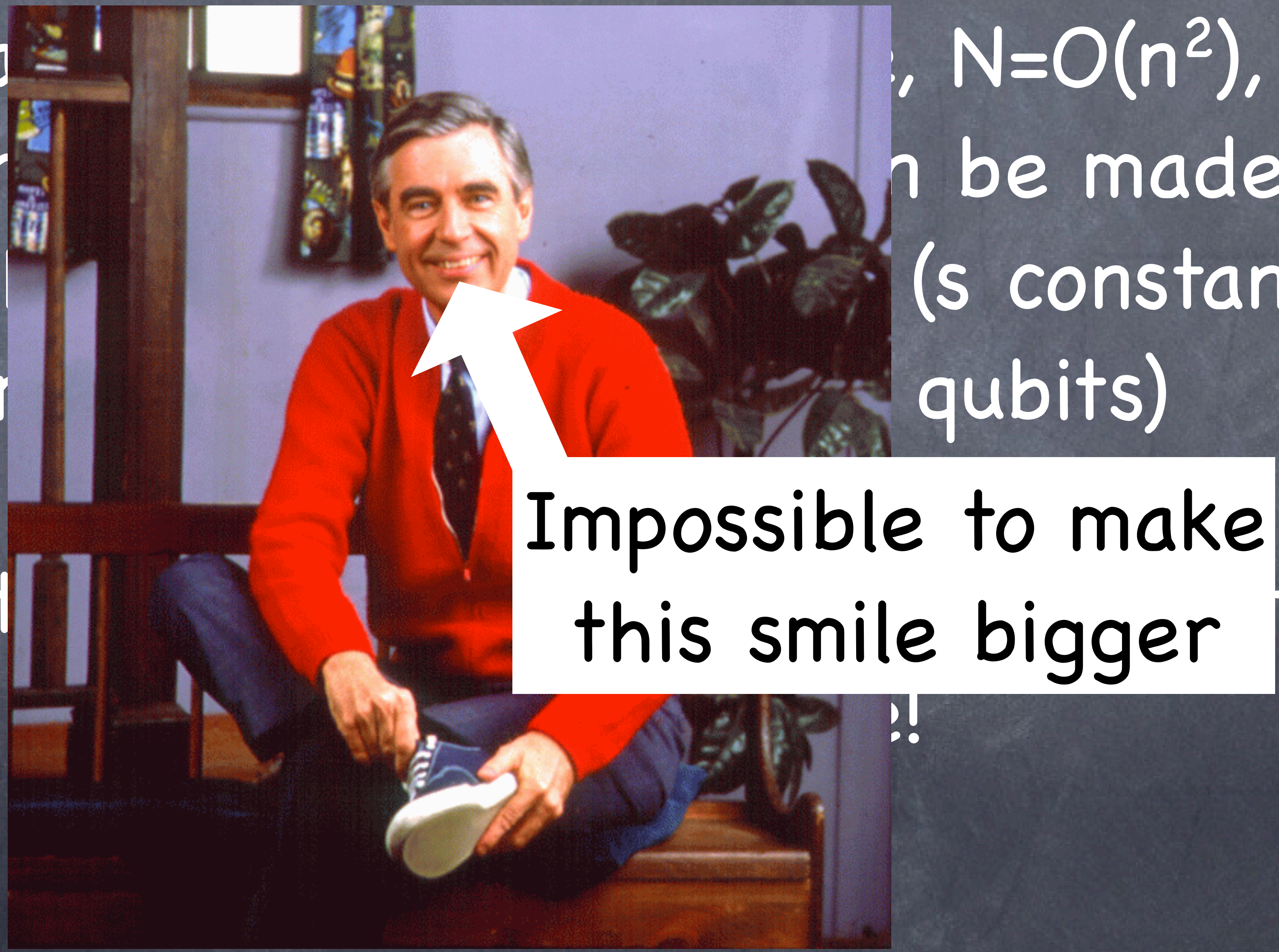
whose syndr … n be made

using s-neig … (s constant) in

>1 spatial di … qubits)

Every st … t

… e!



Main Result

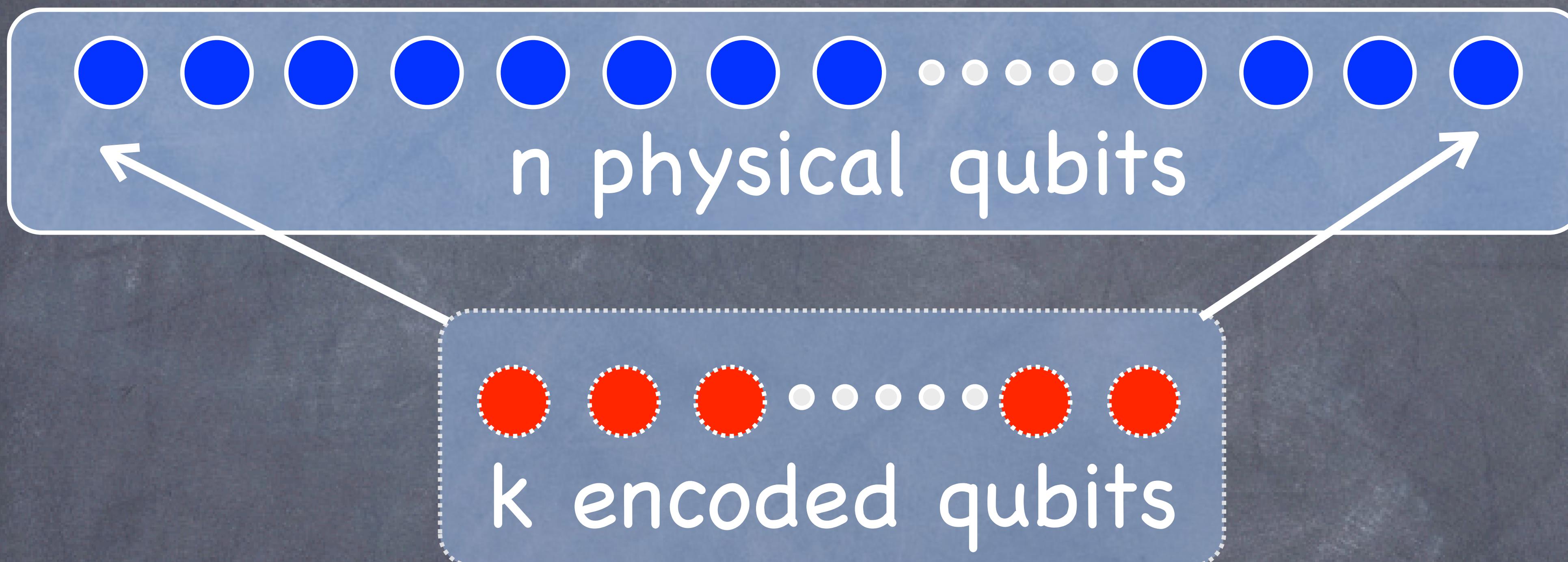
For every [[n,k,d]] stabilizer code, there exists a [[N,k,r,d]] stabilizer subsystem code, $N=O(n^2)$, whose syndrome measurements can be made using s-neighboring measurements (s constant).

To Understand:

1. stabilizer subsystem codes
2. measurement based quantum computing
3. spaced-time locality in QEC

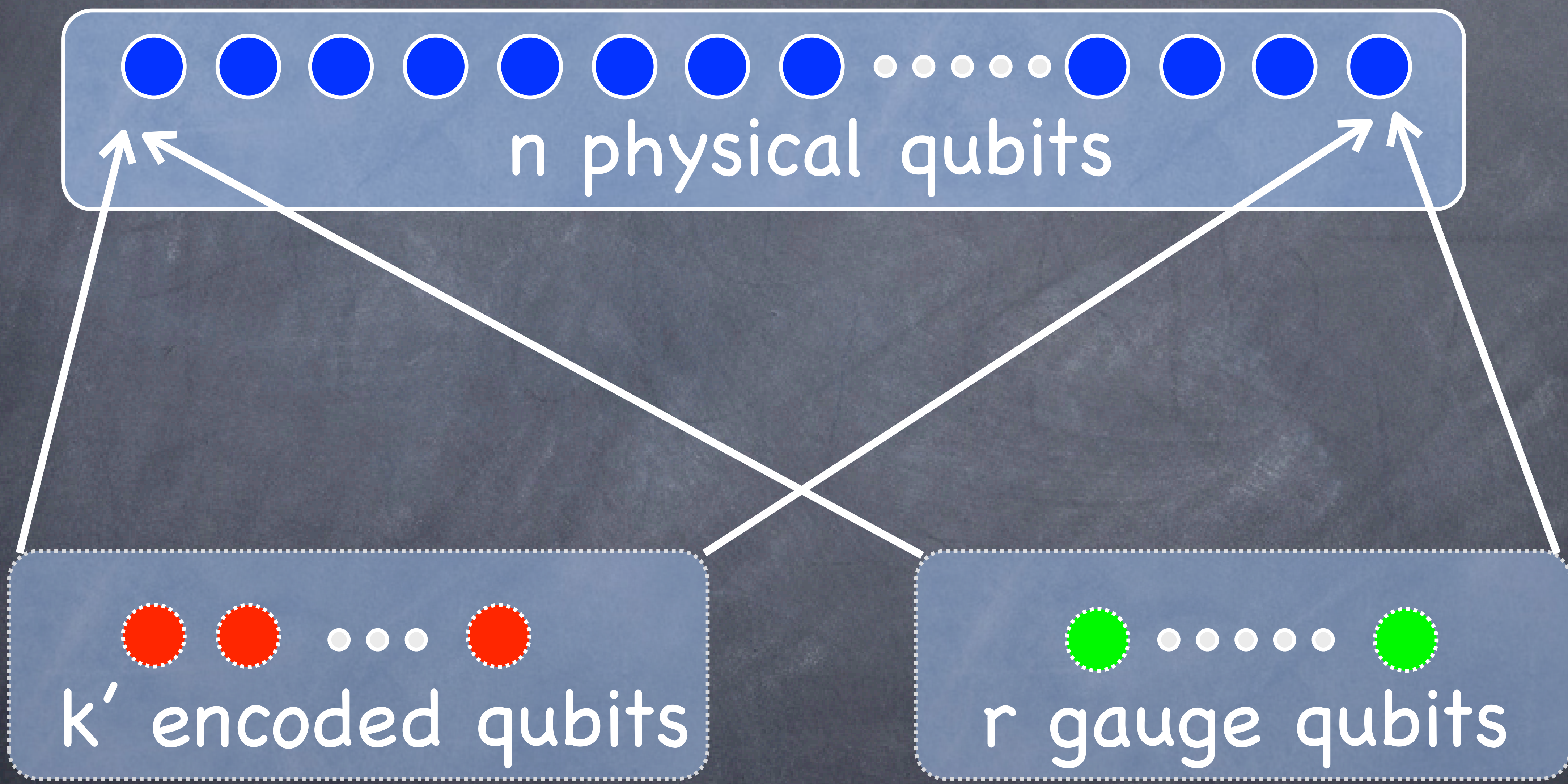
1. Stabilizer Subsystem Code

[[n,k,d]] Stabilizer Subspace Code



1. Stabilizer Subsystem Code

[[n,k,d]] Stabilizer Subspace Code

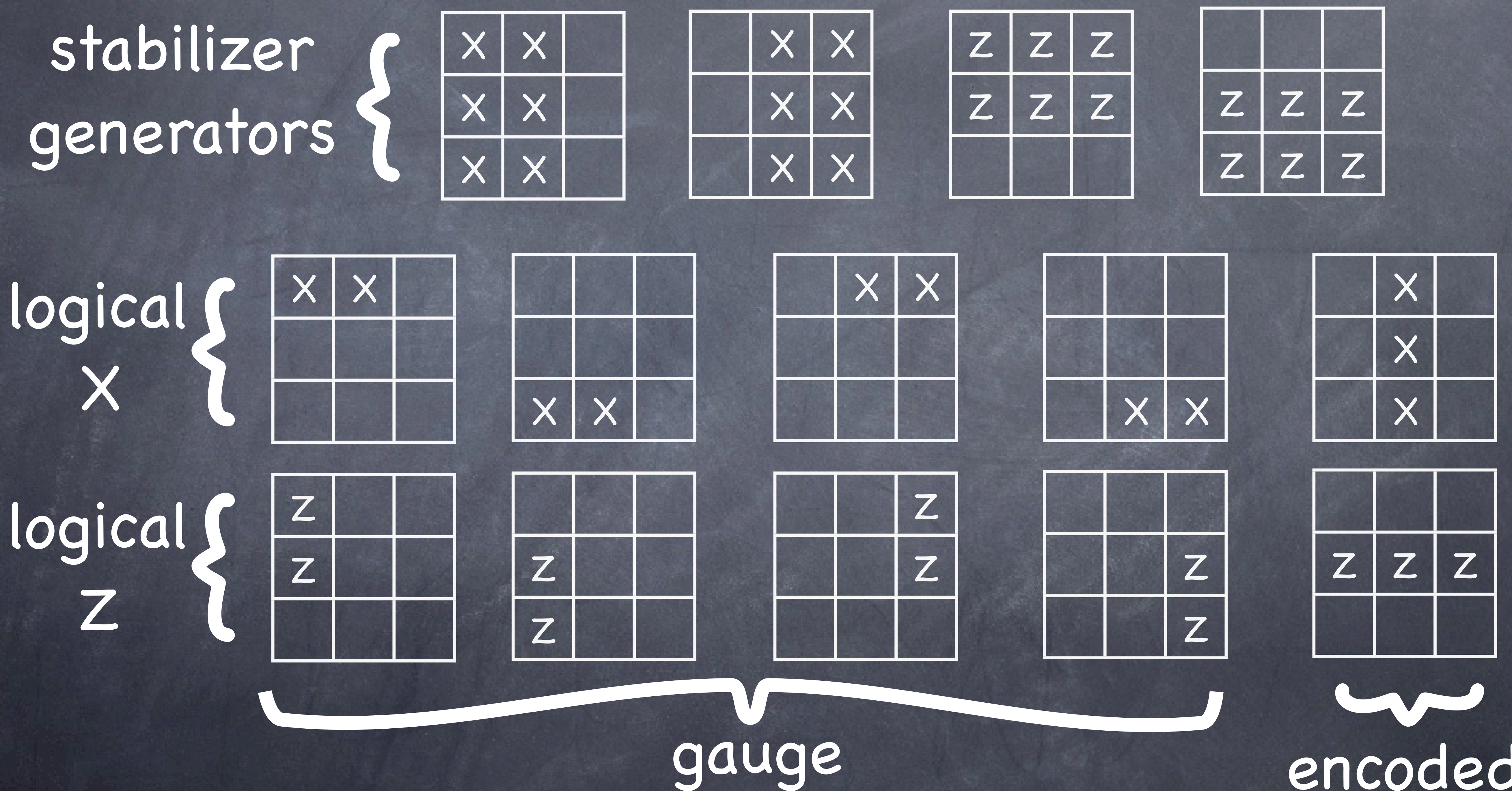


[n,k′,r,d] Stabilizer Subsystem Code

[David Poulin, arXiv:quant-ph/0508131]

