A Globally Controlled Fault-Tolerant Architecture for Quantum Computation

Joe Fitzsimons

Quantum and Nanotechnology Theory Group
Department of Materials
Oxford University

Outline

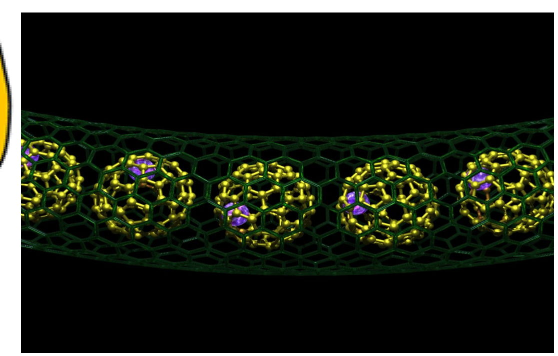
- **Part 1: Quantum State Transfer**
  - Introduction
  - State Transfer
- **Part 2: Quantum Computation**
  - Single Qubit Gates
  - Two Qubit Gates
- **Part 3: Fault Tolerance**
- **Conclusion & Acknowledgements**

Part 1: Quantum State Transfer

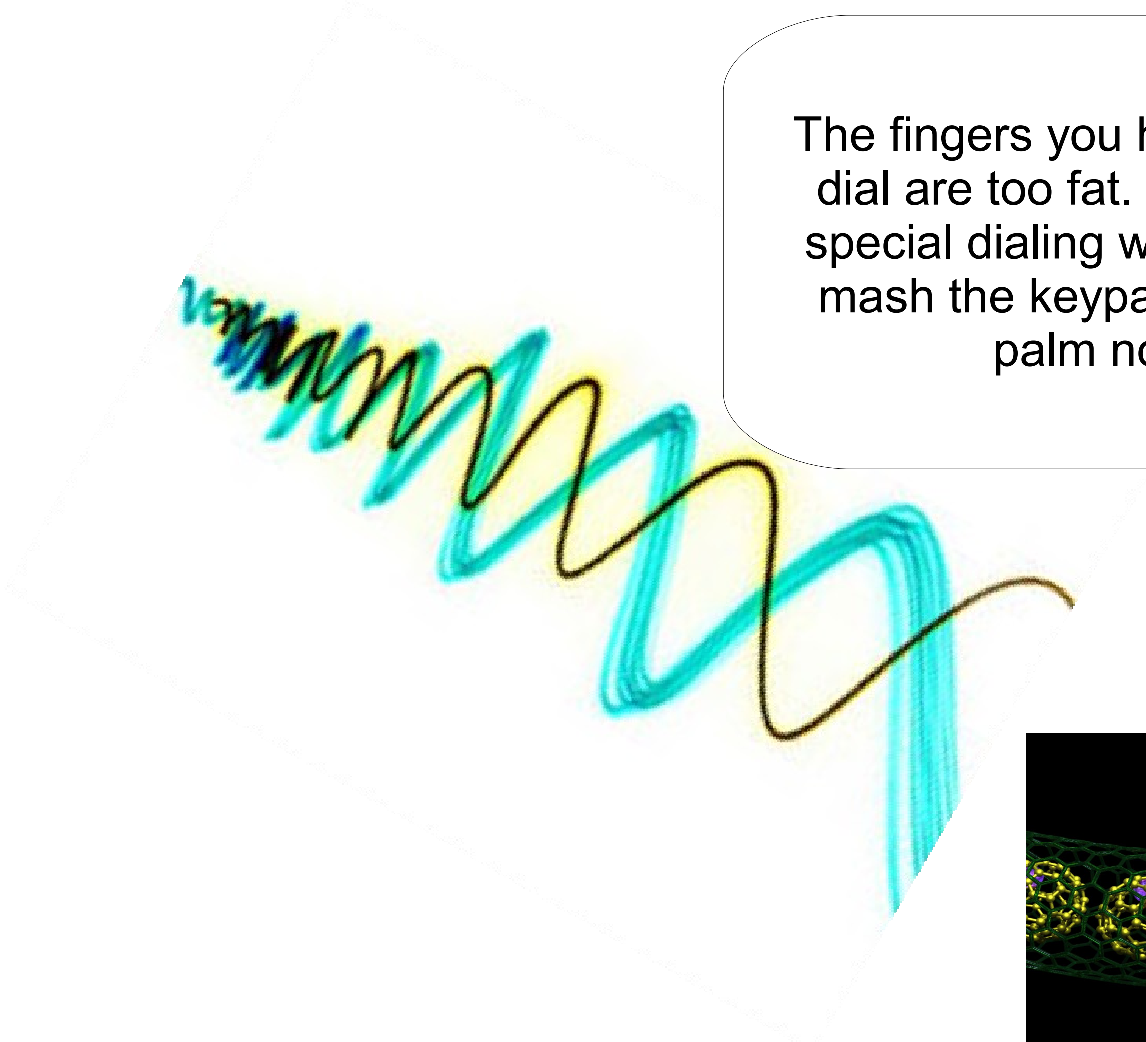
Introduction

Introduction

The fingers you have used to dial are too fat. To obtain a special dialing wand, please mash the keypad with your palm now.



Introduction



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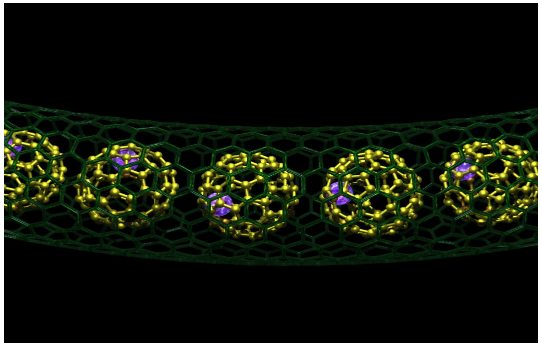
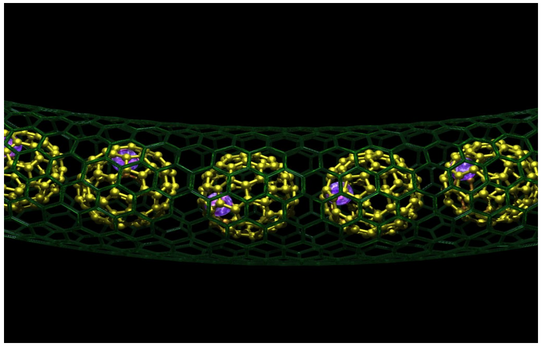


Introduction

Dialing Wand

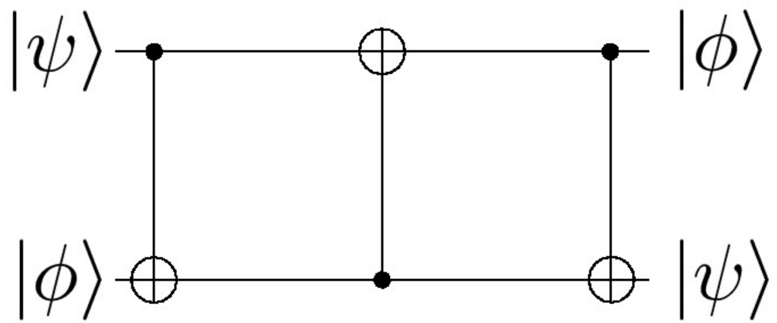
↓

Global Control Scheme



State Transfer

<u>The Swap Circuit:</u>



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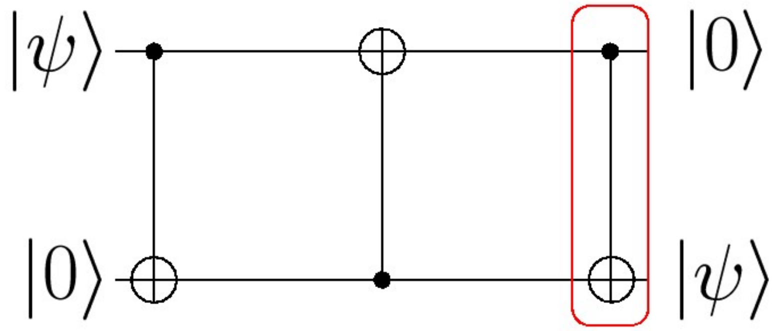

