Pulse techniques for decoupling qubits from noise: experimental tests

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• Bang-bang decoupling $^{31}$P nuclear spins
• Low-decoherence electron-spin qubits and global 1/f noise
• Dynamical decoupling of the qubits
  – Periodic pulse sequences
  – Concatenated pulse sequences
• Summary
Experiments

• 2-pulse Hahn echo

\[ \frac{\pi}{2} \quad \tau \quad \pi \quad \tau \]

Pulses

FID – $T_2^*$

\[ (|0\rangle + |1\rangle) \]

Echo

Signal

• Decoupling

\[ \frac{\pi}{2} \quad \pi \quad \pi \]

\[ (|0\rangle + |1\rangle) \]

Echo

Signal

\[ (|0\rangle + |1\rangle) \]
Dynamical Decoupling

- Replace single $\pi$-pulse with sequence of pulses
  - Refocus spins rapidly ($< \text{noise correlation time}$)
  - “Bang-bang” – fast strong pulses (or 2 different spins)
  - CP (Carr-Purcell) – periodic $\pi$-pulses
    - $\pi_x/2-\tau-X-2\tau-X-2\tau-\ldots-X-\tau$-echo
  - CPMG (Carr-Purcell-Meiboom-Gill) – periodic $\pi$-pulses
    - $\pi_x/2-\tau-Y-2\tau-Y-2\tau-\ldots-Y-\tau$-echo
  - Aperiodic pulse sequences – concatenated sequences
    - Khodjasteh, Lidar, PRL 95, 180501 (2005); PRA 75, 062310 (2007).
      - $\pi_x/2-(p_{n-1}-X-p_{n-1}-Z-p_{n-1}-X-p_{n-1}-Z)-\tau-X-\tau$-echo with $Z=XY$
    - Yao, Liu, Sham, PRL 98, 077602 (2007). – concatenated CPMG
      - $\pi_x/2-(p_{n-1}-Y-p_{n-1}-p_{n-1}-Y-p_{n-1})-\tau-Y-\tau$-echo
  - Experimental pulses $\sim 1\mu s$ (for $\pi$-pulse)
    - Power $\sim 1/(\text{pulse length})^2 \Rightarrow \text{Energy/pulse} \sim \text{power}^{1/2}$
The Qubits: $^{31}\text{P}$ donors in Si

$^{31}\text{P}$ donor: Electron spin ($S$) = $\frac{1}{2}$ and Nuclear spin ($I$) = $\frac{1}{2}$

X-band: magnetic field = $0.35$ T

$\nu_{\mu w1} \sim 9.7$ GHz $\neq$ $\nu_{\mu w2} \sim 9.8$ GHz

$\nu_{\text{rf1}} \sim 52$ MHz $\neq$ $\nu_{\text{rf2}} \sim 65$ MHz

- Blue (microwave) transitions are usual ESR
- All transitions can be selectively addressed
Bang-Bang control

Fast nuclear refocusing

$^{31}\text{P donor: } S = \frac{1}{2} \text{ and } I = \frac{1}{2}$

- $\downarrow e, \downarrow n \quad |3\rangle$
- $\downarrow e, \uparrow n \quad |2\rangle$
- $\uparrow e, \downarrow n \quad |0\rangle$
- $\uparrow e, \uparrow n \quad |1\rangle$

$$\Psi_i = a|0\rangle + b|1\rangle \quad 2\pi \rightarrow \quad \Psi_f = a|0\rangle - b|1\rangle$$

Nuclear refocusing pulse would be $\sim10$ $\mu$s
but electron pulse $\sim30$ ns

(A) Free nuclear spin nutation

(B) One burst of $2\pi$ mw pulses

(C) Two bursts of $2\pi$ mw pulses
Electron spin qubits

- Doping $\sim 10^{15}$/cm$^3$
- Isotopically purified $^{28}$Si:P
- 7K $\Rightarrow$ electron $T_1 \sim$ 100’s milliseconds
- 7K $\Rightarrow$ electron $T_2 \sim$ 60 milliseconds (extrapolating to $\sim$ single donor)
Noise in electron spin echo signals

- Must use single pulses to measure decoherence
  \[ \Rightarrow \text{About 100x sensitivity penalty} \]
B-field noise

Measure noise voltage induced in coil

Origin of noise unclear

Background field in lab?
Domains in the iron?

→ Essentially 1/f
Microwave Field Inhomogeneity

Carr-Purcell (CP) sequence
\[ \pi_x/2 - \tau - X - 2\tau - X - 2\tau - \ldots - X - \tau - \text{echo} \]
Periodic (standard) CPMG

$\pi_x/2 - \tau - Y - 2\tau - Y - 2\tau - \ldots - Y - \tau$-echo

Self correcting sequence
Coherence after N pulses

Standard CPMG

$T_2 = 8.5\text{ms}$
Concatenated CPMG

\[ \pi/2 \text{ pulse} \quad \pi \text{ pulse} \quad \pi \text{ pulse} \]

![Graph showing microwave signal with labeled time intervals.](image)
Coherence vs. concatenation level

Concatenated CPMG

- $l = 2$ (2 pulses)
- $l = 4$ (10 pulses)
- $l = 6$ (42 pulses)

$T_2 = 5.8\text{ms}$
Concatenated and periodic CPMG

Concatenated CPMG
42 pulses

Periodic CPMG
32 pulses
Fault-Tolerant Dynamical Decoupling

- $\pi_x/2 - (p_{n-1}X - p_{n-1}X - Y - p_{n-1}X - p_{n-1}X - Y) - \tau - X - \tau$-echo

- Not obvious that it self-corrects

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**Concatenated XZXZ (p2)**

**CPMG**

Time (ms)
Coherence vs. concatenation level

Concatenated XZXZ pulse sequence

- $p_1$ (4 pulses)
- $p_2$ (14 pulses)
- $p_3$ (60 pulses)
- $p_4$ (242 pulses)
- $p_5$ (972 pulses)

$T_2 = 15\text{ms}$
Sanity check: collapse adjacent pulses

- Effect of combining pairs of adjacent pulses
  - Ex. $Z-Z \rightarrow I$
  - $n^{\text{th}}$ level concatenation without combining $\Rightarrow 2 \cdot 4^n - 2 = 510$ for $n=4$
  - $n^{\text{th}}$ level concatenation with combined pulses $= 306$ for $n=4$

![Graph showing concatenated XZXZ Echo Decay](image)
Sanity check: white noise

Si:P at 10 K

\[ T_2 = 330 \, \mu s \]
\[ T_1 = 420 \, \mu s \]
\[ XZXZ(p3) = 410 \, \mu s \]
Summary

• Dynamical decoupling can work for electron spins
• Through the hyperfine interaction with the electron can generate very fast bang-bang control of nucleus
• CPMG preserves initial $\pi_x/2$ with fewest pulses
  – But does not deal with pulse errors for $\pi_y/2$
  – CPMG cannot protect arbitrary state
    • Concatenated CPMG does no better
• Can utilize concatenated XZXZ sequence out to at least 1000 pulses
  – Situation with $\pi_y/2$ initial states is more complex
    • Not clear fidelity improves monotonically with level
    • But much better than CP
    • May need to combine XZXZ with composite pulses