A Subsystem Code

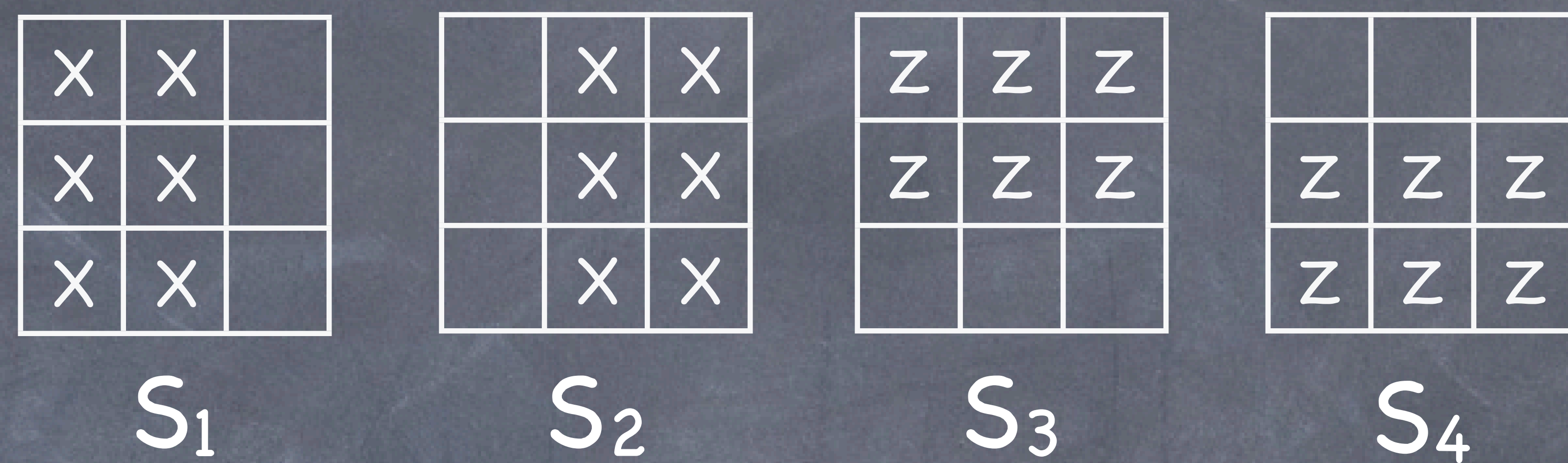
[Bacon, arXiv:quant-ph/0506023]

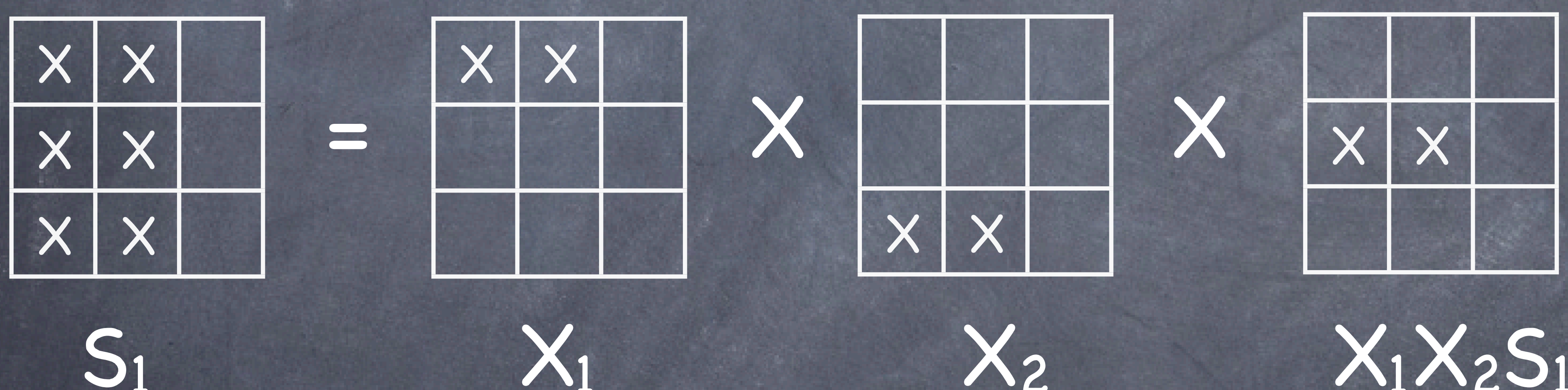


[[9,1,4,3]] stabilizer subsystem code

A Subsystem Code

stabilizer generators





We can measure S_1 by measuring 2-local operators that are products of gauge and stabilizer operators

6-local → 2-local

Stabilizer Moves

S_1	S_2	....	S_{n-k-r}	z_1	z_2	...	z_r	Z_1	Z_2	....	Z_k
				x_1	x_2	...	x_r	X_1	X_2	....	X_k
stabilizer				gauge				logical			

Stabilizer Moves

S	S_2	....	S_{n-k-r}	z_1	z_2	...	z_r	S_{n-k-r}	Z_1	Z_2	....	Z_k
				x_1	x_2	...	x_r	X_0	X_1	X_2	....	X_k

turn a stabilizer into a logical operator

S_1	S_2	....	S_{n-k-r}	z_1	z_2	...	z_r	Z_1	Z_2	....	Z_k
				x_1	x_2	...	x_r	X_1	X_2	....	X_k

stabilizer gauge logical

Stabilizer Moves

S	S_2	....	S_{n-k-r}	z_1	z_2	...	z_r	S_{n-k-r}	Z_1	Z_2	....	Z_k
				x_1	x_2	...	x_r	X_0	X_1	X_2	....	X_k

turn a stabilizer into
a logical operator

S_1	S_2	....	S_{n-k-r}	z_1	z_2	...	z_r	Z_1	Z_2	....	Z_k
				x_1	x_2	...	x_r	X_1	X_2	....	X_k

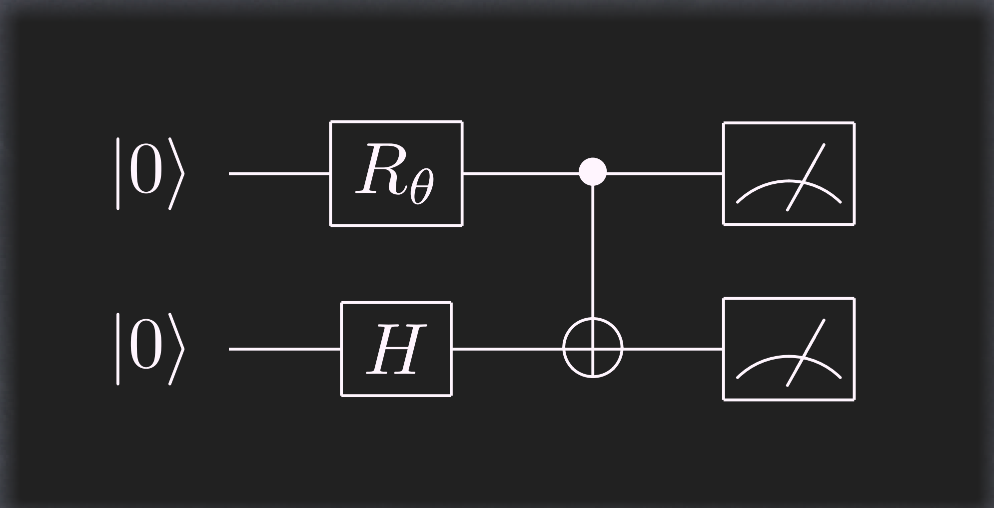
stabilizer gauge logical

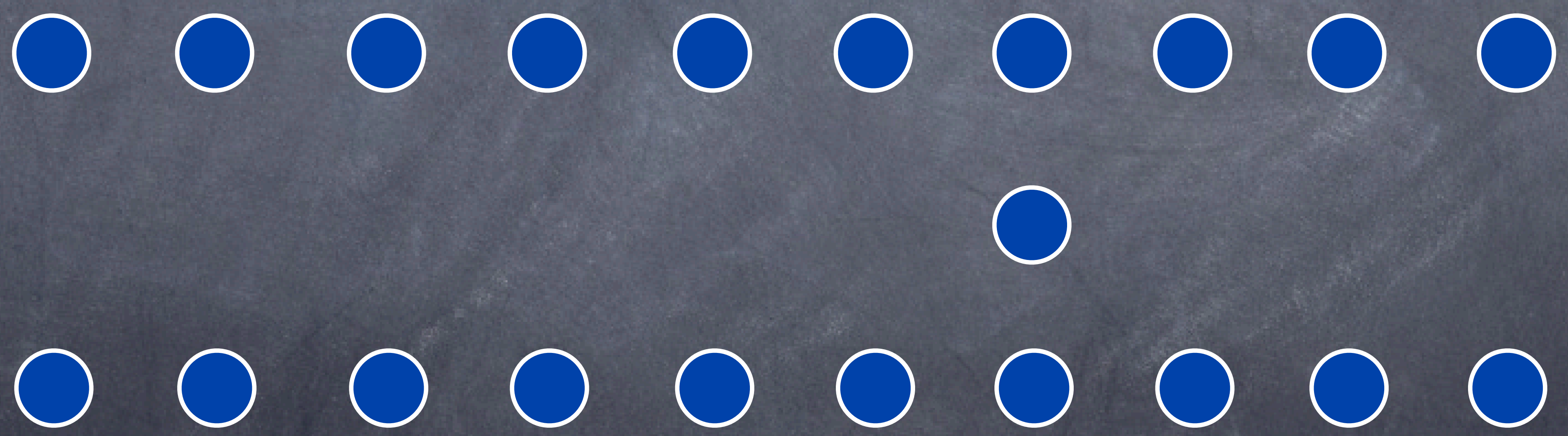
turn a stabilizer into
a gauge operator

S	S_2	....	$S_{n-k-r-1}$	S_{n-k-r}	z_1	z_2	...	z_r	Z_1	Z_2	....	Z_k
				x_0	x_1	x_2	...	x_r	X_1	X_2	....	X_k

2. Measurement Based QC

[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]





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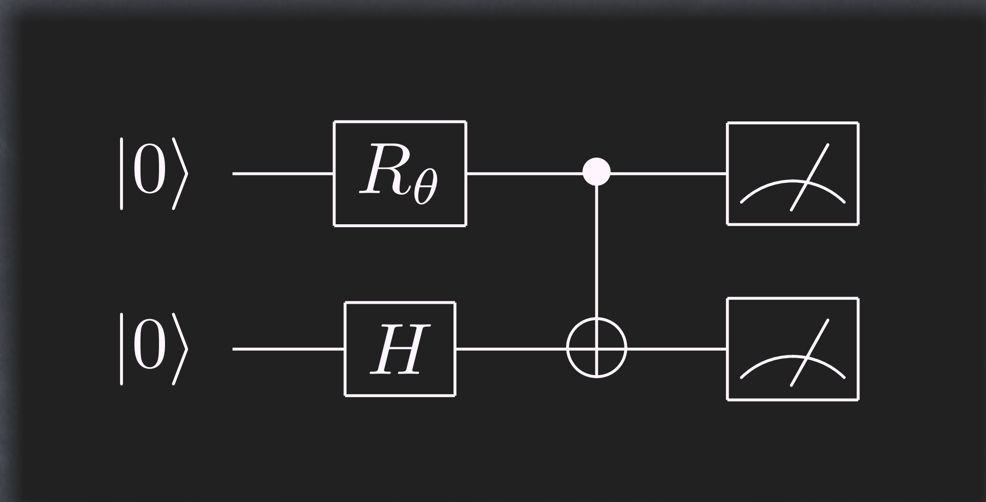
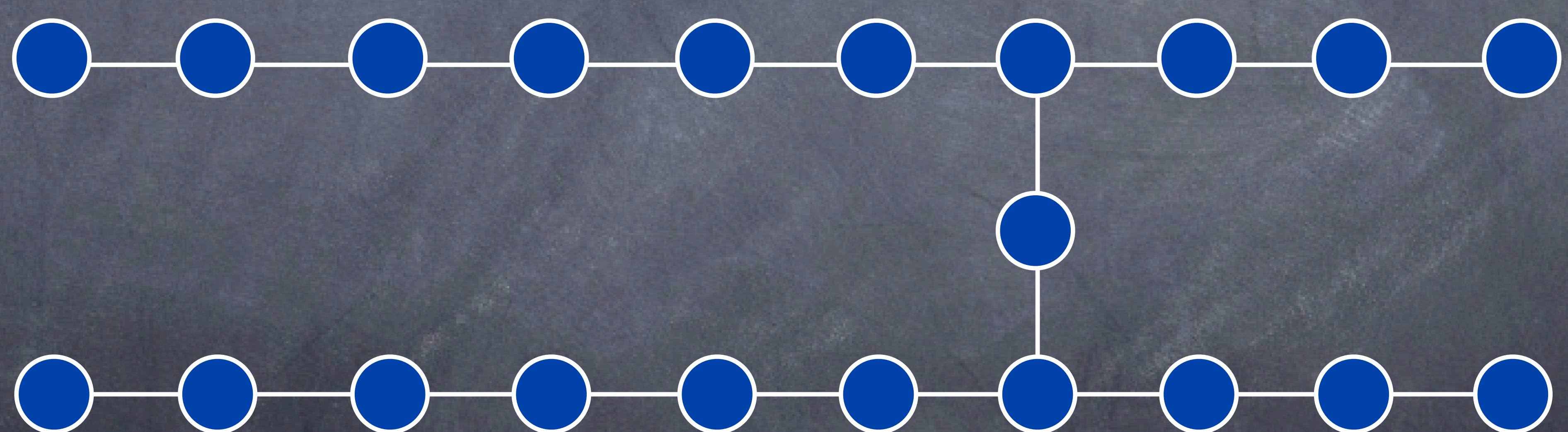
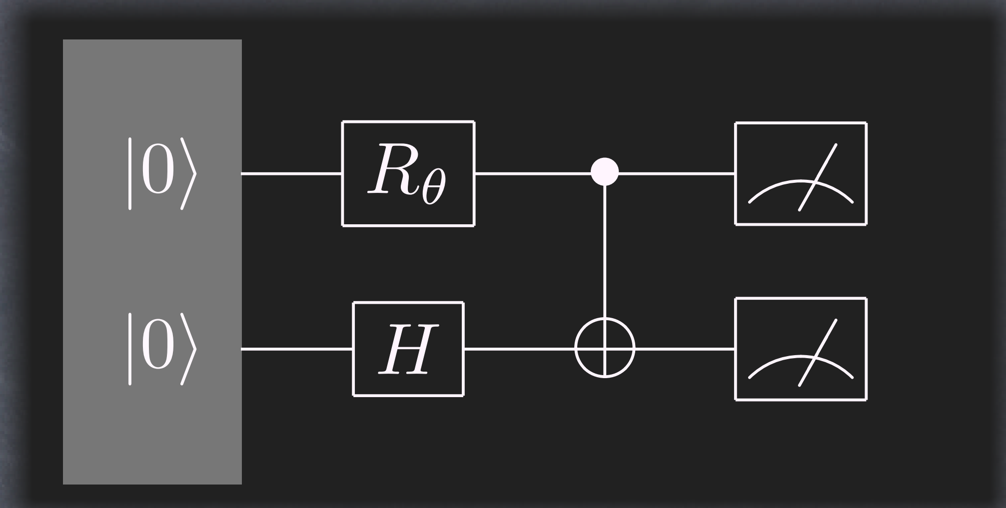


