Quantum Error Correction in correlated quantum noise

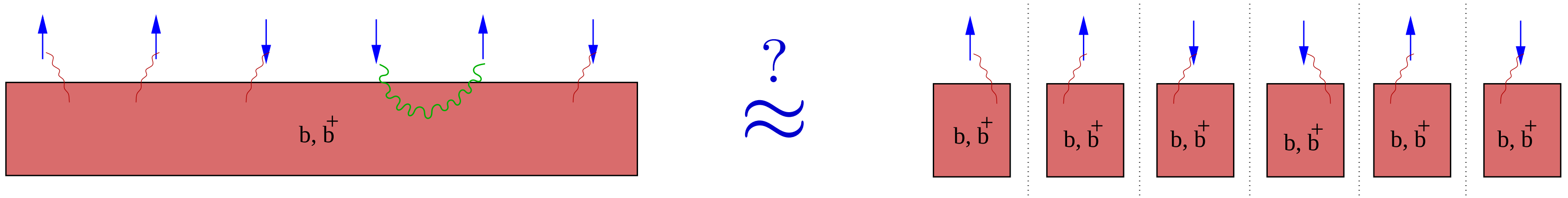
Rochus Klesse

Institut für Theoretische Physik, Universität zu Köln

In collaboration with:

- Sandra Frank ($\longrightarrow$ FZ Jülich)
- Stefan Borghoff
- Thomas Zell

Supported by DFG (German Research Foundation)



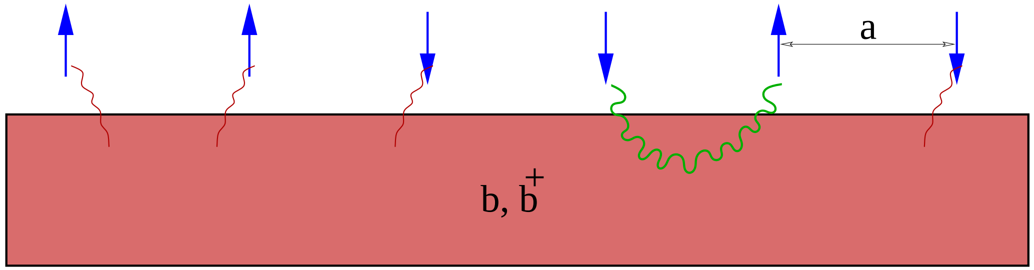
Outline:

I. Physical model

II. Effect of noise correlations on QEC: leading-order estimate

III. Results of exact calculation

I. Physical model :



- n (effective) spin-$\frac{1}{2}$
- common bosonic bath
- linear and local spin-boson coupling : $\sum_l \; g \; S_l \otimes A_l$

$S_l = X_l, \; Y_l$ or Z_l

$A_l \;=\;$ bosonic field strength at spin l

- no direct coupling between spins

noise correlations due to *boson exchange* !

QEC :

- CSS codes for $\leq \; t$-qubit errors
- instantaneous
- ideal

To which extent do noise correlations interfere with QEC ?

II. Leading-order estimate :

$$\psi \xrightarrow{\quad \mathcal{N}_\tau \quad} \psi' \xrightarrow{\quad \mathcal{R} \quad} \psi''$$

$\mathcal{N}_\tau$: noise on n qubit register during time τ

$\mathcal{R}$: $\leq t$ -qubit error correcting operation

residual error :

$$\Delta \equiv \| \psi'' - \psi \| \overset{!}{\sim} \binom{n}{t+1} \left(\alpha g^2 \right)^{t+1} \langle A_{l_1}^2 \dots A_{l_{t+1}}^2 \rangle ,$$

g : coupling constant

$\alpha = \alpha(\tau)$

$$\Delta \quad \sim \quad \binom{n}{t+1} \, (\, \alpha g^2 \,)^{t+1} \, \langle A_{l_1}^2 \ldots A_{l_{t+1}}^2 \rangle \; ,$$

t : number of correctable errors

independent noise :

$$\langle A_{l_1}^2 \ldots A_{l_{t+1}}^2 \rangle = \langle A_{l_1}^2 \rangle \ldots \langle A_{l_{t+1}}^2 \rangle = \langle A^2 \rangle^{t+1} \, .$$

infinite spin distance

correlated noise :

$$\langle A_{l_1}^2 \ldots A_{l_{t+1}}^2 \rangle = \quad \langle A^{2(t+1)} \rangle \quad = \quad \underline{(2t+1)!!} \; \langle A^2 \rangle^{t+1}$$

vanishing spin distance

$$\Longrightarrow \qquad \frac{\Delta_{corr}}{\Delta_{ind}} \quad = \quad (2t+1)!!$$