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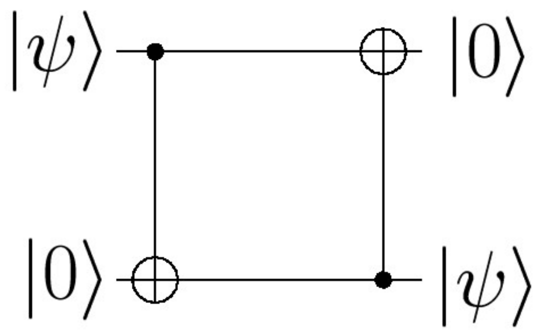

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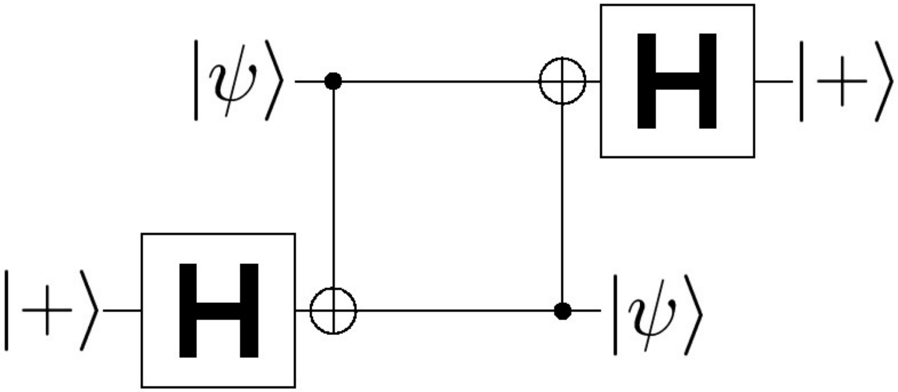

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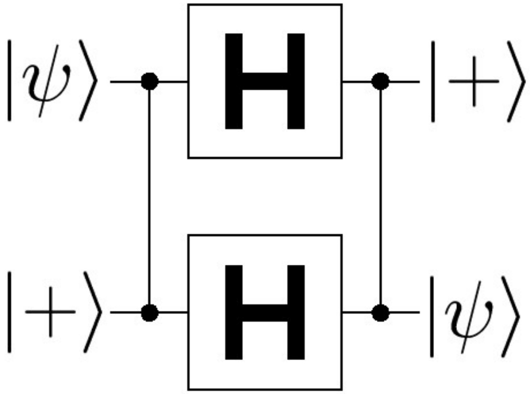
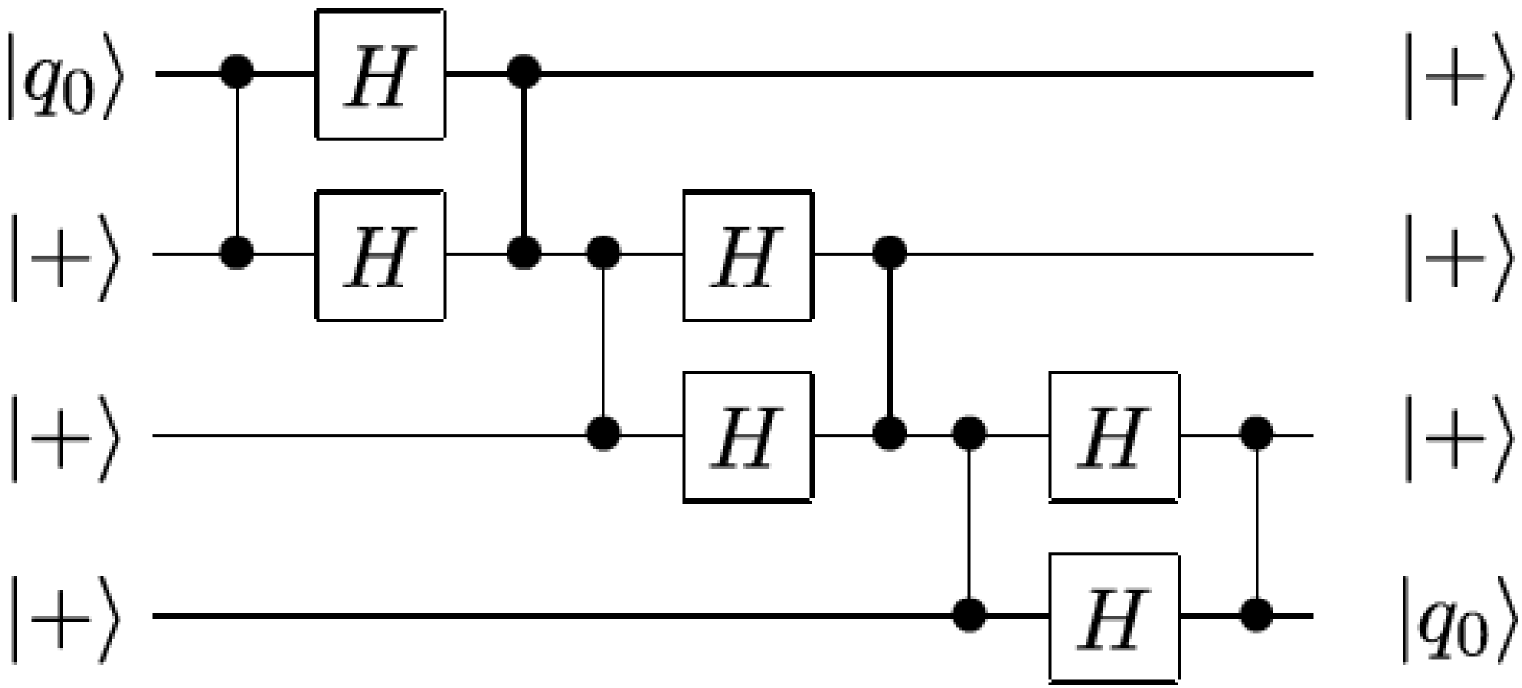