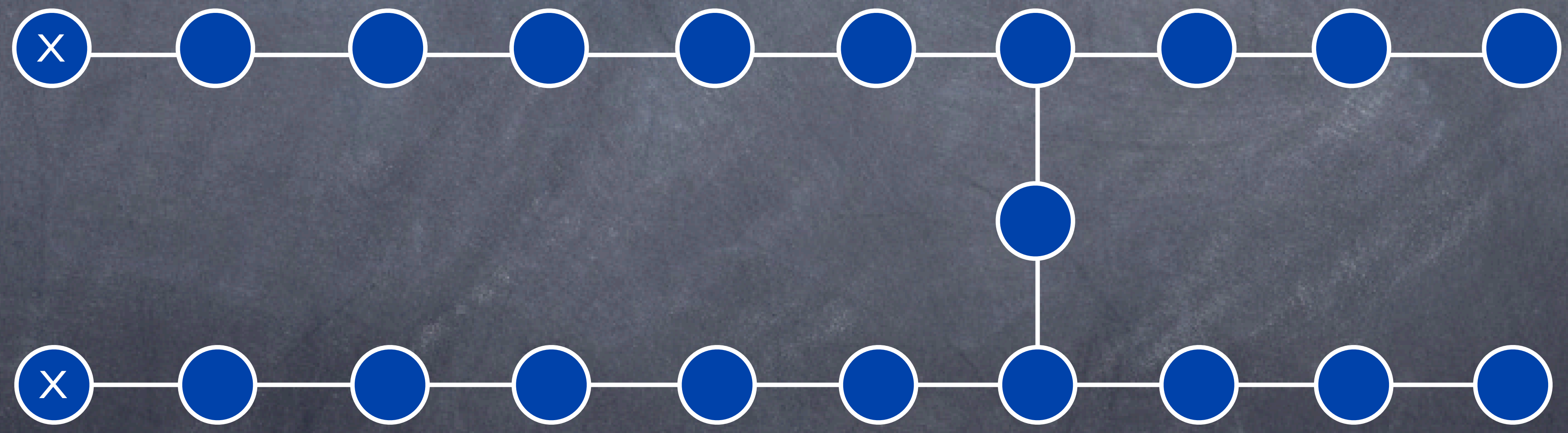


Create Entangled State

2. Measurement Based QC

[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]





Adaptively measure to enact circuit

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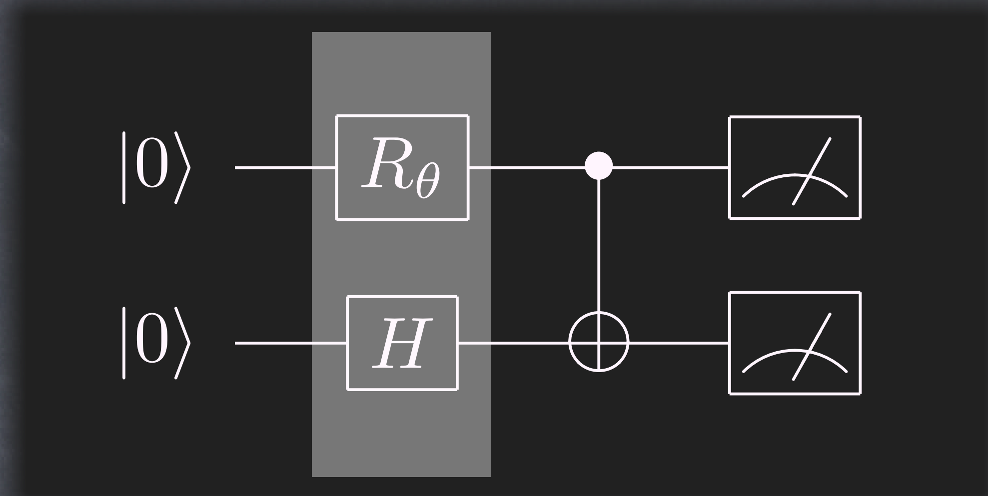

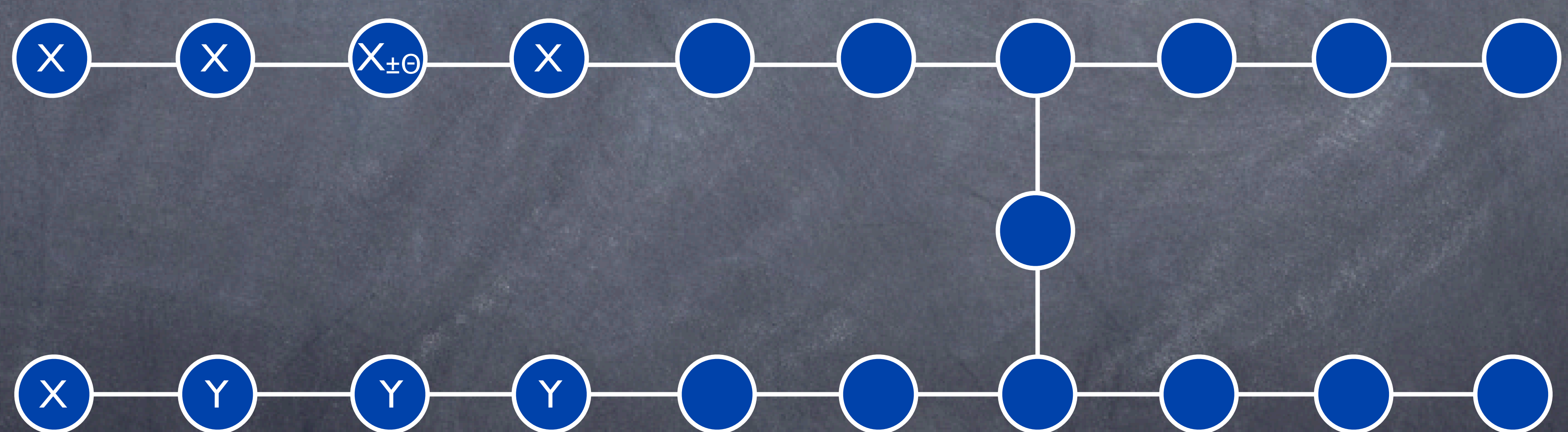

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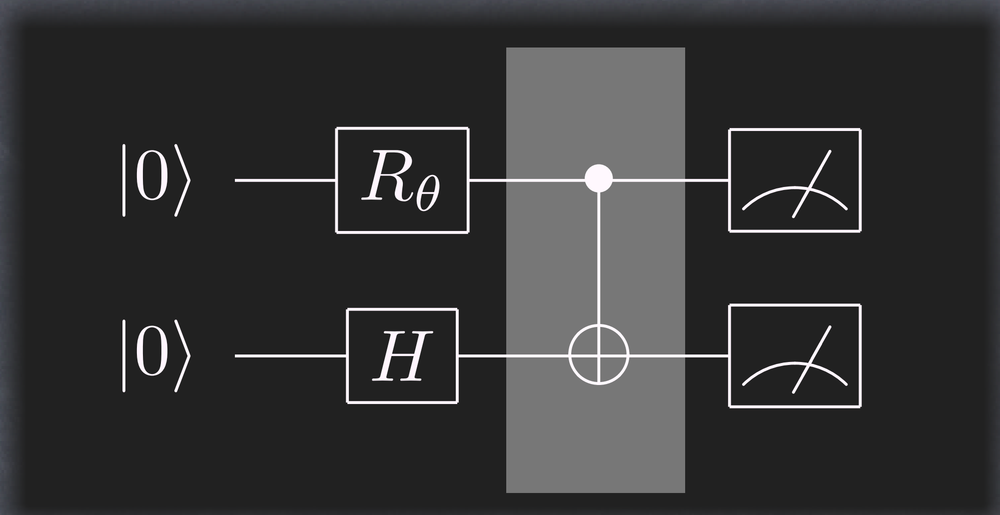