$\longrightarrow$ $n,\ t \gg 1$, constant correctable error rate $t/n = q$:

independent noise:

$$\Delta \sim \binom{n}{q\,n} \left(\alpha g^2 \langle A^2 \rangle \right)^{qn} \quad \approx \quad \left(c_q \, \alpha \, g^2 \langle A^2 \rangle \right)^{qn} \quad \xrightarrow{n \to \infty} \quad 0$$

if $g^2 < g_0^2 \equiv (c_q \, \alpha \, \langle A^2 \rangle)^{-1}$

correlated noise:

$$\Delta \sim \binom{n}{q\,n} (2qn+1)!! \left(|g|^2 \langle A^2 \rangle \right)^{qn} \quad \approx \quad \left(c_q \, \alpha \, n \, g^2 \langle A^2 \rangle \right)^{qn} \quad \xrightarrow{n \to \infty} \quad \infty$$

for all $g^2 > 0$!

$\binom{n}{q\,n} \approx e^{n H_2(q)}$, $\qquad (2n+1)!! \sim (2n)^n$

Conclusion:

- correlations due to boson exchange do matter

$$\frac{\Delta_{corr}}{\Delta_{ind}} \simeq \frac{\langle A^{2t} \rangle}{\langle A^2 \rangle^t} \simeq (2t)^t$$

- perturbative series in g^2 problematic

$$\lim_{n \to \infty} \Delta_{corr} = \infty \, !?$$

- unboundness of A^2 essentially

otherwise $\lim_{t \to \infty} \frac{\langle A^{2t} \rangle}{\langle A^2 \rangle^t} = 1$

$\longrightarrow$ *investigation of exactly solvable model: III.*

III. Dissipationless n-spin-boson model:

$$H = \frac{\varepsilon}{2}\sum_{l=1}^{n} Z_l \quad + \quad \sum_k \omega_k b_k^\dagger b_k \quad + \quad \sum_{k,l} Z_l (g_k\, e^{ik\, r_l}\;\; b_k^\dagger \quad + \quad h.c.)$$

Unruh '95; Palma, Suominen, Ekert '96

↷ exact noise operation $\mathcal{N}_\tau$ on n-qubit register: $\rho \xrightarrow{\tau} \mathcal{N}_\tau(\rho)$

Ensemble of all $[[n,k]]$ CSS codes:

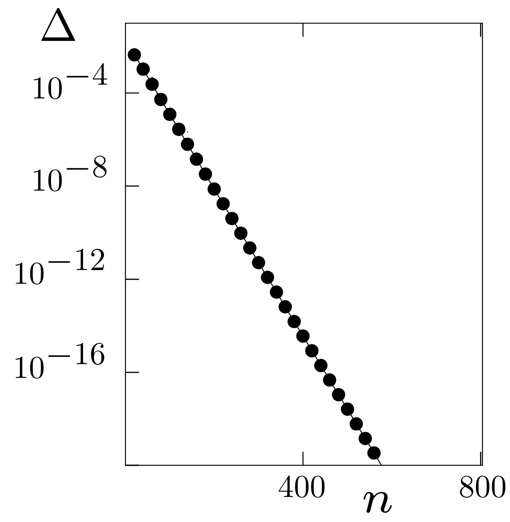
- n large
- fixed information rate $R = k/n$
- correctable error rate $q = t/n$ determined by $R = 1 - 2H_2(2q)$

