# Criticism of Fault-Tolerant Quantum Information Processing

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Robert Alicki

University of Gdańsk

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- Does II-Law implies BCP?

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- Challenges to BCP:

  Superconducting qubits

  Fault-tolerant quantum error correction

  Topological phase transitions

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- Dynamical picture - rapid decoherence of superpositions into mixtures

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- Simplex is a classical figure of states

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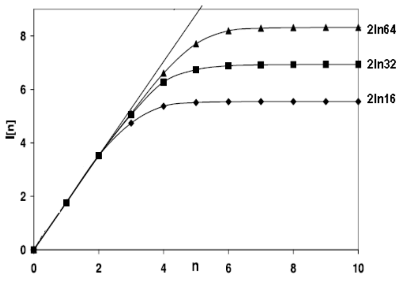
- 

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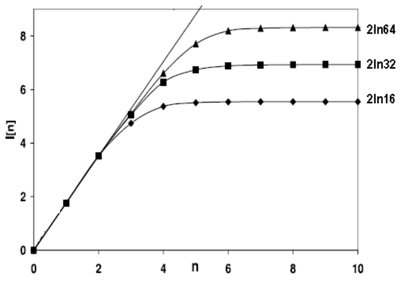
- 

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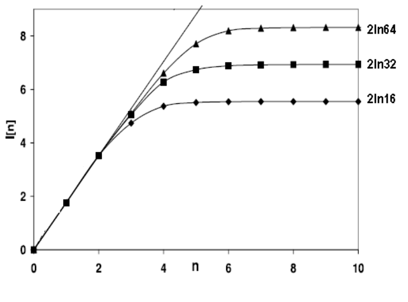
- 

- Logarithmic time scale $t \simeq \frac{\log \dim \mathcal{H}}{h_{KS}}$
- No difference in sensitivity between quantum and classical systems.

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- Evolution of QC

$$\rho_{out} = \Lambda(K\tau)\rho_{in} = \Lambda_K \mathcal{U}_K \cdots \Lambda_2 \mathcal{U}_2 \Lambda_1 \mathcal{U}_1 \rho_{in}$$

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- Threshold Theorem
  If $\eta < \eta_c$ then the efficient quantum computation is possible for arbitrarily long input.

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- Convolutionless Master equation

$$\frac{d}{dt}\rho(t) = \mathcal{L}(t)\rho(t) \ , \ \mathcal{L}(t) = \Big(\frac{d}{dt}\Lambda(t)\Big)\Lambda(t)^{-1} \ .$$

- Unitary maps

$$\mathcal{U}_k = \mathcal{U}(t_k, t_{k-1}) = \mathbf{T}\exp\Big\{-i\int_{t_{k-1}}^{t_k}\mathcal{H}_Q(t)dt\Big\}\ ,\ \mathcal{H}_Q(t)\rho \equiv [H_Q(t), \rho]$$

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- Autocorrelation function for the bath and spectral density

$$F(\tau) = \mathrm{Tr}\big(\rho_B B(\tau) B\big)\ ,\ R(\omega) = \int_{-\infty}^{\infty} F(t) e^{i\omega t} dt$$

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- KMS condition ($\hbar$) $R(-\omega) = e^{-\hbar\omega/kT}R(\omega)$ implies for $t >> \hbar/kT$

$$|F(t)| \simeq \frac{R(0)}{2\pi kT}t^{-2}$$

Non-exponential decay –> A2) not satisfied !

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- Compute Lindblad operators $a_j(\omega)$, $b_j(\omega)$, $j = 1, 2, ..., N$

$$[H_N, a_j(\omega)] = -\omega a_j(\omega) \quad , \quad \sum_{\{\omega\}} a_j(\omega) = \sigma_j^x$$

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- Construct thermal generator in Heisenberg picture (Davies-WCL)

$$\mathrm{L}_N(\cdot) = \sum_{j=1}^{N} \mathrm{L}_j^a(\cdot) + \sum_{j=1}^{N} \mathrm{L}_j^b(\cdot)$$

$$\mathrm{L}_j^a(\cdot) = \frac{1}{2} \sum_{\{\omega \geqslant 0\}} \left( a_j^\dagger(\omega)[\cdot, a_j(\omega)] + h.c. + e^{-\omega/kT} a_j(\omega)[\cdot, a_j^\dagger(\omega)] + h.c. \right)$$

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- Then, the encoded qubit becomes metastable
- However, qubit observables can be computationally accessible or computationally non-accessible

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- Most probable result: There exist exponentially stable qubit observables with algorithmically hard encoding

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- There is a strong evidence in favor of BCP.
- No ultimate experimental or even theoretical counterexample to BCP has been provided.