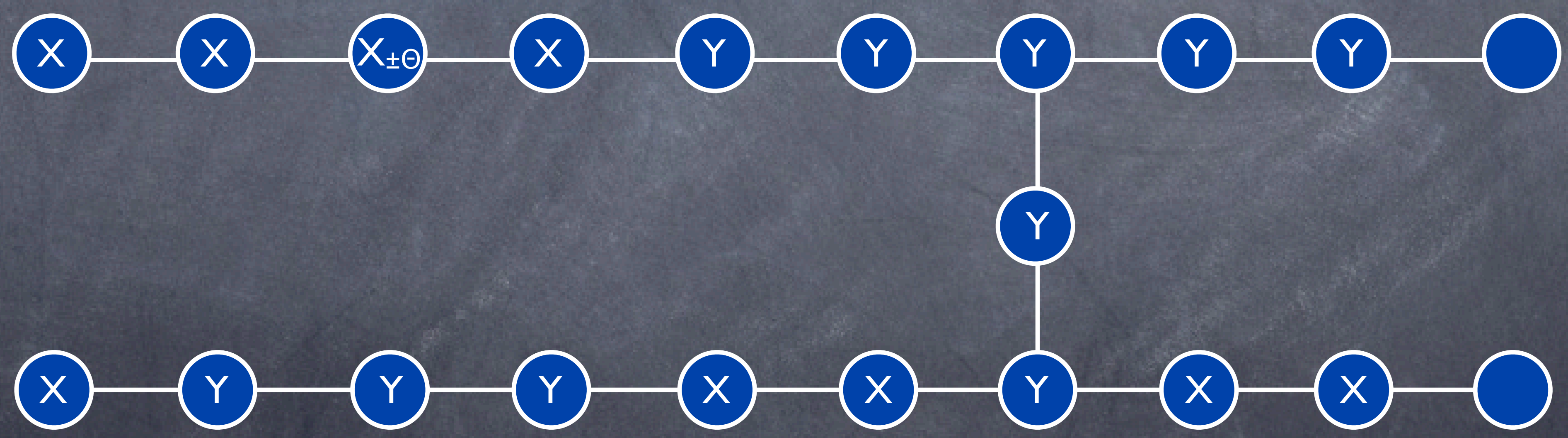

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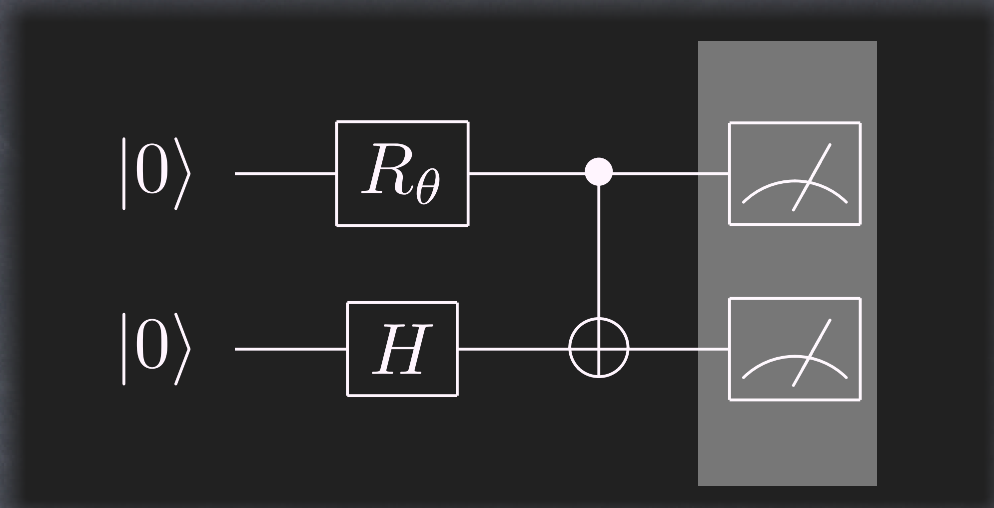
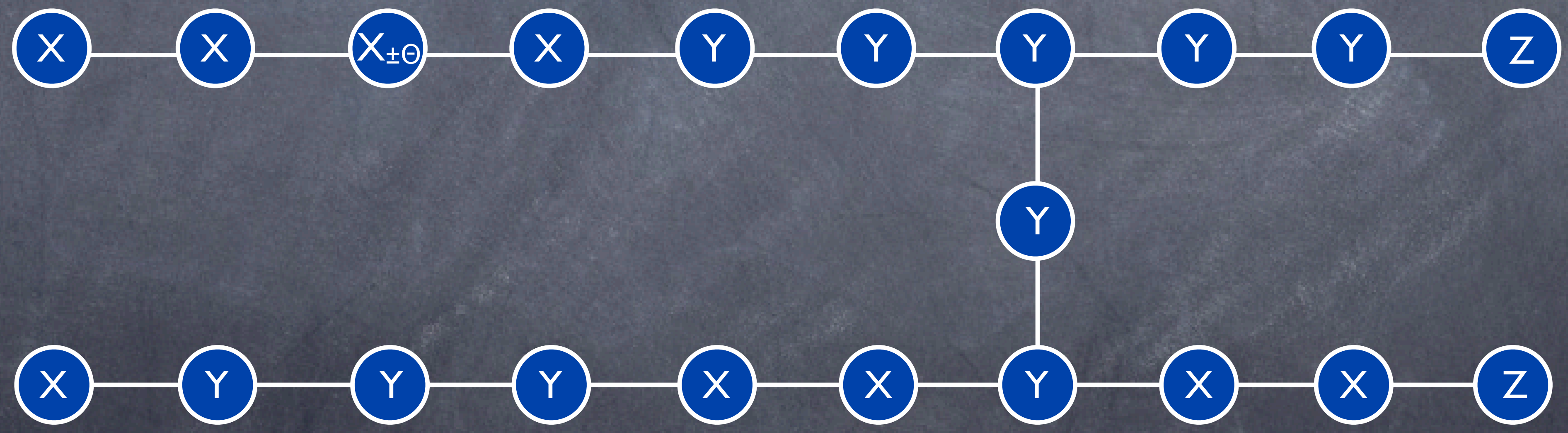
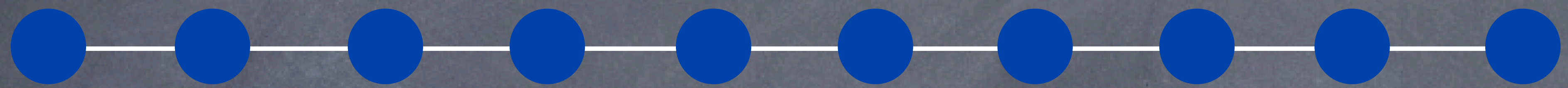




Adaptively measure to enact circuit

Initial state described locally + measurements are all local

MBQC Wire to Code



Graph state 1D line: common +1 eigenstate of graph state vertex operators

S_1=XZII....

S_2=ZXZI....

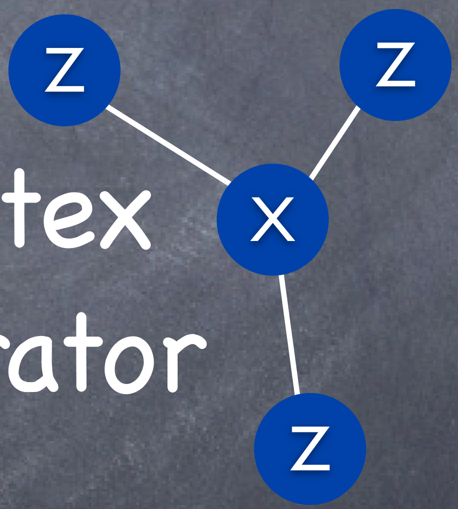
S_3=IZXZ....

...

S_{n-1}=....IZXZ

S_n =....IIXZ

vertex operator



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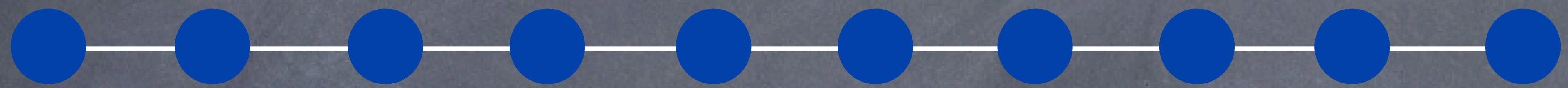


Remove end stabilizer from stabilizer group and promote it to a logical operator

Stabilizer generators

~~S_1=XZII....~~

S_2=ZXZI....

S_3=IZXZ....

...

S_{n-1}=....IZXZ

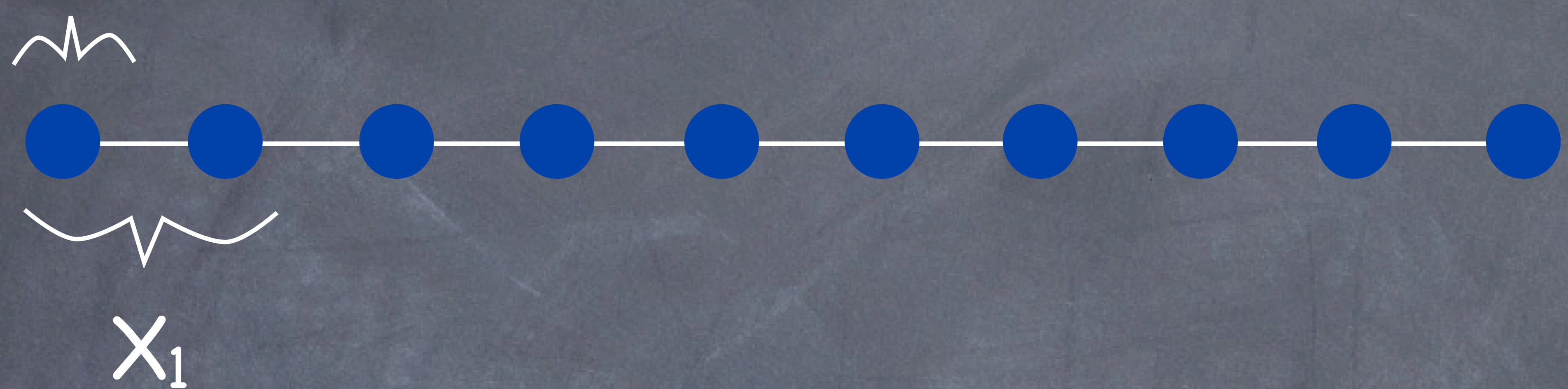
S_n =....IIXZ

Logical operators

X_1=XZIIII, Z_1=ZIIIII

One encoded distance 1 qubit

Qubit Where?



<u>Stabilizer generators</u>

~~S_1=XZII....~~

S_2=ZXZI....

S_3=IZXZ....

...

S_{n-1}=....IZXZ

S_n =....IIXZ

<u>Logical operators</u>

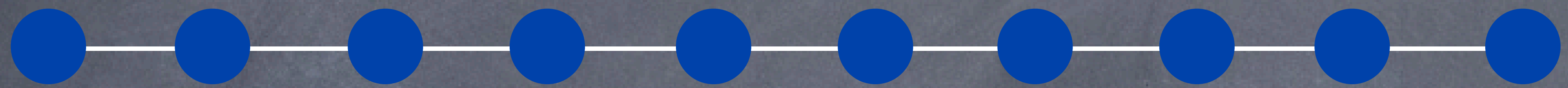
X_1=XZIIII..., Z_1=ZIIIII...

S_2Z_1=IXZIII...

S_4Z_1=IXIXZI...

modulo stabilizer information can be accessed across many different r-local measurements

MBQC Wire to Code



Make S_2 a gauge qubit

<u>Stabilizer generators</u>

~~S_1=XZII....~~

~~S_2=ZXZI....~~

S_3=IZXZ....

...

S_{n-1}=....IZXZ

S_n =....IIXZ

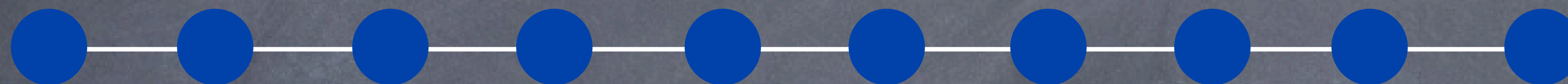
<u>Logical operators</u>

X_1=XZII..., Z_1=ZIII...

<u>Gauge operators</u>

x_1=XIII..., z_1=ZXZI...

MBQC Wire to Code



Make S_2 a gauge qubit

<u>Stabilizer generators</u>

~~S_1=XZII....~~

~~S_2=ZXZI....~~

S_3=IZXZ....

...

S_{n-1}=....IZXZ

S_n =....IIXZ

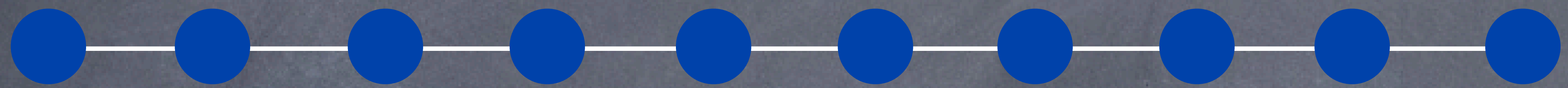
<u>Logical operators</u>

~~X_1=XZII..., Z_1=ZIII...~~

<u>Gauge operators</u>

x_1=XIII..., z_1=ZXZI...

MBQC Wire to Code



Make S_2 a gauge qubit

<u>Stabilizer generators</u>

~~S_1=XZII....~~

~~S_2=ZXZI....~~

S_3=IZXZ....

...

S_{n-1}=....IZXZ

S_n =....IIXZ

<u>Logical operators</u>

X_1=XZII..., Z_1=IXZI...

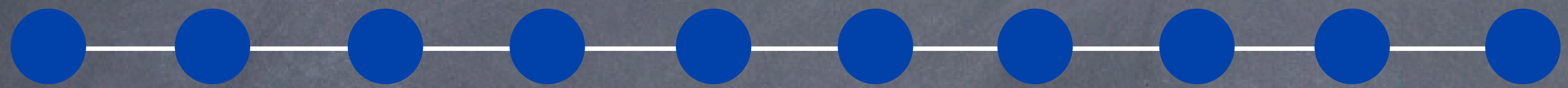
<u>Gauge operators</u>

x_1=XIII..., z_1=ZXZI...

X_1x_1=IZII...

Z_1 moved over a bit...

MBQC Wire to Code



<u>Stabilizer generators</u>

none

<u>Logical operators</u>

X_1=XIXI…XIX, Z_1=IXIX…IXZ

Modulo gauge operators
localized to last qubit

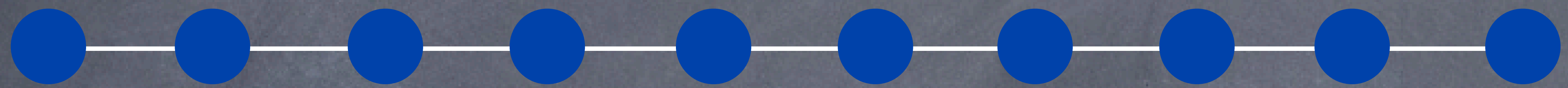
<u>Gauge operators</u>

Generated by

XIII..II, IXII…II, … , IIII.,.XI

ZXZI..II, IZXZ…II, … , IIII…ZX

MBQC Wire to Code



<u>Stabilizer generators</u>

S_1=XIXI…XIX

<u>Gauge operators</u>

Generated by

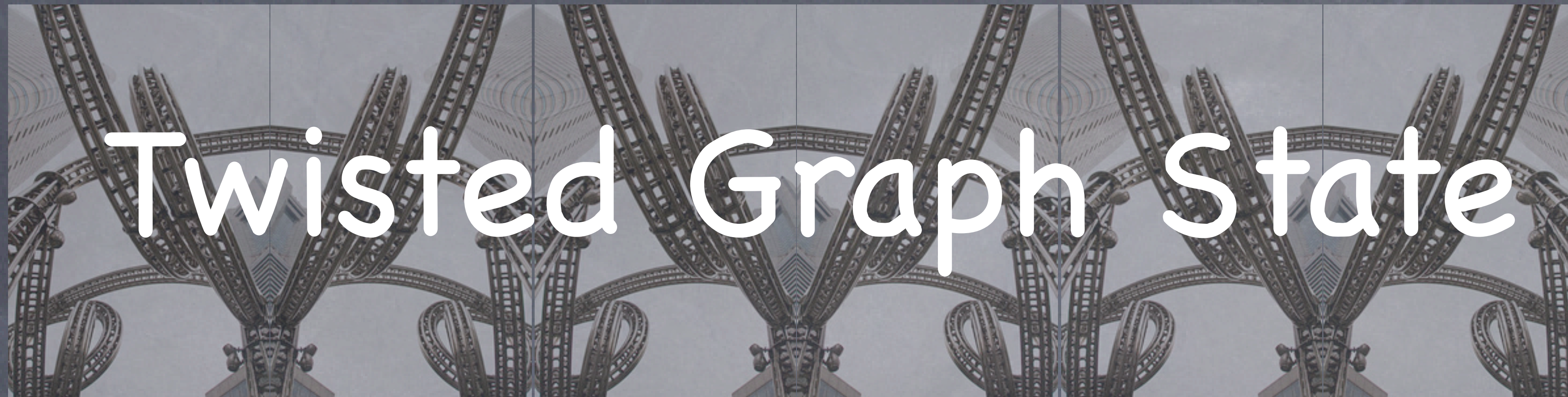
XIII..II, IXII…II, … , IIII.,.XI

ZXZI..II, IZXZ…II, … , IIII…ZX

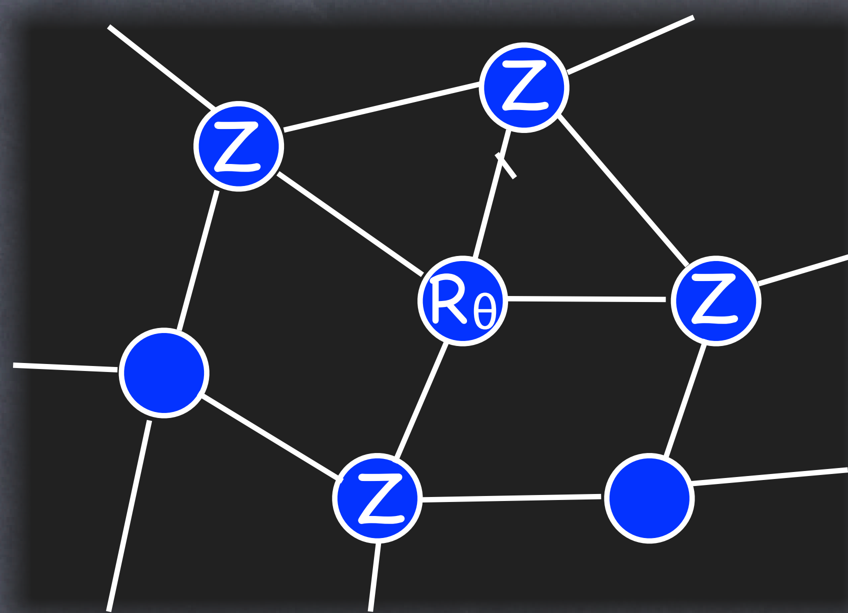
<u>Logical operators</u>

none

Measure S_1 by series of single qubit X measurements (gauge)