↷ ensemble averaged residual error :

$$\Delta_{n,q} \;=\; 1 - \left\langle \int_C d\psi \;\; F(\psi, \mathcal{R}_C \circ \mathcal{N}(\psi)) \right\rangle_{n,q}$$

Independent noise ($a = \infty$):

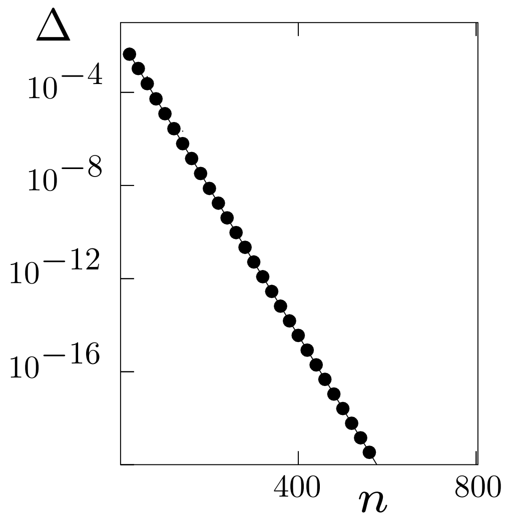
$$\Delta_{n,q} \quad \simeq \quad \exp\left[-\, n \, \frac{(q - K(0,\tau))^2}{2K(0,\tau)}\right] \; ,$$



$K(r,\tau)$: time- and distance-dependent decoherence coefficient

Independent noise ($a = \infty$):

$$\Delta_{n,q} \quad \simeq \quad \exp\left[-\, n \,\frac{(q-K(0,\tau))^2}{2K(0,\tau)}\right] \;,$$

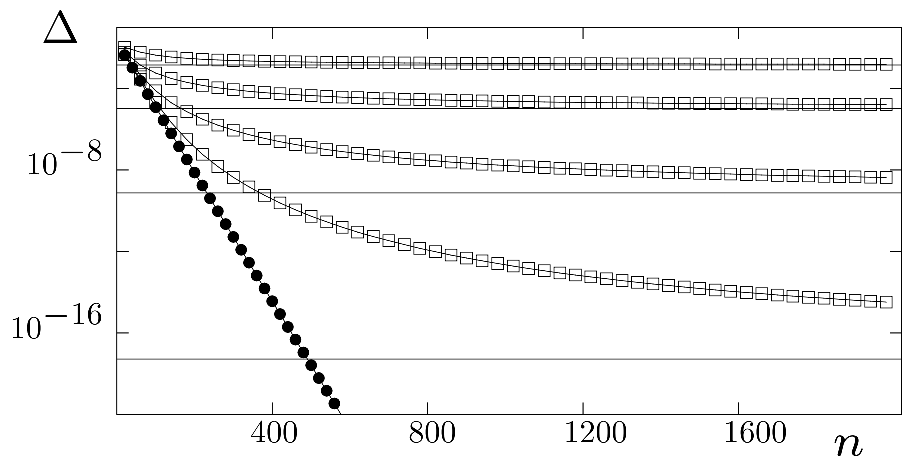


$K(r,\tau)$: time- and distance-dependent decoherence coefficient

Correlated noise ($a < \infty$, $n \gg 1$):