State Transfer

The Transfer Circuit: Concatenate Swaps



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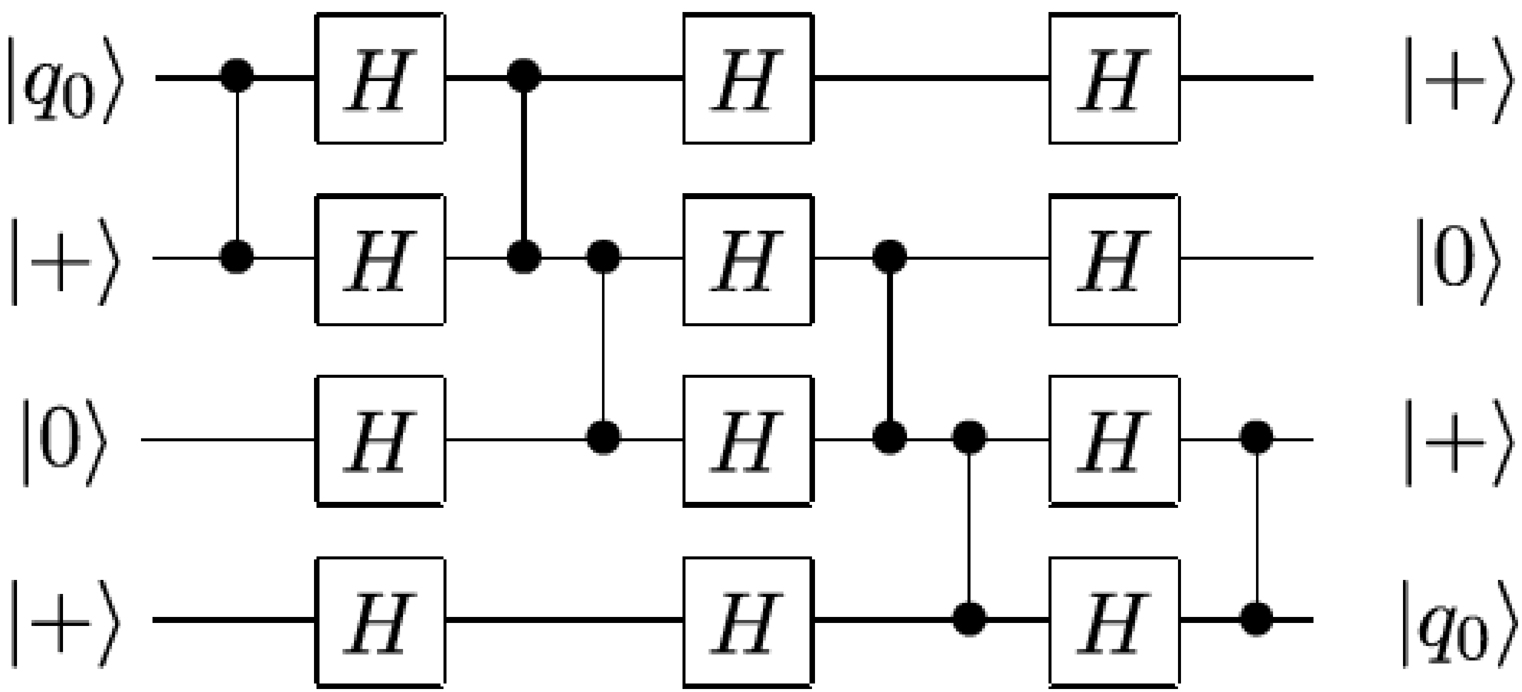

State Transfer

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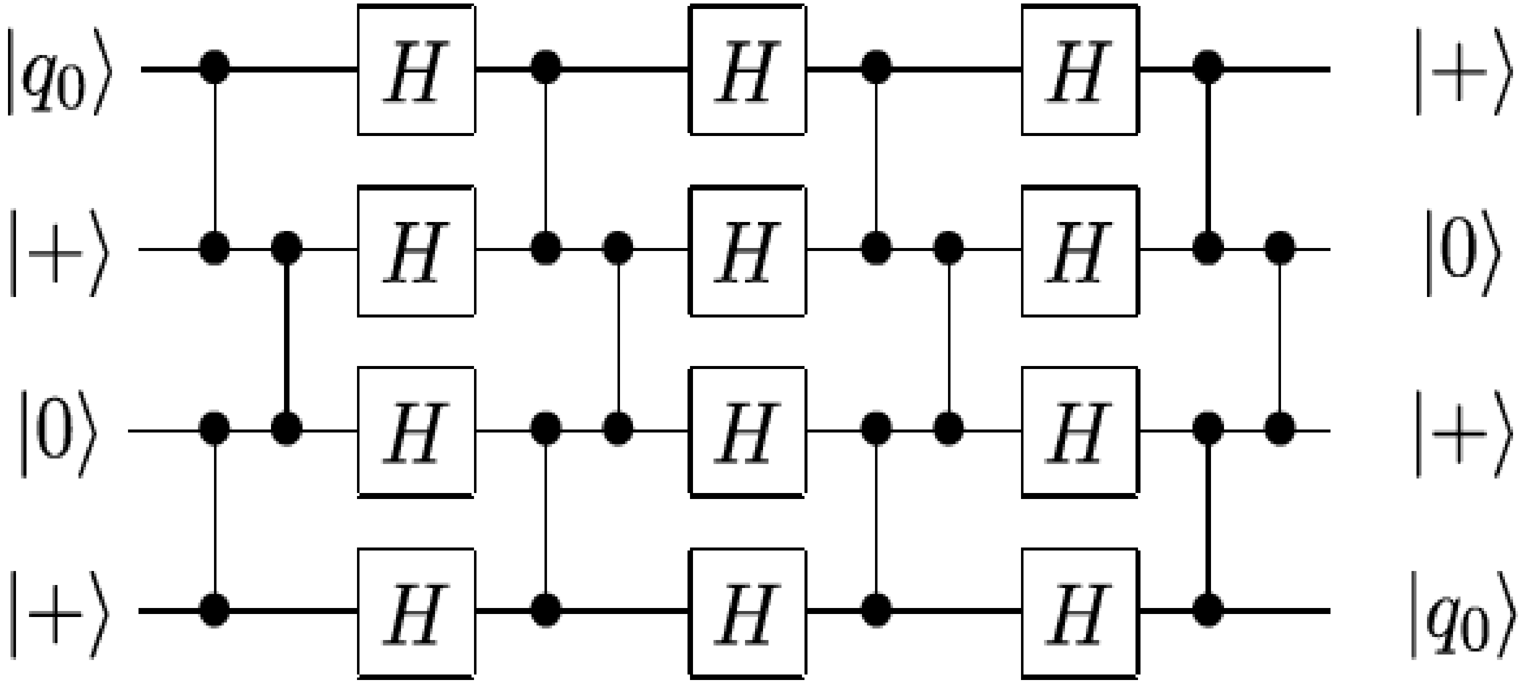


See B. Schumacher and R.F. Werner, quant-ph/0405174 (2004)

State Transfer

$$S = \prod_a U_{CZ}^{(a,a+1)} = \prod_a \frac{1}{2}\left(I + \sigma_z^{(a)} + \sigma_z^{(a+1)} - \sigma_z^{(a)}\sigma_z^{(a+1)}\right)$$

So S can be rewritten as

$$S = \exp\left(-i\,\pi \sum_a \frac{1-\sigma_z^{(a)}}{2}\,\frac{1-\sigma_z^{(a+1)}}{2}\right)$$

which can then be expanded, giving

$$S = \underline{\exp\left(-i\frac{\pi}{4}\sum_a \sigma_z^{(a)}\sigma_z^{(a+1)}\right)} \times \prod_a \underline{\exp\left(-i\frac{\pi}{4}\right)}\,\underline{\exp\left(-i\frac{\pi}{4}\sigma_z^{(a)}\right)\exp\left(-i\frac{\pi}{4}\sigma_z^{(a+1)}\right)}$$

