Fault-tolerant holonomic computation on quantum error-correcting codes

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Outline

- Holonomic quantum computation (HQC)
- Why combine HQC with EC?
- Basic fault-tolerant operations
- The scheme
- Properties
- Future research

Holonomic Computation

Let $\{H_\lambda\}$, $\lambda \in \mathcal{M}$, be a family of isodegenerate Hamiltonians:

$$H_\lambda = \sum_{n=1}^{R} \varepsilon_n(\lambda)\Pi_n(\lambda)$$

In the adiabatic limit, the evolution driven by $H(t) := H_{\lambda(t)}$ is

$$U(t) = \mathcal{T}\exp(-i\int_0^t d\tau H(\tau)) = \bigoplus_{n=1}^{R} e^{i\omega_n(t)} U_{A_n}^{\lambda}(t),$$

where $\omega_n(t) = -\int_0^t d\tau \varepsilon_n(\lambda(\tau))$ and $U_{A_n}^{\lambda}(t) = \mathcal{P}\exp(\int_{\lambda(0)}^{\lambda(t)} A_n)$.

The adiabatic connection is

$A_n = \sum_\mu A_{n,\mu} d\lambda^\mu$, where $(A_{n,\mu})_{\alpha\beta} = \langle n\alpha;\lambda|\frac{\partial}{\partial\lambda^\mu}|n\beta;\lambda\rangle$.

When the path forms a loop: $U_n^\gamma \equiv U_{A_n}^{\lambda}(T) = \mathcal{P}\exp(\oint_\gamma A_n)$.

Why combine HQC with EC?

1. Scalability of any computational method requires fault tolerance.

2. HQC is compatible with the Weak Coupling Limit derivation of Markovian dynamics (Alicki, Lidar, Zanardi).

3. Combine the holonomic approach with protection from decoherence.

Basic fault-tolerant operations

A scheme is fault-tolerant if a single error during an operation introduces at most one error per block of the code.

- Transversal unitary operations:

  - single-qubit unitaries
  - transversal C-NOT

- Preparation and use of a cat state $(|0...0\rangle + |1...1\rangle)/\sqrt{2}$:

  - preparation
  - verification (measurement of the parity of the state)
  - transversal C-NOT gates from logical states to the cat state

- Single-qubit measurements in the computational basis.

The scheme

Consider an [[n,1,3]] stabilizer code, $S = \langle G_1, G_2, ..., G_{n-1} \rangle$.

- Single-qubit operations:

Take a stabilizer element acting non-trivially on the qubit:

$\widehat{G} = Z \otimes \widetilde{G}$, where $\widetilde{G} \in \mathcal{G}_{n-1}$.

(For subsystem codes, an operator on the noisy subsystem.)

The starting Hamiltonian: $\widehat{H}(0) = -Z \otimes \widetilde{G}$

Slowly vary this Hamiltonian: $\boxed{\widehat{H}(s) = -H(s) \otimes \widetilde{G}}$, $s = t/T$.

If the change of $H(s)$ is adiabatic, $\boxed{\widehat{U}(s) \approx -U(s) \otimes \tilde{I}}$.

The scheme

- Single-qubit operations (continued):

Define $V_{\theta\pm} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \mp e^{-i\theta} \\ \pm e^{i\theta} & 1 \end{pmatrix}$: $\begin{array}{l} |0\rangle \to (|0\rangle \pm e^{i\theta}|1\rangle)/\sqrt{2} \\ |1\rangle \to (|1\rangle \mp e^{-i\theta}|0\rangle)/\sqrt{2} \end{array}$

Consider the single-qubit Hamiltonian $V_{\theta\pm} Z V_{\theta\pm}^{\dagger} \equiv H_{\theta\pm}$.

The unitaries are implemented by interpolations of the type:

$\widehat{H}(s) = -H_{\theta\pm}(s) \otimes \widetilde{G}$, where $H_{\theta\pm}(s) = (1-s)Z + sH_{\theta\pm}$.

The same instantaneous spectrum (minimum gap $\Delta = \sqrt{2}$).

The adiabatic condition:

$$T \gg \frac{\varepsilon}{\Delta^2}$$

, where $\varepsilon = \max_{0 \le s \le 1} |\langle 1; s| \frac{dH(s)}{ds} |0; s\rangle|$

The scheme

- Example:

  The X gate: $Z \otimes \widetilde{G} \rightarrow H_{\pi/2+} \otimes \widetilde{G} \rightarrow -Z \otimes \widetilde{G}$

  The Z gate:

  $$Z \otimes \widetilde{G} \rightarrow H_{0+} \otimes \widetilde{G} \rightarrow -Z \otimes \widetilde{G} \rightarrow H_{\pi/2-} \otimes \widetilde{G} \rightarrow Z \otimes \widetilde{G}$$

- Controlled NOT:

  1) Apply a phase gate on the control qubit.

  2) Apply $Z \otimes \widetilde{G} \rightarrow H_{\pi/2+} \otimes \widetilde{G}$ on the target qubit.

  3) Apply $I^c \otimes H_{\pi/2+} \otimes \widetilde{G} \rightarrow Z^c \otimes Z \otimes \widetilde{G}$.

The scheme

- Preparation of the cat state $(|0...0\rangle + |1...1\rangle)/\sqrt{2}$:

  1) prepare all qubits in state $(|0\rangle + |1\rangle)/\sqrt{2}$; $Z \to H_{0+}$

  2) apply $H^{1j}(s) = -[(1-s)I^1 \otimes X^j + sZ^1 \otimes Z^j]$.

  This state is invariant under $Z^j Z^{j+1}$.

  3) single-qubit Hadamard on each qubit.

- Verification of the cat state:

  We need an ancillary qubit to measure the parity.

  It turns out theat we can use a single qubit $|0\rangle$, $S = \langle Z \rangle$

  (the same procedure as before, but with $\widetilde{G} = 1$).

Properties of the scheme

- Single-qubit errors do not propagate.

  By construction, the operations are transversal in each eigenspace!

  The dynamical phase is irrelevant, since we project on the eigenspaces when we correct.

  (For subsystem codes the dynamical error is on the noisy subsystem.)

- Hamiltonians of weight 2+1 (optimal for HQC on qubits).

  E.g., the 9-qubit Shor code or the Bacon-Shor codes.

Properties of the scheme

- Effects on the threshold:

The Hamiltonians act on a few qubits in the same block (at the lowest level of concatenation). This affects parallelism, but only by a constant factor.
As a result, the threshold for the decoherence rate decreases (e.g., for the 9-qubit Bacon-Shor code $\longrightarrow$ a factor of 1.5).

Adiabatic gates are slow. From computational perspective this is only a constant overhead, but it decreases further the decoherence threshold (Bacon-Shor code $\longrightarrow$ a factor of 65).

The overhead can be beneficial where precise control is harder than isolation from the environment.

Future research

- Can we implement the scheme using 2-local Hamiltonians?
- Can we exploit the protection form the gap for designing hybrid correction-detection codes?
- Physical implementation?