Twisted Graph State



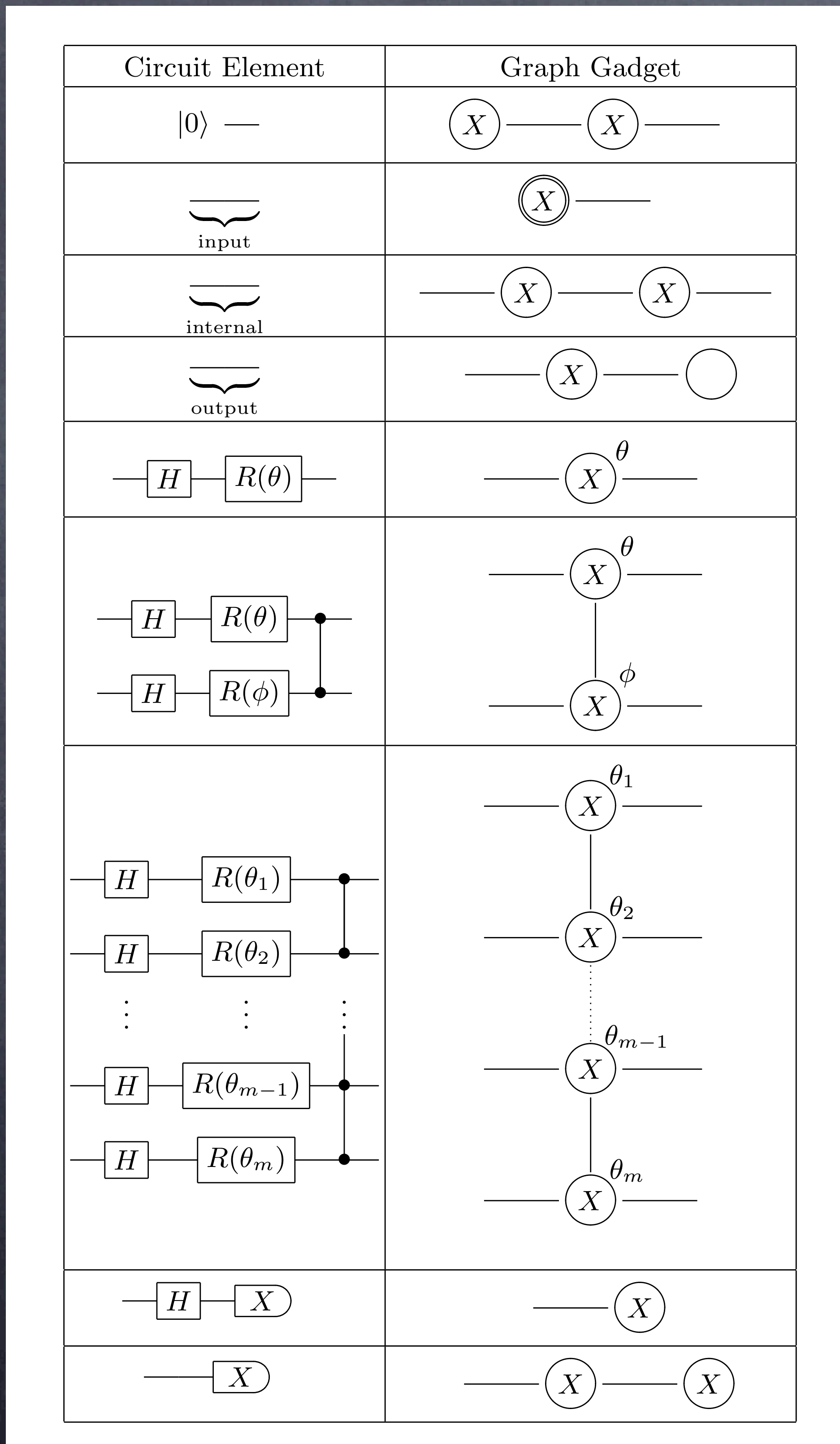
$$= \; [R_\theta]_v \prod_{w|(v,w)\in\mathcal{E}} [Z]_w$$

$$R_\theta = \cos\theta X + \sin\theta Y$$

"Vertex operator with angle θ"

<u>Twisted Graph State</u>: is common +1 eigenstate of all vertex operators (one per vertex)

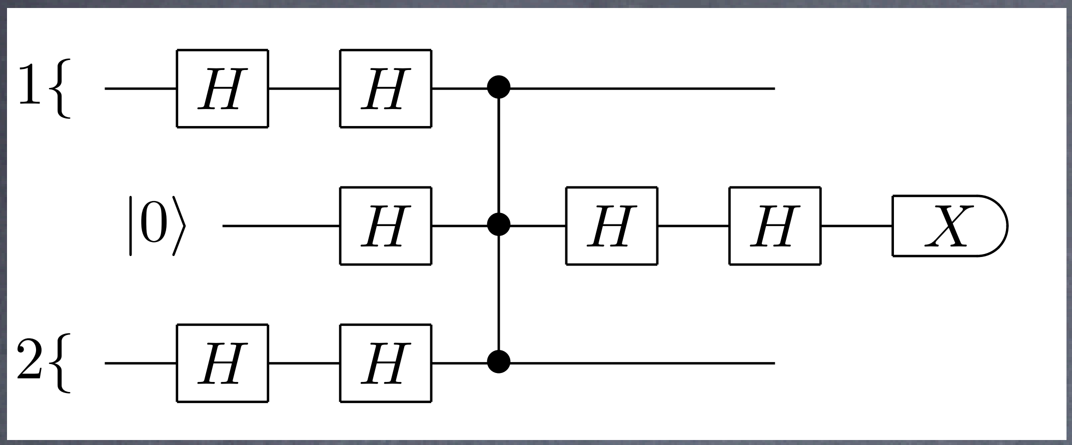
QC to Twist Graph State

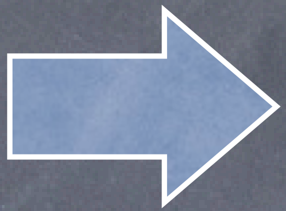


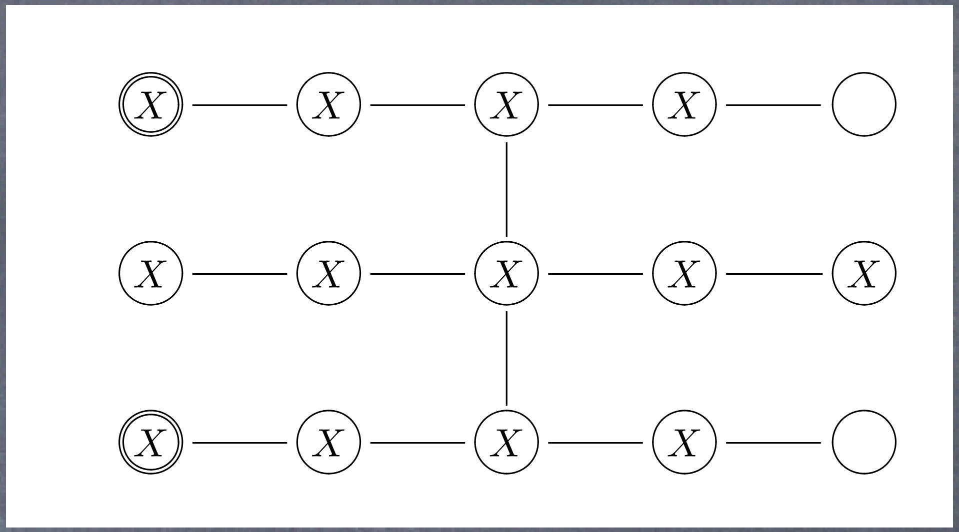
Quantum circuit made up of preparations, gates, and measurements

Graph describing vertex operators and places where X measurements would be made to enact MBQC for the quantum circuit