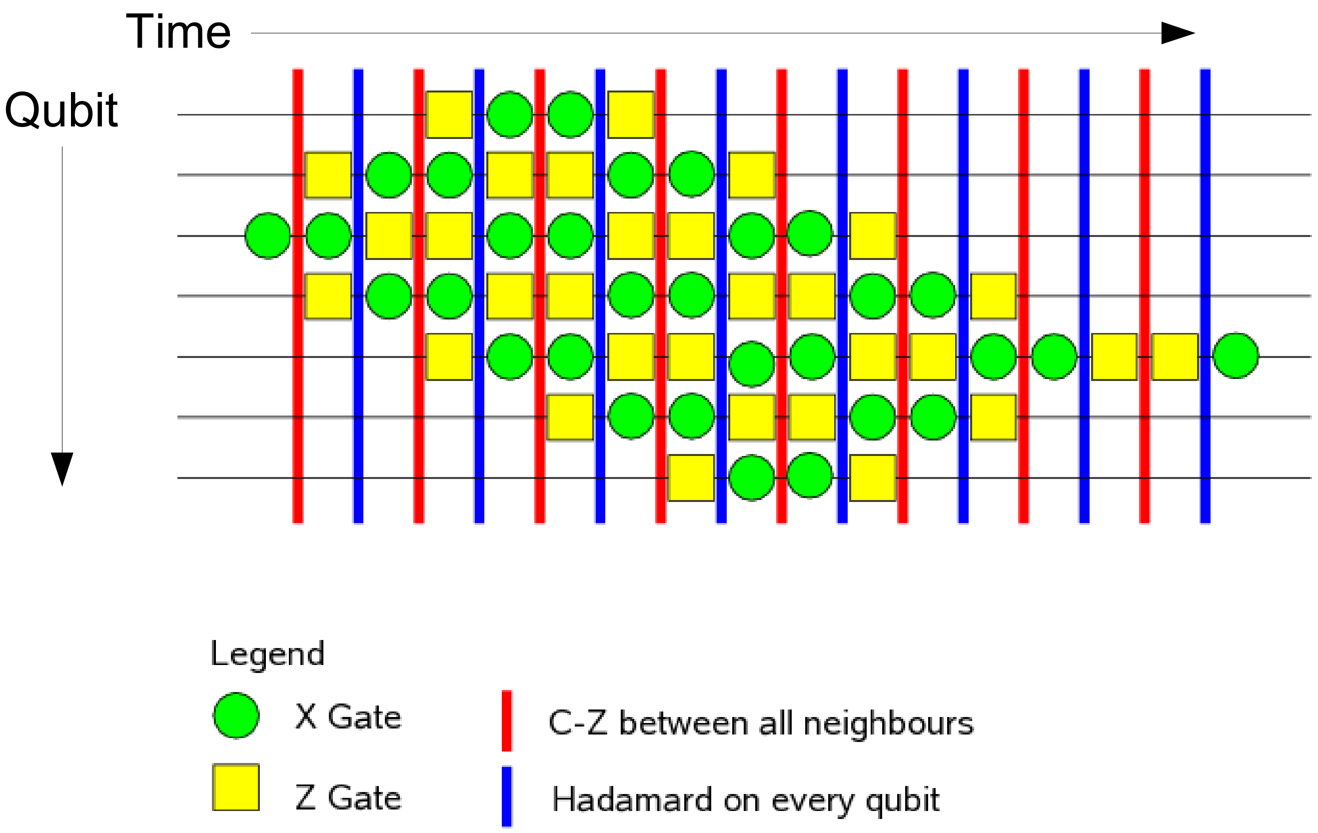
The first term of which corresponds to an Ising interaction, and the remainder correspond to a global phase and to local Z rotations. This can be rewritten as

$$S = \underline{\exp\left(+i\frac{\pi}{4}\left(\sigma_z^{(1)} + \sigma_z^{(N)}\right)\right)}\,\underline{\exp\left(-i\frac{\pi}{4}\sum_a \sigma_z^{(a)}\sigma_z^{(a+1)}\right)} \times \underline{\prod_a \exp\left(-i\frac{\pi}{2}\sigma_z^{(a)}\right)}$$

which contains terms corresponding to an Ising interaction between neighbours, a -π/2 Z rotation on all spins and a π/4 Z rotation on the spins at either end of the chain.

State Transfer

For example, the following operations are all equivalent:



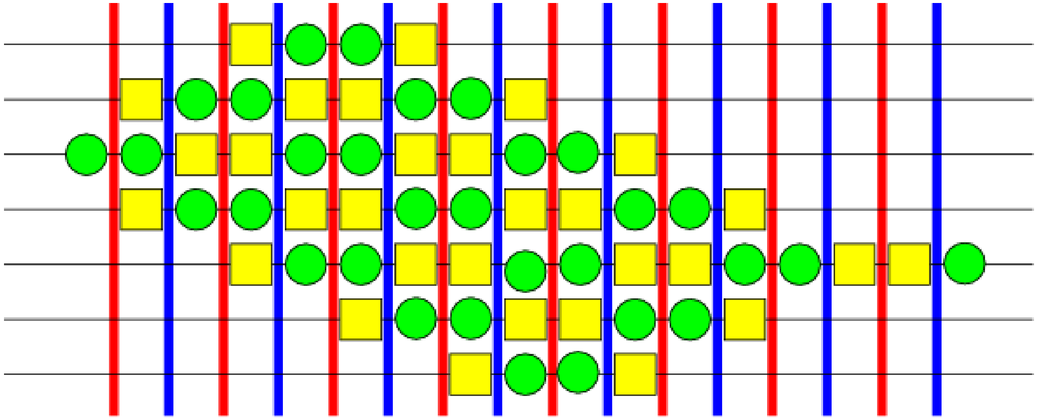
Part 2: Quantum Computation

Single Qubit Gates

To build upon this scheme, to allow single qubit operations to be performed, it is necessary to separate the logical qubits, adding a $|+\rangle$ state between each. So

$$|\psi\rangle = |\psi_0\rangle \otimes |+\rangle \otimes |\psi_1\rangle \otimes |+\rangle \otimes |\psi_2\rangle \otimes |+\rangle \otimes \cdots \otimes |\psi_N\rangle$$

This ensures that the state of the physical qubit, at a given end, is only affected by one logical qubit.



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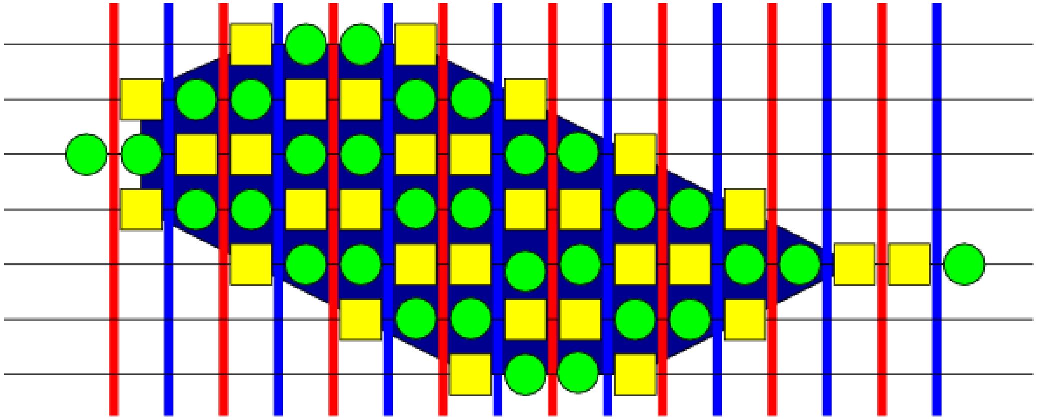

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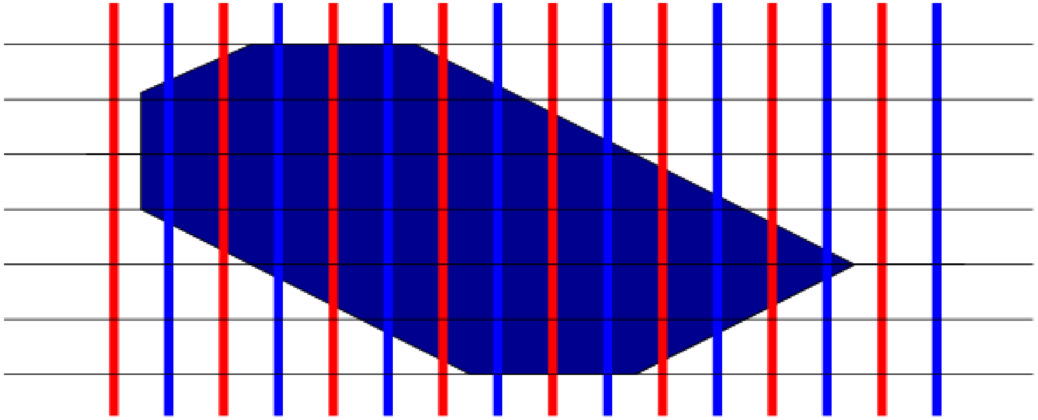