$$\Delta_{n,q} \quad \sim \quad \exp\left[\frac{-q}{K(r,\tau)}\right]$$

r : spatial extension of register

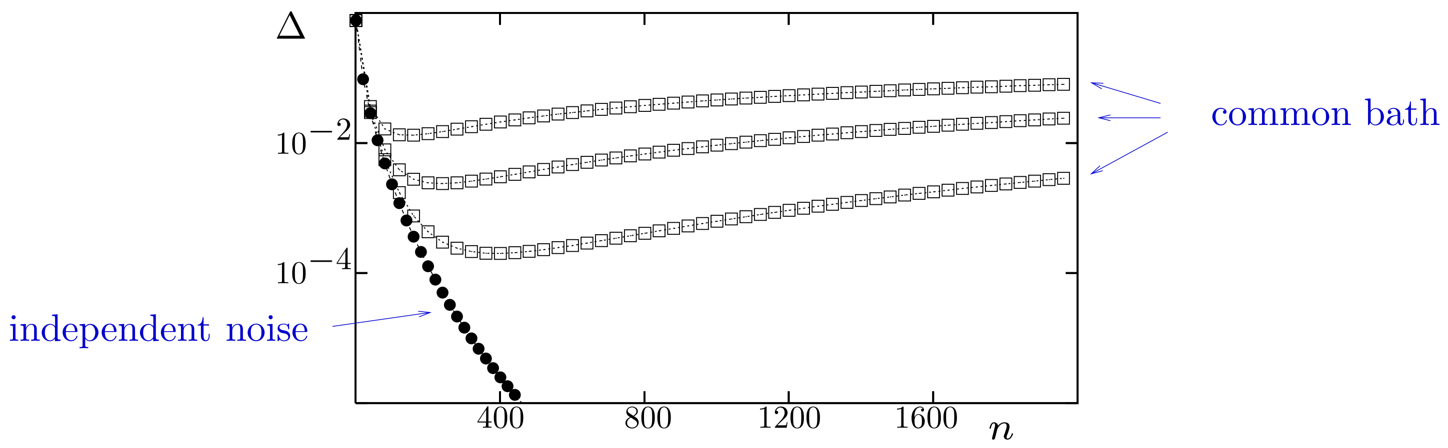


$q = 0.05$, fixed $K_{r,\tau} =$ 0.01, 0.005, 0.0025, and 0.00125, $K_o = 0.01$

Example:

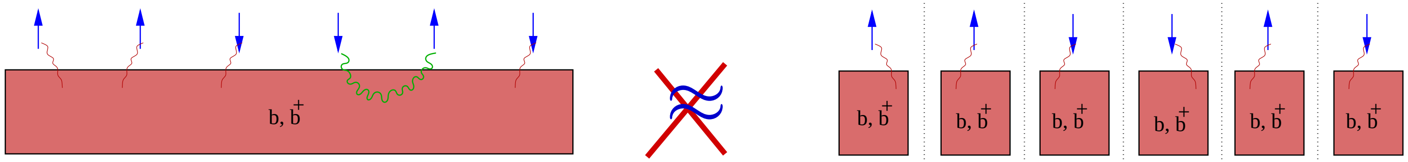
- n spins on cubic array
- spatial extension $r = n^{1/3} r_0$
- observation time $\tau = n^{1/3} \tau_0$ (since $\tau \propto r$!?)
- ohmic bosonic bath:

$$K(n) \equiv K(n^{1/3} r_0,\ n^{1/3} \tau_0,\ T) \stackrel{!}{=} n^{1/3} K(r_0, \tau_0, T)$$



CSS error correction significantly reduced by boson-exchange correlations

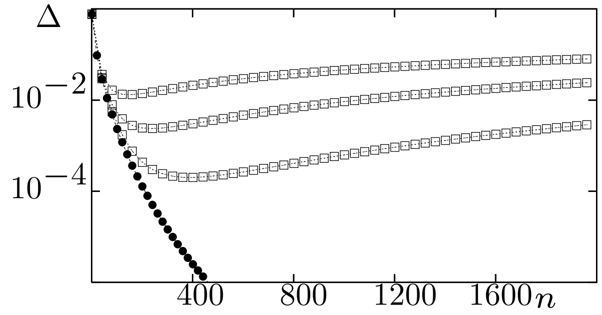
Summary/Conclusion

- 

$\langle A^{2t} \rangle \gg \langle A^2 \rangle^t$

- spin-boson model + CSS error correction:

$$\Delta = 1 - \langle F_{av}(C, \mathcal{R} \circ \mathcal{N}) \rangle_{[[n,qn]]}$$



Open questions

- improvement by use of "decoherence-free-subspaces" ?
- impact on FT-QC?
- relation to improved Threshold Theorems?

cf. B. Terhal *et al.* '05, P. Aliferis *et al.* 05, D. Aharonov *et al.* 06

Lit.: RK, S. Frank, PRL **95**, 230503 '05
RK, PRA **75**, 062315 '07
S. Borghoff, RK, PRA **76** '07