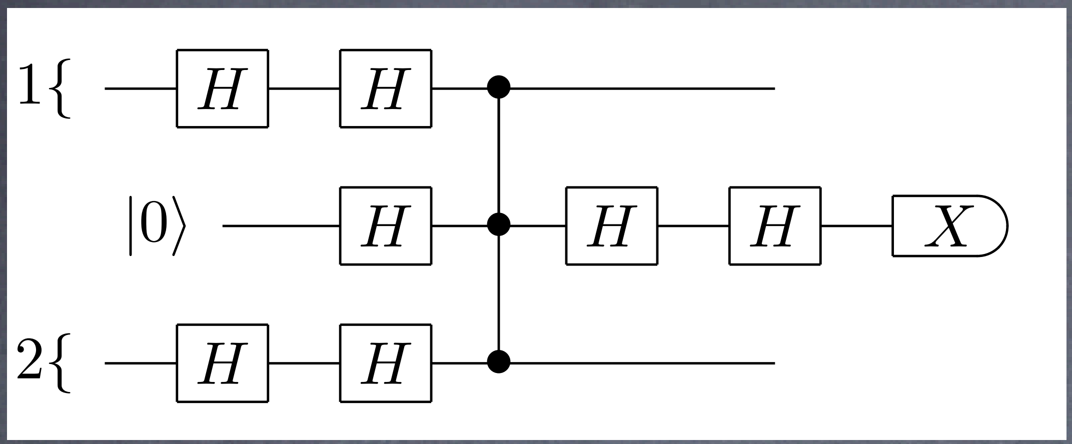
Encoded MBQC Example

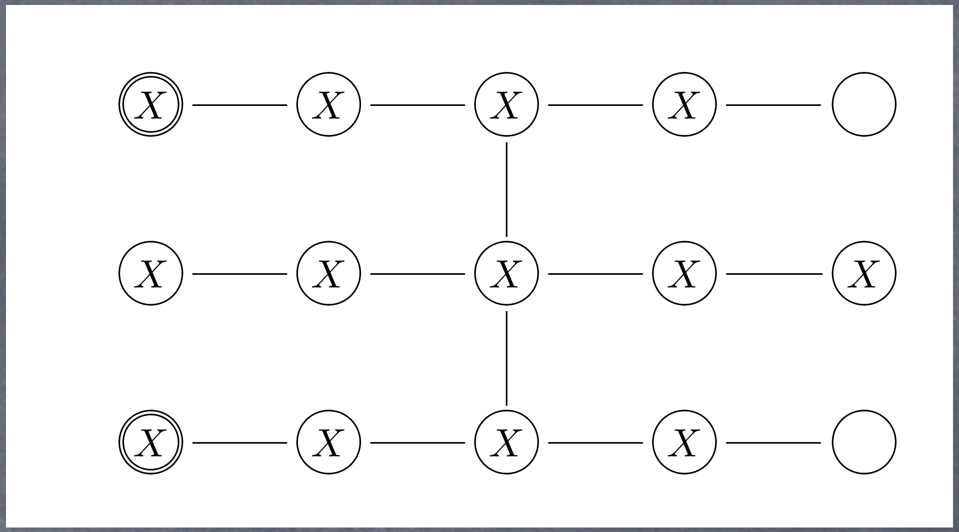






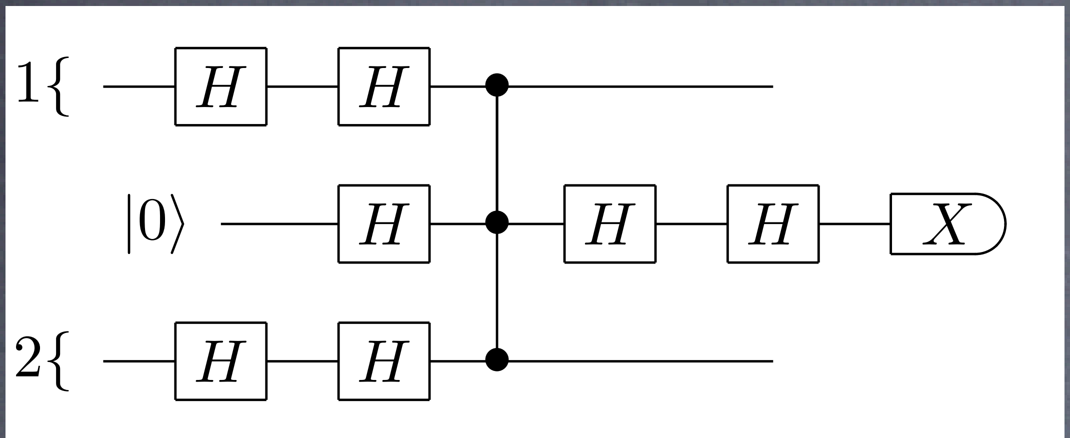
Encoded MBQC Example

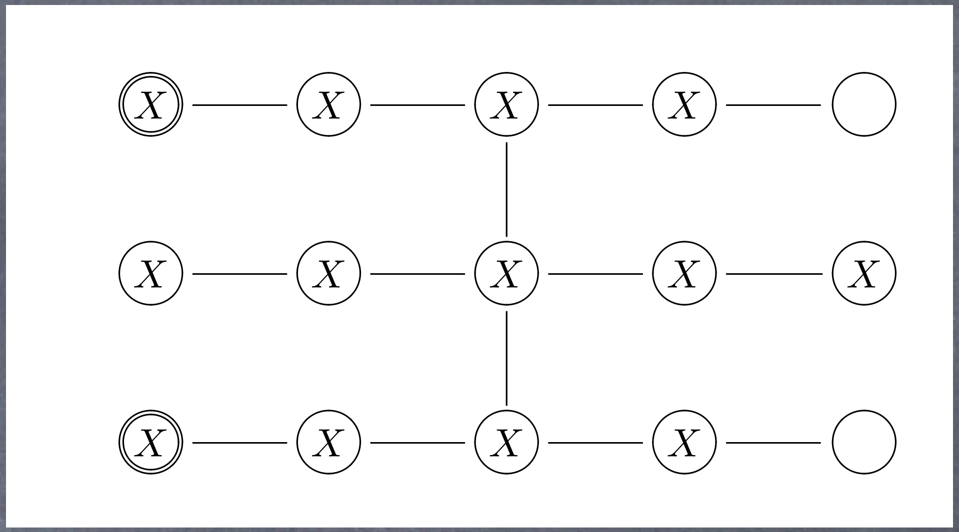




1. vertex operator for every non-double-circled node

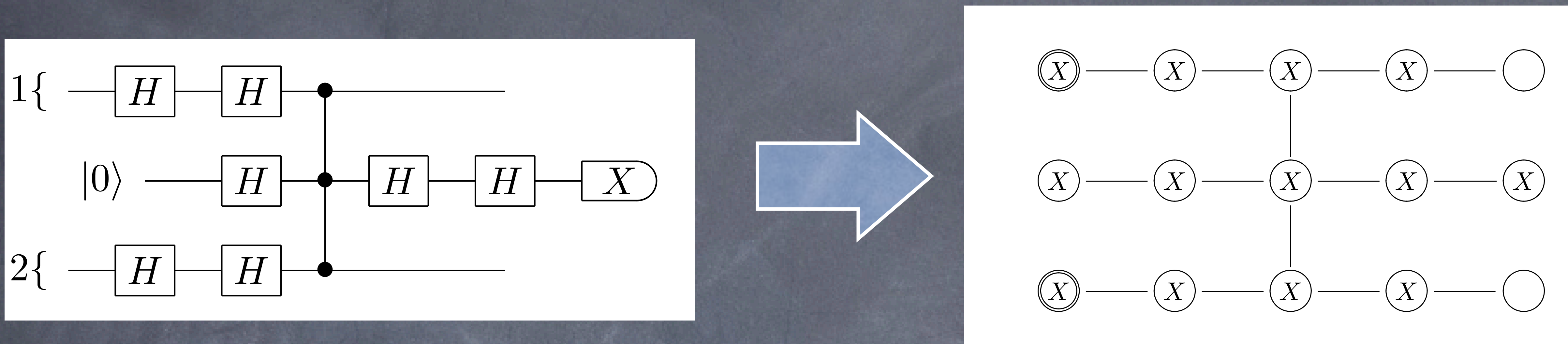
Encoded MBQC Example





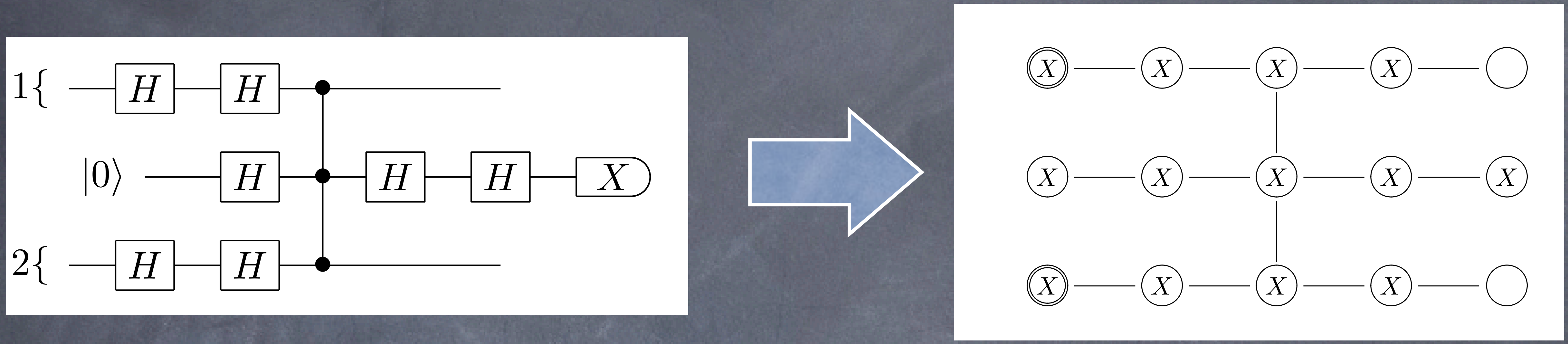
1. vertex operator for every non-double-circled node
2. starting state is +1 common eigenspace of these vertex operators

Encoded MBQC Example



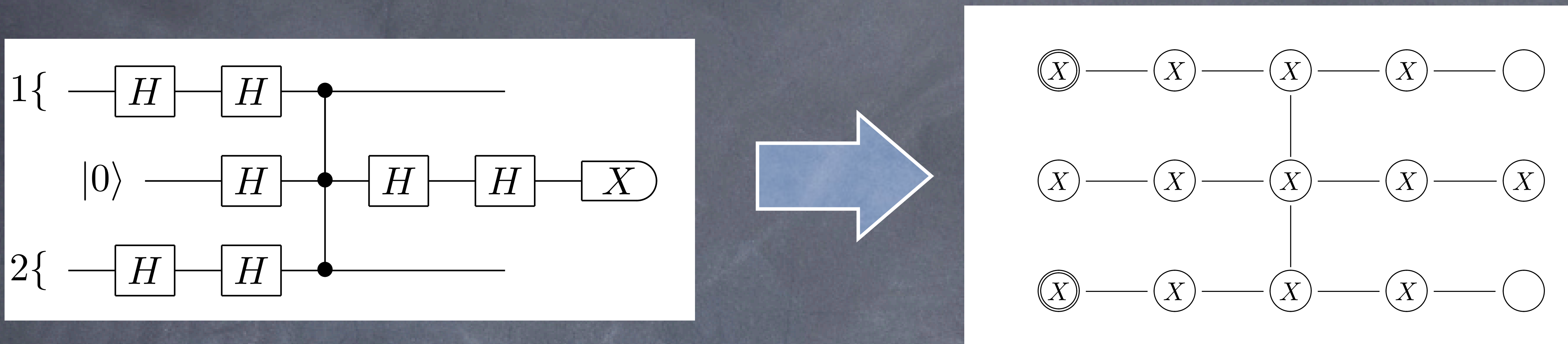
1. vertex operator for every non-double-circled node
2. starting state is +1 common eigenspace of these vertex operators
3. input information encoded @ double-circled nodes

Encoded MBQC Example



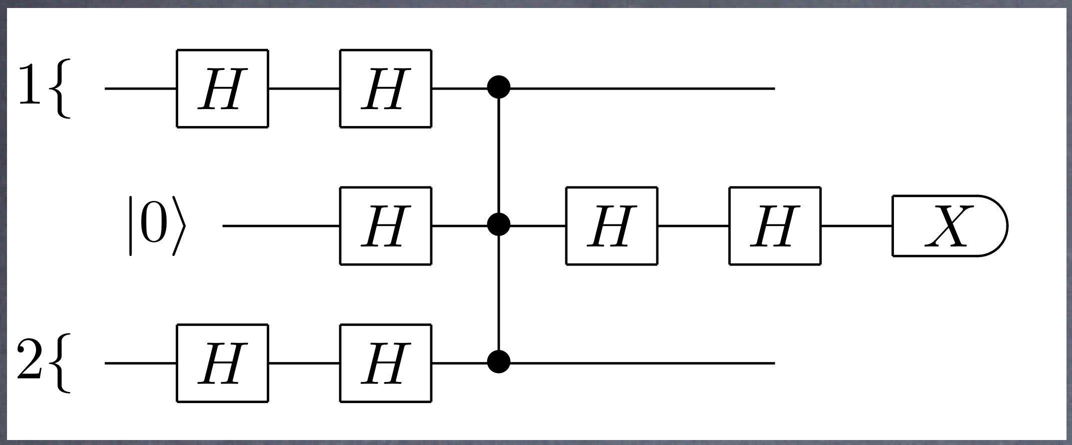
1. vertex operator for every non-double-circled node
2. starting state is +1 common eigenspace of these vertex operators
3. input information encoded @ double-circled nodes
4. X measurements occur on all vertices labeled X

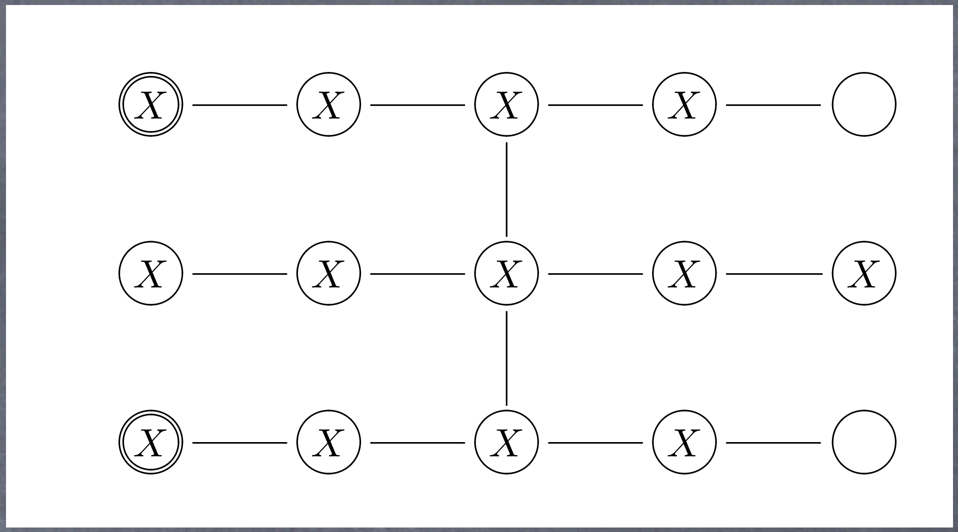
Encoded MBQC Example



1. vertex operator for every non-double-circled node
2. starting state is +1 common eigenspace of these vertex operators
3. input information encoded @ double-circled nodes
4. X measurements occur on all vertices labeled X
5. Output measurements at X's, unmeasured output qubits at non-X-ed nodes

Encoded MBQC Example

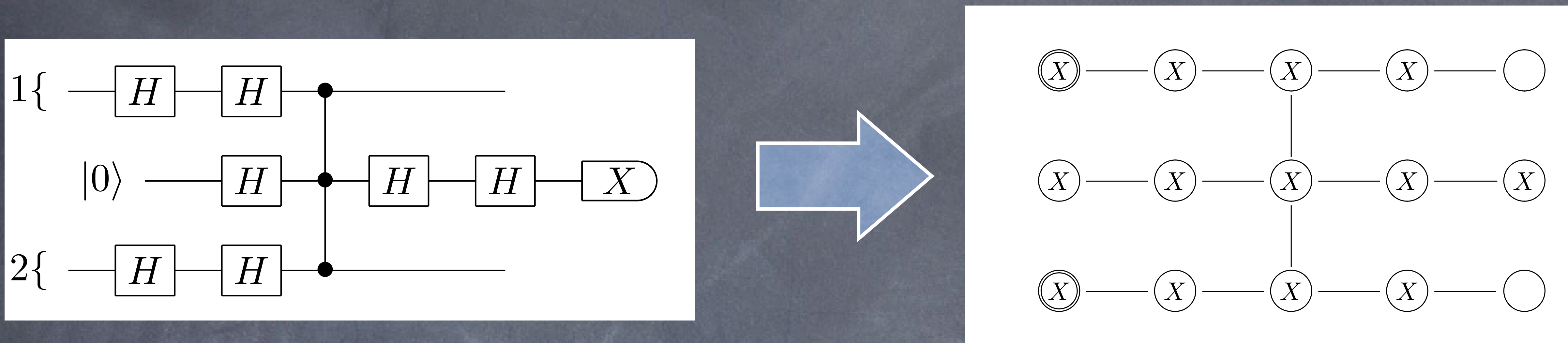


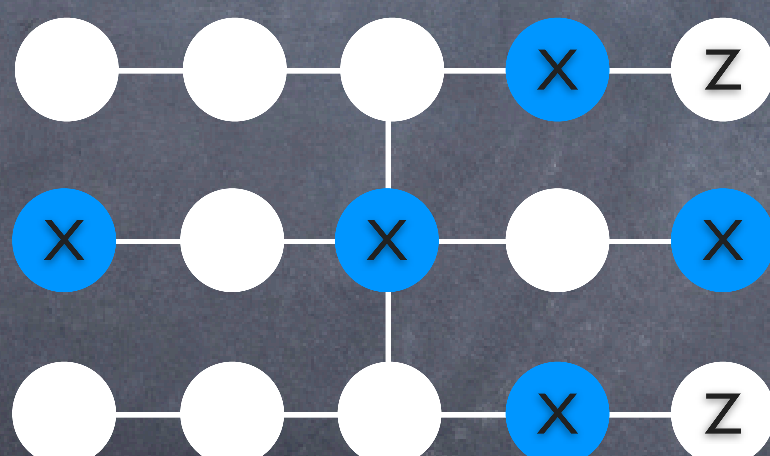


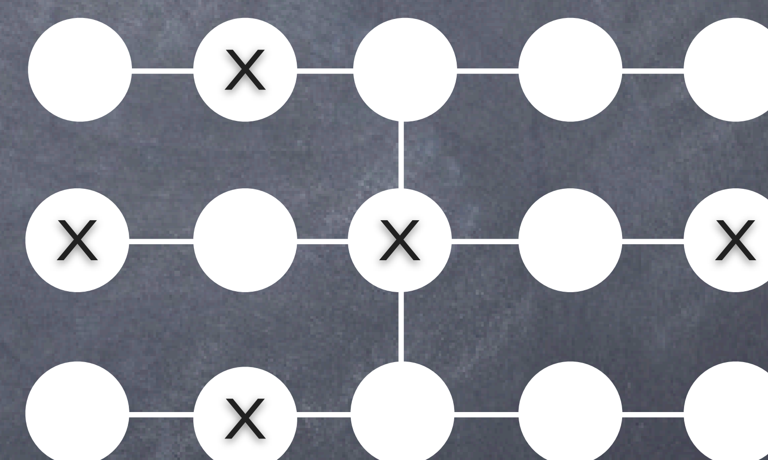
1. vertex operator for every non-double-circled node
4. X measurements occur on all vertices labeled X

Instead of performing MBQC we can instead consider the code generated by these two sets of operators.