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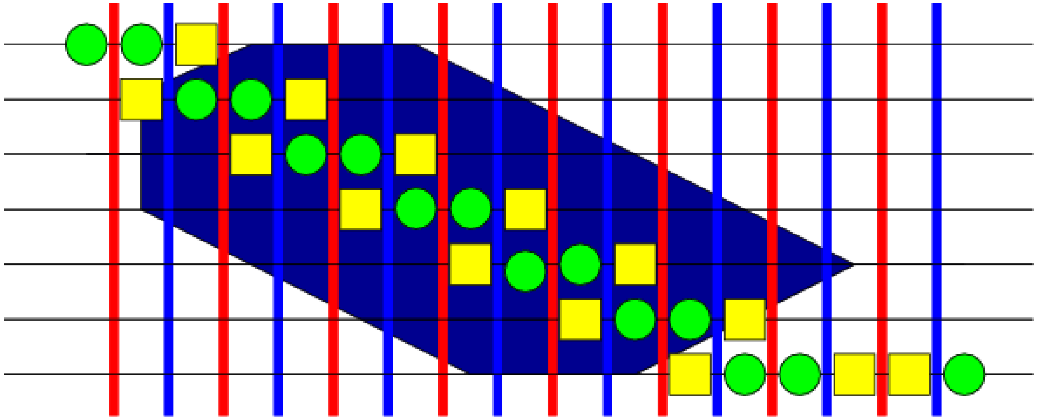

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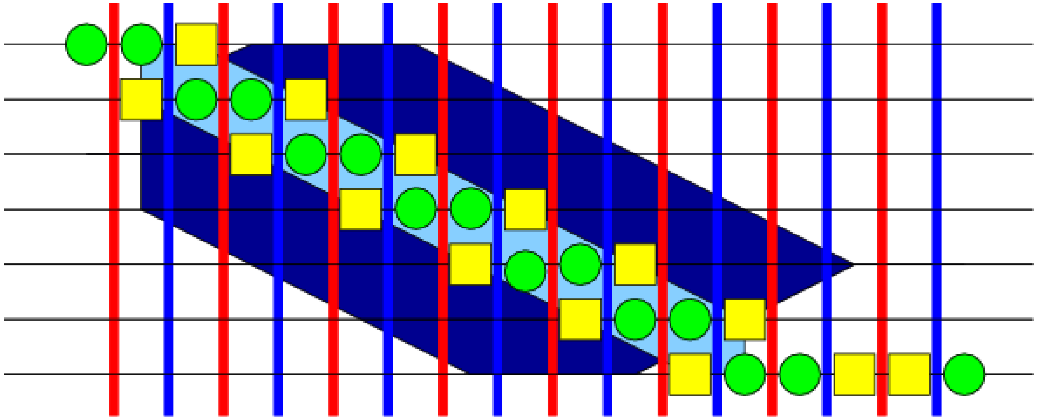

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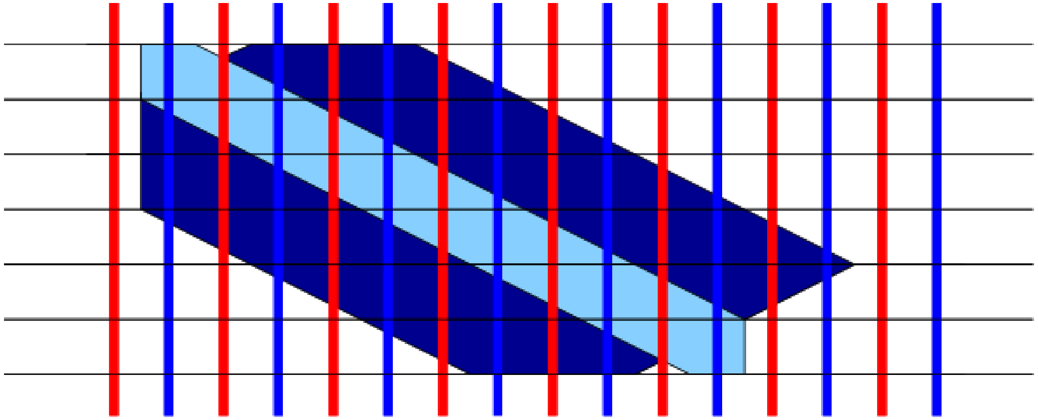
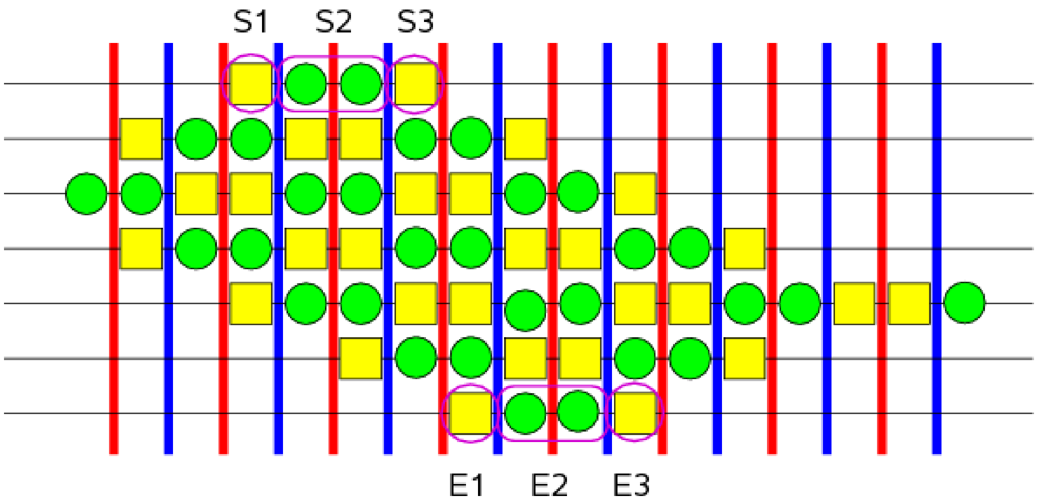


Single Qubit Gates

At times S2 and E2 a z-rotation on the physical qubit at the indicated end of the chain will result in the same z-rotation on the corresponding logical qubit.



Single Qubit Gates

At the end of each cycle, a ±π/2 X rotation is performed on all qubits.

This will leave the buffer qubits unchanged, but rotate the logical qubits.

$$R_x(\theta)|+\rangle = |+\rangle$$

By repeating the procedure for a single qubit Z rotation, we can now also perform Y rotations.

$$R_x\left(\frac{\pi}{4}\right) R_z(\theta) R_x\left(-\frac{\pi}{4}\right)|\phi\rangle = R_y(-\theta)|\phi\rangle$$

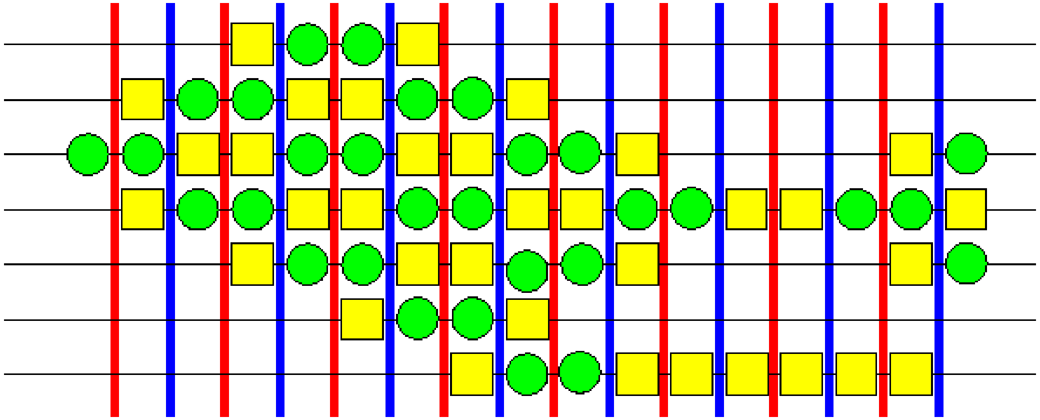
Since an arbitrary rotation can be written as $R_z(a)R_y(b)R_z(c)$, arbitrary single qubit unitaries can be performed.

Two Qubit Gates

For universal quantum computation, it is also necessary to be able to perform a two qubit gate, such as a C-Z or a CNOT.

To accomplish this, we need to decouple one of the end spins. This can be done by applying stroboscopicly an X gate half way between the Hadamard gates.

When the state of the spin, at a chosen end of the chain, is only affected by the state of the CONTROL qubit, it is decoupled as shown.



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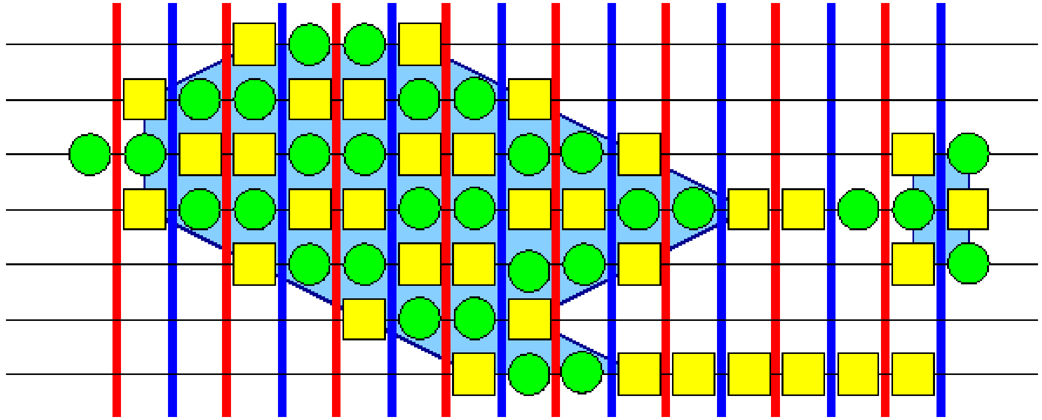
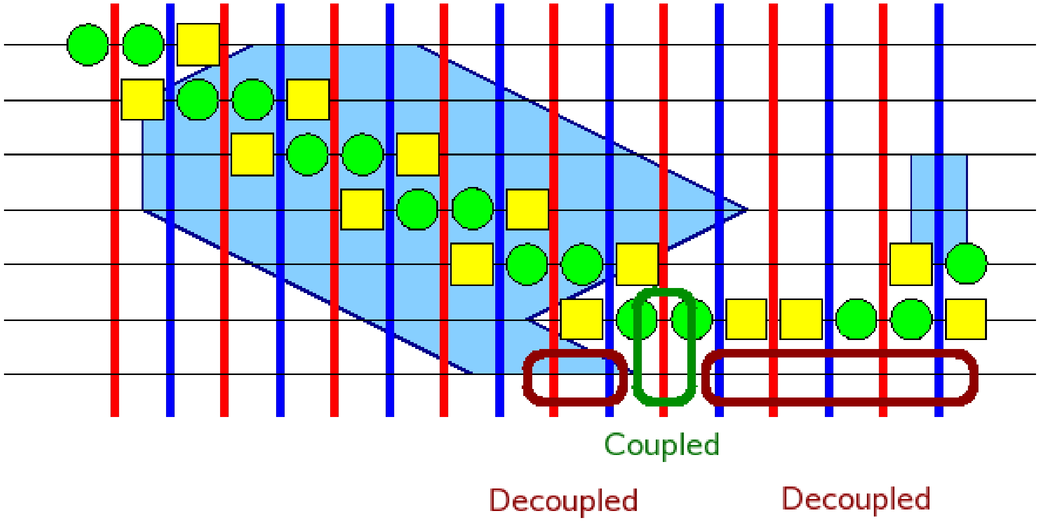


Two Qubit Gates

To interact the TARGET qubit with the CONTROL, we stop decoupling the end spin for one round of the Ising interaction, allowing a C-Z to be performed.

The state of the neighbouring qubit is $|q_{N-1}\rangle = |0\rangle$ if $|\psi_a\rangle = |0\rangle$

and $|q_{N-1}\rangle = |1\rangle$ if $|\psi_a\rangle = |1\rangle$

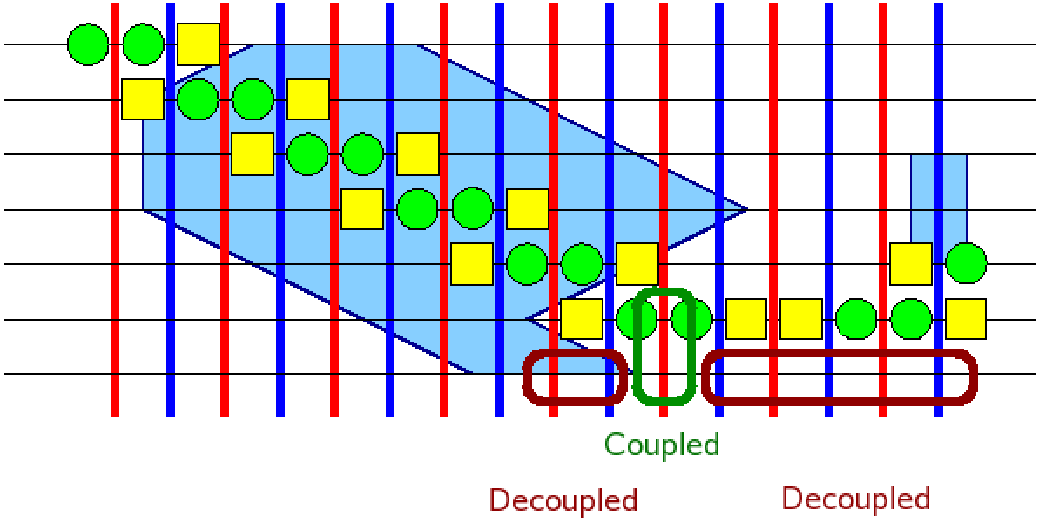


Two Qubit Gates

The state of the end qubit is $|q_N\rangle = |0\rangle$ if $|\psi_b\rangle = |0\rangle$

and $|q_N\rangle = |1\rangle$ if $|\psi_b\rangle = |1\rangle$

By allowing a CZ between these two physical qubits we obtain a CZ between the logical qubits. To localise the logical qubits again, we skip a set of Hadamard gates, and run the process backwards, this time skipping the isolated coupling.