In Action

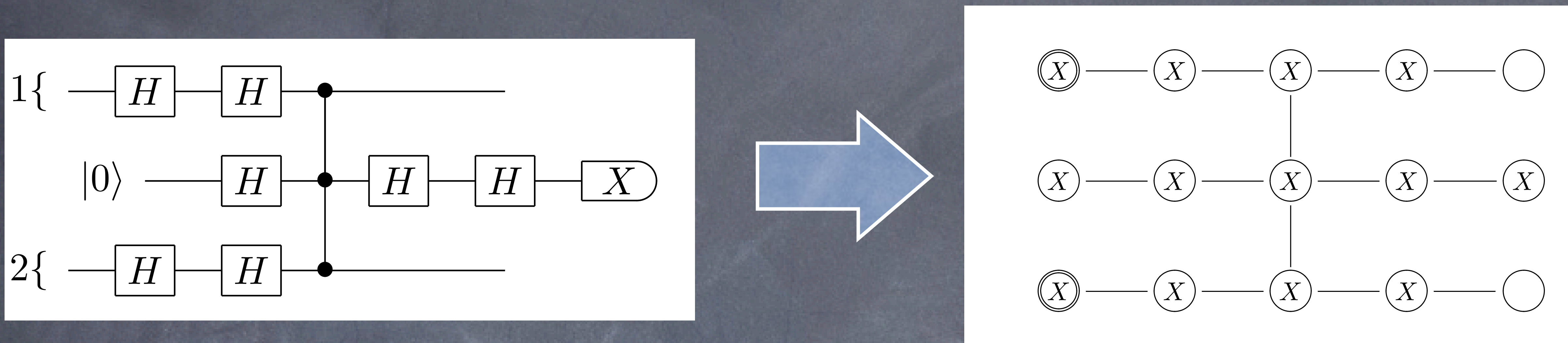


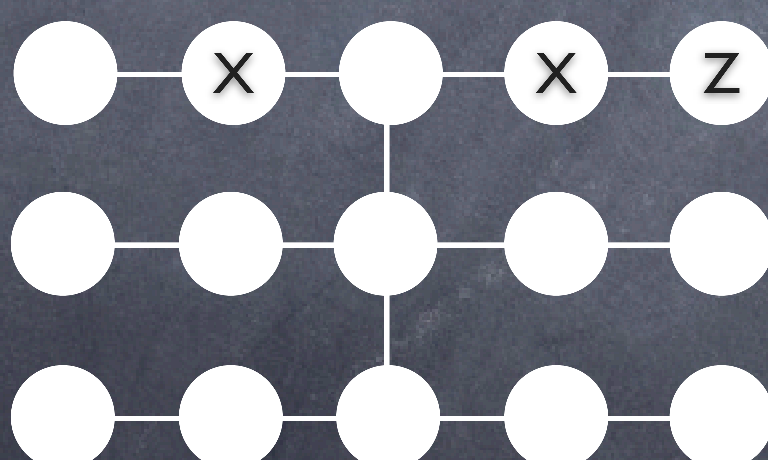
$S_1 =$ 

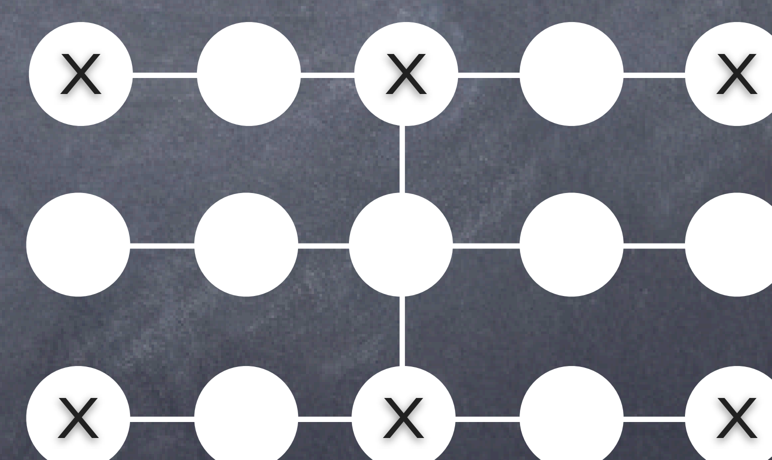
$S_2 =$ 

S_1 measured by measuring blue vertex operators.
S_2 represents that preparation and measurement coincide.

In Action



$\bar{Z} =$ 

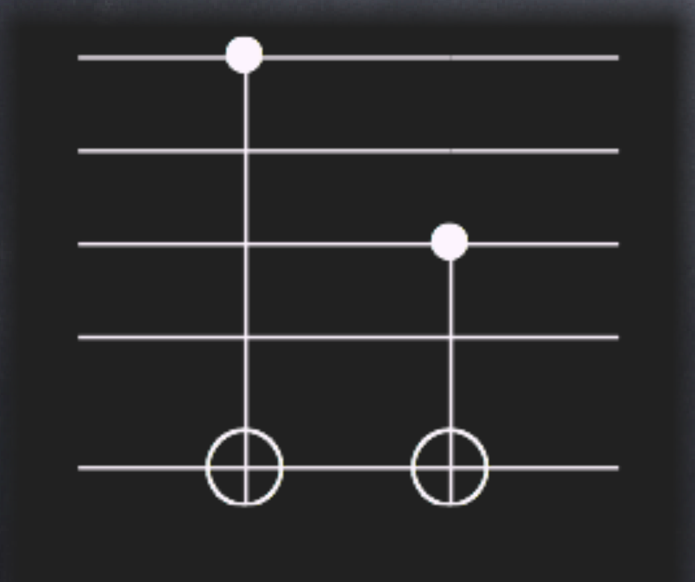
$\bar{X} =$ 

Modulo gauge and stabilizer group logical Z can be made weight 1, but logical X can only be made weight 2

3. Sidebar: Space-Time Neighboring

Any quantum circuit can be made to be neighboring in space-**time** for any spatial dimension D

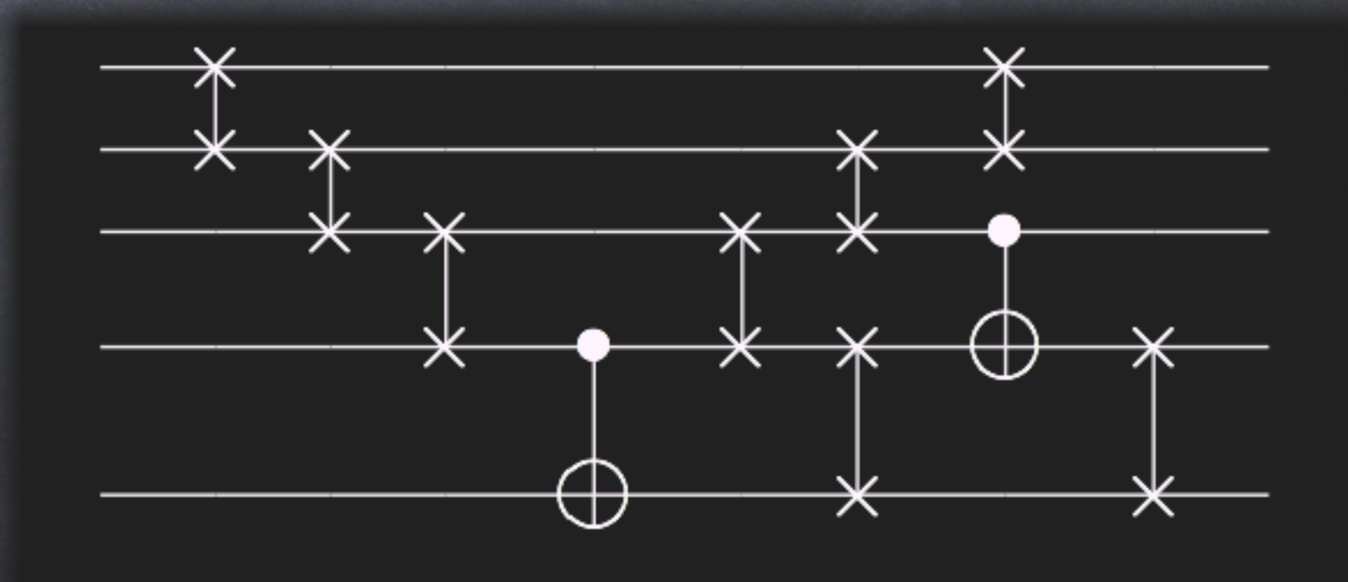
Example in 1D



space

time

$\cong$



space

time

Space-Time QEC

Fault-tolerant quantum error correction can be done using s-neighboring locality in space-time while maintaining a threshold.

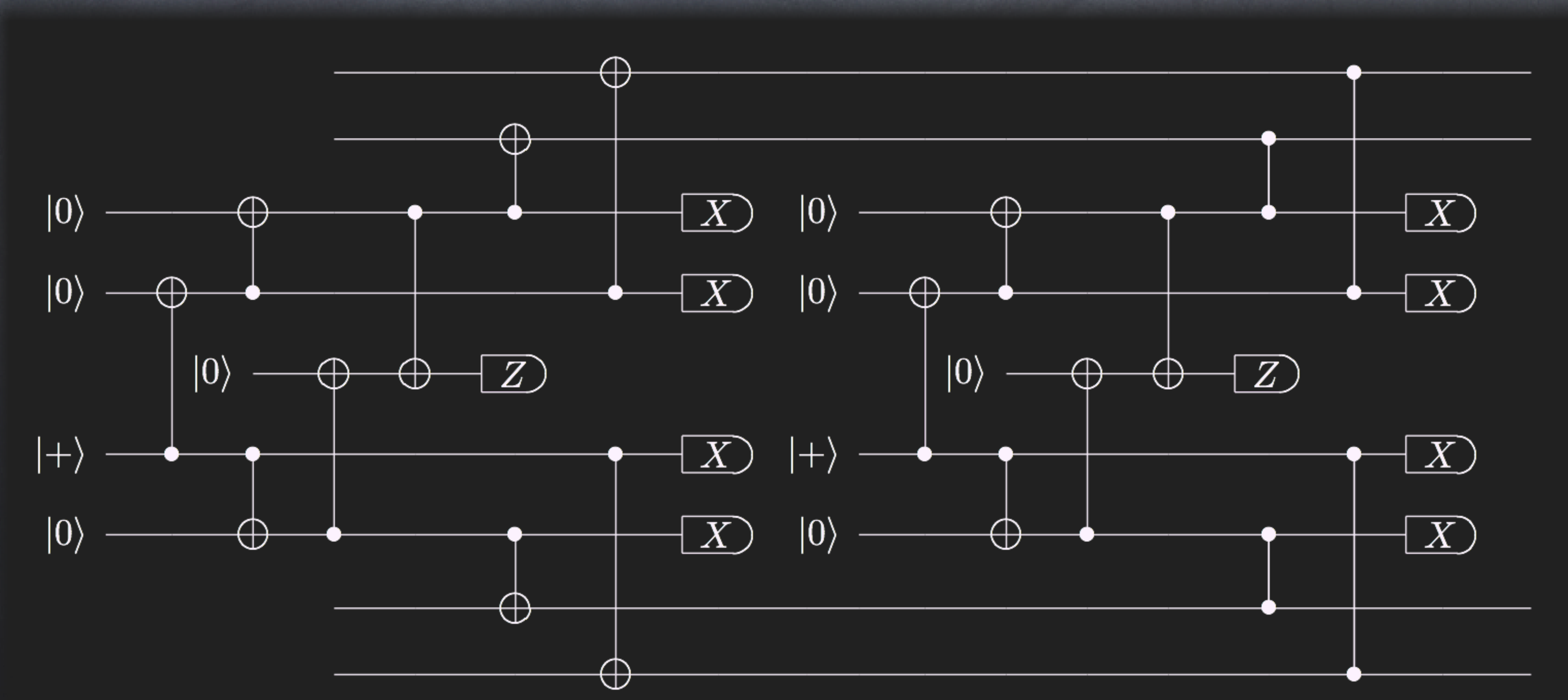
(1D requires next-nearest neighbor gates)

[Aharonov, Ben-Or, arXiv:quant-ph/9906129]
[Gottesman, arXiv:quant-ph/9903099]

THE CONSTRUCTION

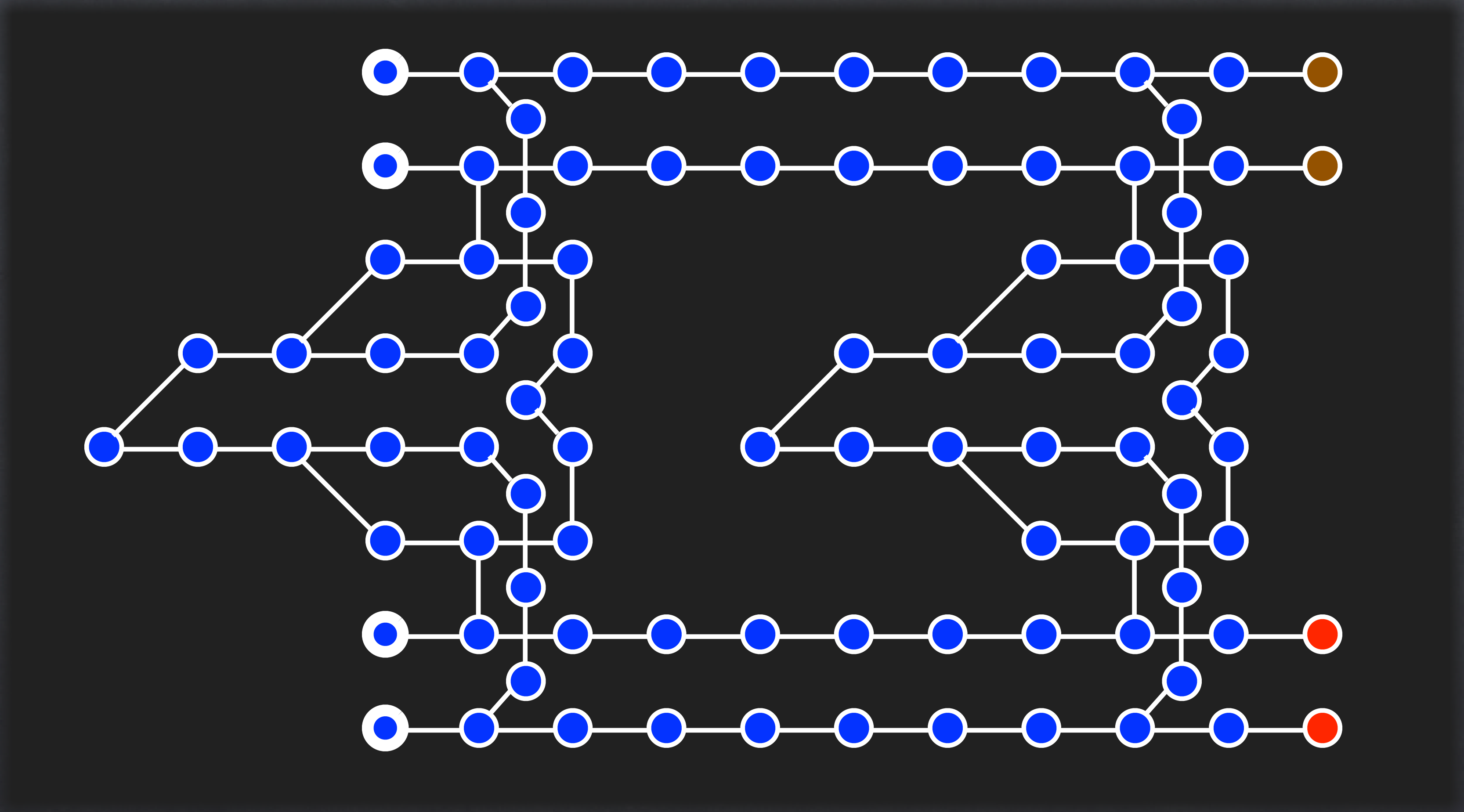
Input: [[n,k,d]] stabilizer code

1. Construct fault-tolerant syndrome measuring quantum circuit that is s-neighboring in space-time on lattice in dimension D (D>1)



CONSTRUCTION

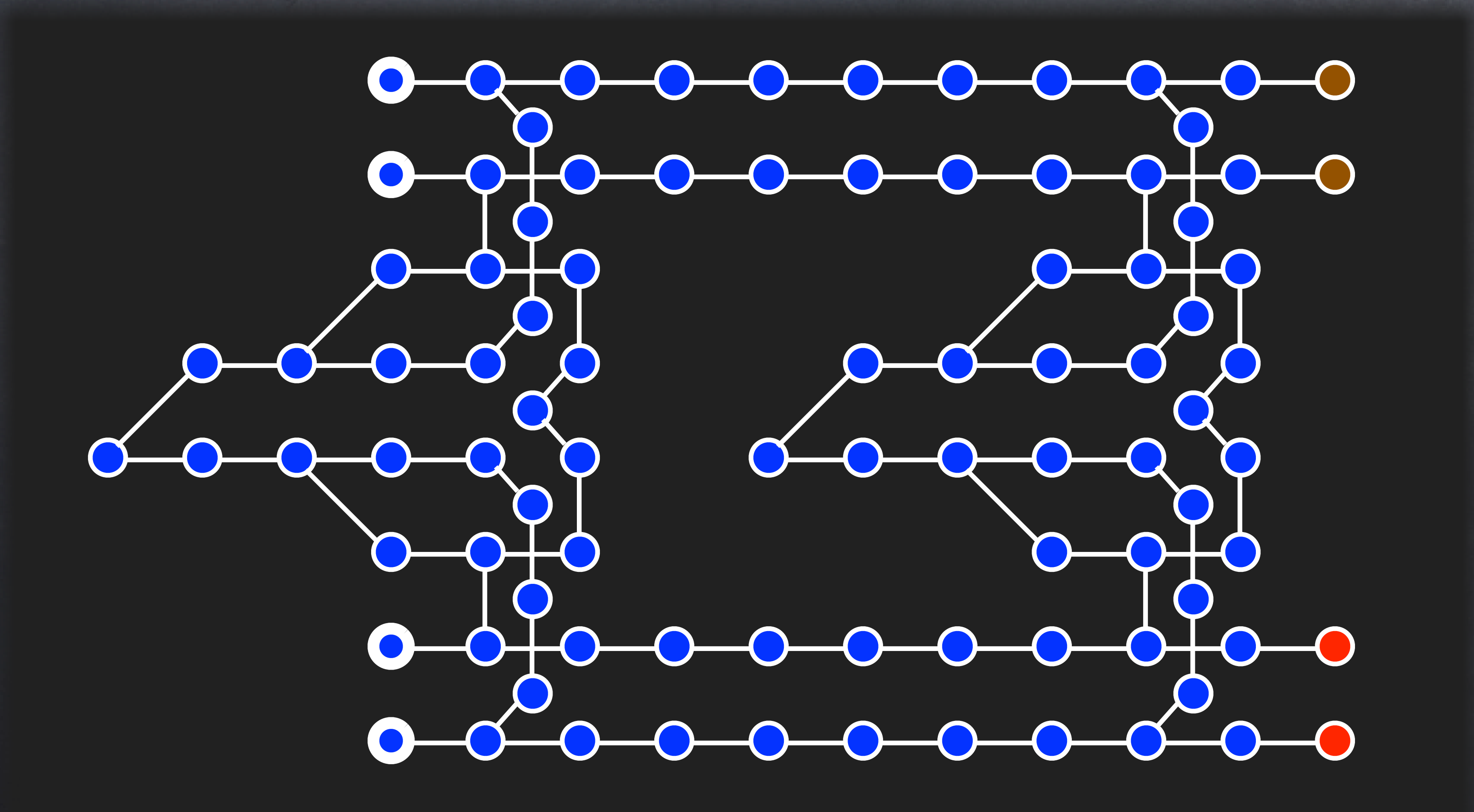
2. Convert this circuit to the scheme that would be used to implement it in MBQC



[Related: Rausendorf, Harrington, Goyal 2006]

CONSTRUCTION

3. Take operators from MBQC construction (vertex operators and X operators for measurements) and identify (a) stabilizer subgroup of these operators and (b) gauge qubits

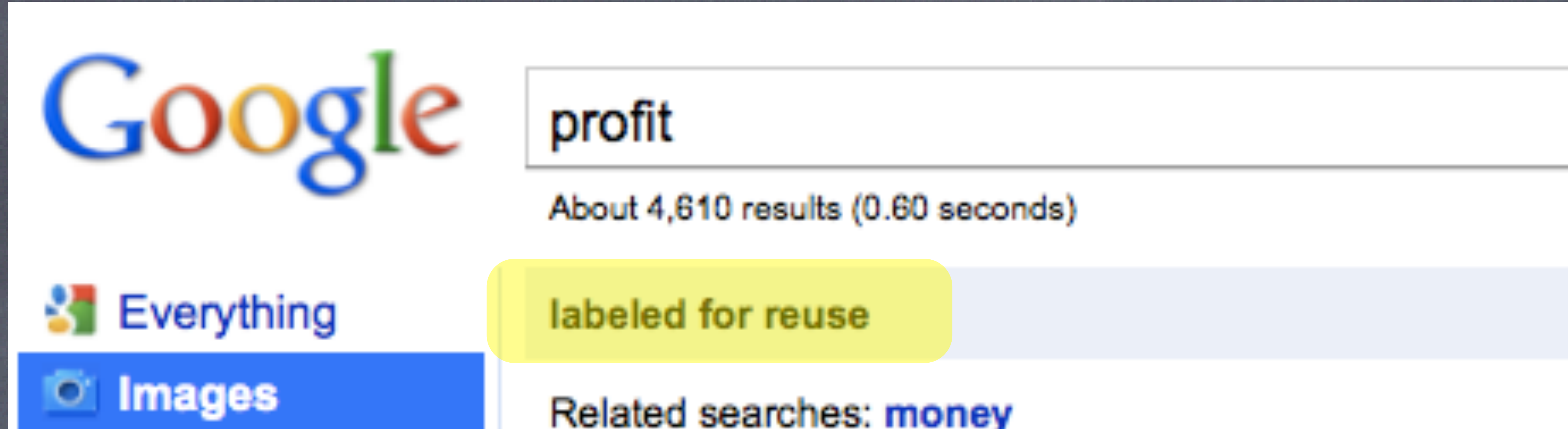


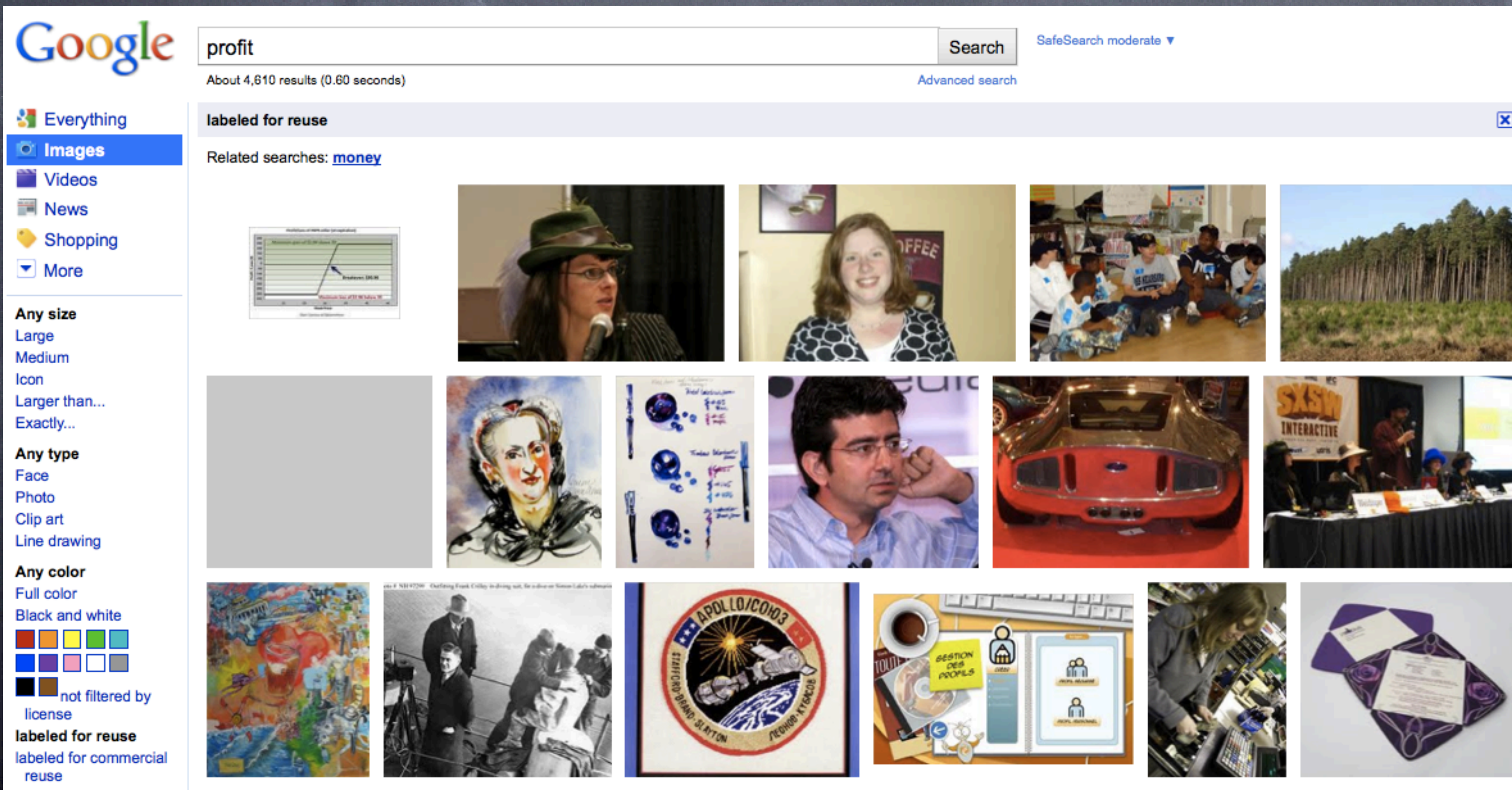
$X^{\otimes 4}$ and $Z^{\otimes 4}$ stabilizer localized on red qubits

Profit

For every [[n,k,d]] stabilizer code, there exists a [[N,k,r,d]] stabilizer subsystem code, $N=O(n^2)$, whose syndrome measurements can be made using s-neighboring measurements (s constant).

$$$$


profit
About 4,610 results (0.60 seconds)
Everything
Images
labeled for reuse
Related searches: money


profit
Search
SafeSearch moderate ▼
About 4,610 results (0.60 seconds)
Advanced search
Everything
Images
Videos
News
Shopping
More
Any size
Large
Medium
Icon
Larger than...
Exactly...
Any type
Face
Photo
Clip art
Line drawing
Any color
Full color
Black and white
not filtered by license
labeled for reuse
labeled for commercial reuse
labeled for reuse
Related searches: money

FAQ

Q: Why fault-tolerant syndrome measuring circuits?

- These maintain the distance of the code

Q: But aren't fault-tolerant circuits probabilistic?

- Often yes. Use the circuit as if the measurement result turned out in your favor

Q: Do these codes have thresholds?

- Unknown, but likely given their relation to fault-tolerant circuits

Q: s-local for what s?

- Can be made 3-local

Consequences

arXiv:soon (after we launch)

Q-Dub Group Still Lives!

Grads

Isaac Crosson (Physics)
Lukas Svec (Physics)
Jijiang Yan (Physics)
Kamil Michnicki (Physics)
David Rosenbaum (CS)
Paul Pham (CS)
Kevin Zatloukal (CS)

Faculty

Aram Harrow (Pseudo-prof)
Steve Flammia (Scientist!)

Undergrads

Zakk Webb (Physics/CS/Math)
Kate Liotta (Physics/CS)
Jonathan Shi (Physics/CS)
Melanie Jensenworth (Math/CS)
Jeffrey Booth (CS)
Rob McClure (CS)
Harshad Petwe (CS)

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The Test Dog

The Successful Test Dog