Part 3: Fault Tolerance

Fault Tolerance

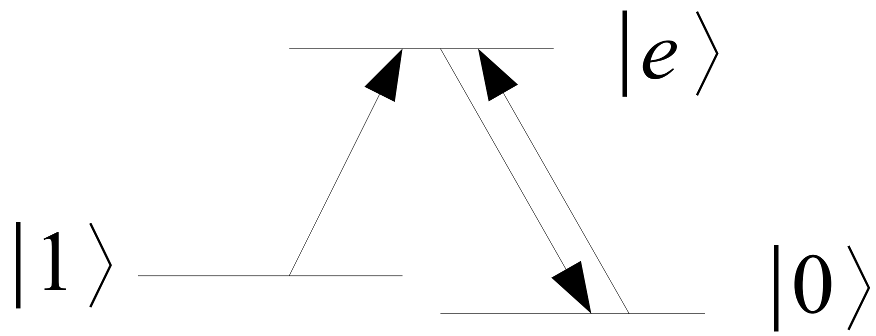
Spin chain layout

- Need at least two species so that ancillas can be reset without destroying the states of the logical qubits.

- Subchains can be decoupled by applying X operator to one species. This will commute with the interaction everywhere except at the boundaries between species.

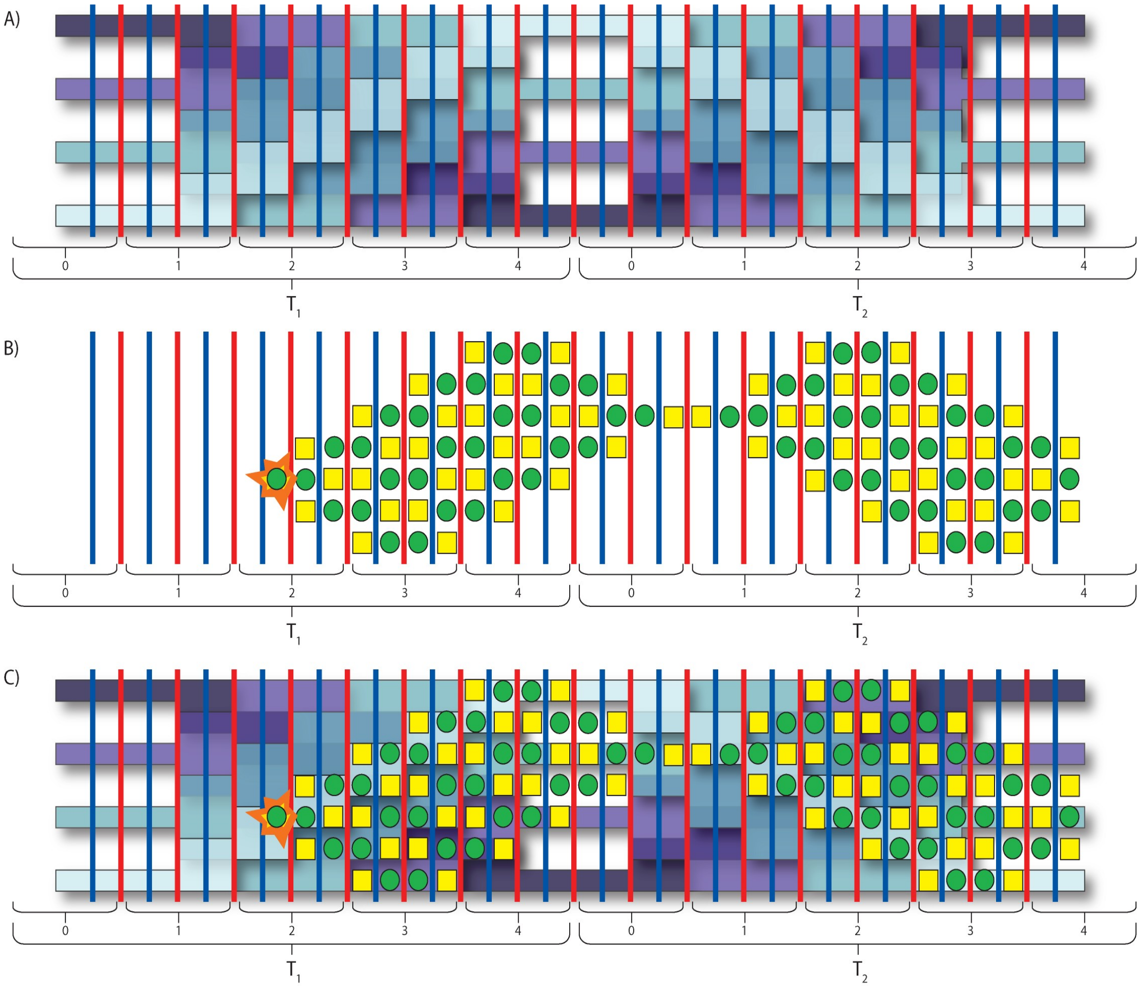
- Need to be able to reset ancillas without knowing their state

  → Need a lambda level structure for at least one species



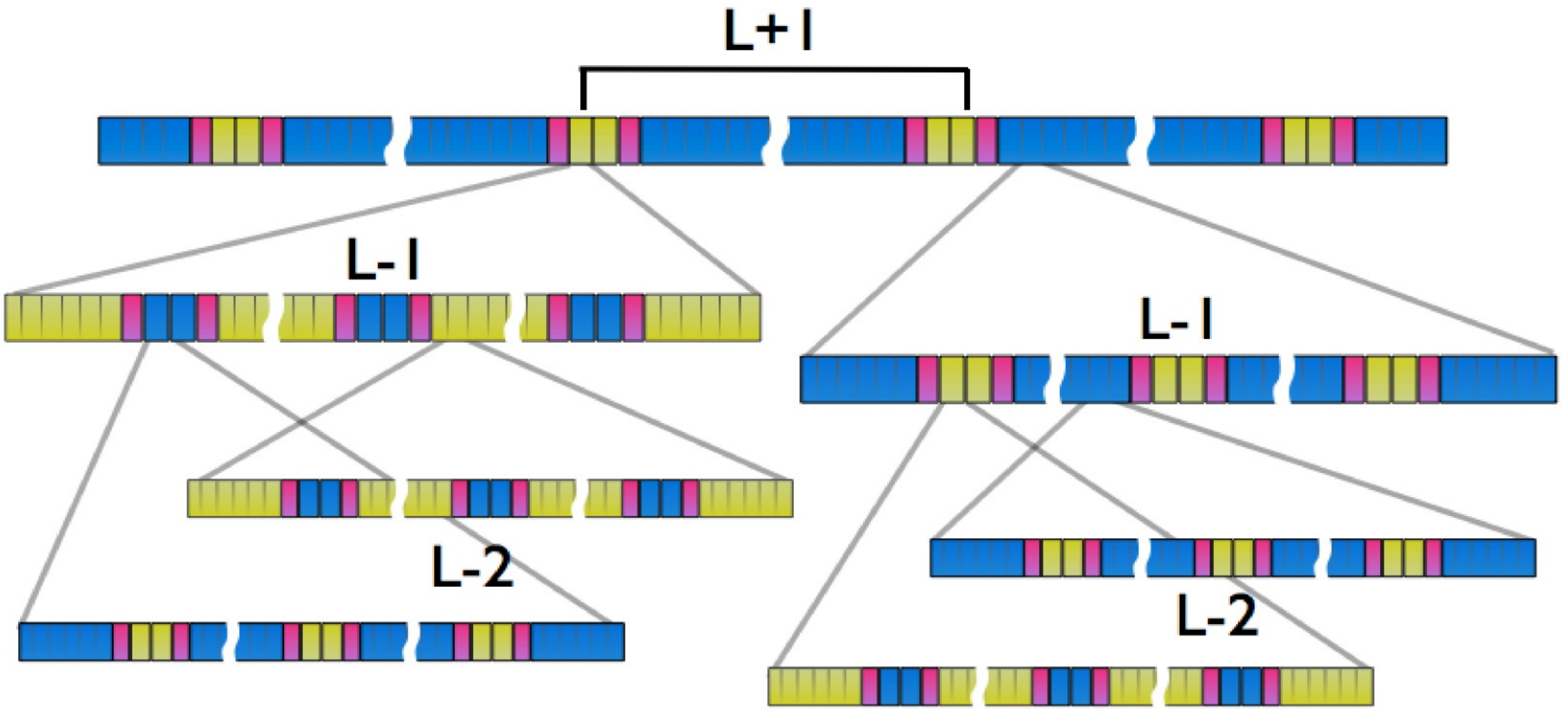
Fault Tolerance

Error propagation



Fault Tolerance

Spin chain layout



Species A level m encoded qubit

Species B level m encoded qubit

Species C physical qubit

Species A level m+1 encoded qubit

Species B level m+1 encoded qubit

Fault Tolerance

Universal QC on subchains

Procedure:

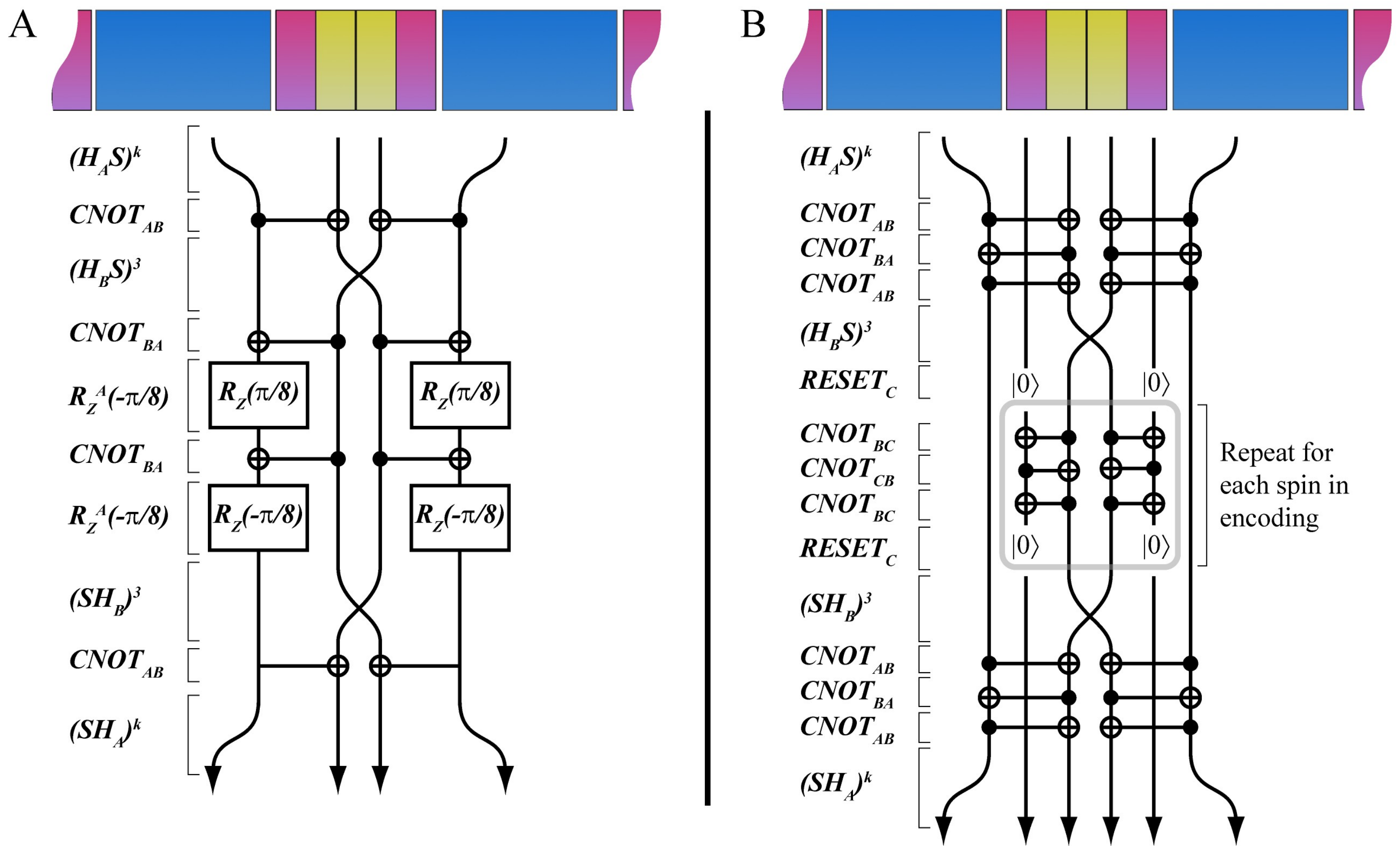
- Use species C qubit to set last qubit on species B to $|0\rangle$
- Interaction between species becomes a local z-rotation on the last spin of species A.

$$e^{i\theta z_A z_B}|0\rangle_B = e^{i\theta z_A}$$

- Decoupling can be used to control angle of rotation
- Single spin rotations and entangling gates between spins are then accomplished as described in Part 2.
- Switch A and B, above to do rotations on species B.

Fault Tolerance

Gates between neighbouring blocks of species A



Fault Tolerance

We can now :

- Perform all operations required to perform error correction
- Perform arbitrary rotations on each qubit, as long as each logical qubit has the same rotation applied
- Interact spins between logical qubits, and hence perform logic gates between qubits in a fault-tolerant manner.

Back at the start! We now have a meta-Ising spin chain where each of the species A chains takes the place of a spin in the original chain.

QuNaT

Conclusion & Acknowledgements

Collaborators

Theory: Jason Twamley, Simon Benjamin

Experiments: Li Xiao, Jonathan Jones

References

- Theory: *Phys. Rev. Lett. 97, 090502* quant-ph/0601120,
  - *Joseph Fitzsimons and Jason Twamley*
- Experiment: , *Phys. Rev. Lett. 99, 030501* quant-ph/0606188
  - *Joseph Fitzsimons, Li Xiao, Simon C. Benjamin, Jonathan A. Jones*
- Fault-tolerance: arXiv:0707.1119
  - *Joseph Fitzsimons and Jason Twamley*

Extra Slides

State Transfer

Any pure state of a qubit *a* can be written as

$$|\Psi_a\rangle = \alpha_a|0\rangle + \beta_a|1\rangle = (\alpha_a + \beta_a\sigma_x^{(a)})|0\rangle$$

So, for any operator *M*,

$$M|\Psi_a\rangle = [M(\alpha_a + \beta_a\sigma_x^{(a)})]|0\rangle = \alpha_a M|0\rangle + \beta_a M\sigma_x^{(a)}|1\rangle$$

The initial state for the spin chain is:

$$|\phi\rangle = |0\rangle \otimes |+\rangle \otimes |0\rangle \otimes |+\rangle \otimes |0\rangle \otimes |+\rangle \otimes \cdots \otimes |0\rangle$$

This satisfies $S|\phi\rangle = |\phi\rangle$ and $SH|\phi\rangle = H|\phi\rangle$

So for any $|\psi_a\rangle = (\alpha_a + \beta_a\sigma_x^{(a)})|\phi\rangle$

$$(HS)^m|\psi\rangle = \alpha_a H^m|\phi\rangle + \beta_a (HS)^m \sigma_x^{(a)}|\phi\rangle$$

State Transfer

To see how the transport circuit actually performs a multi-qubit swap gate, it is important to note the following identities:

$$S\,\sigma_z^{(a)} = \sigma_z^{(a)}\,S$$

$$S\,\sigma_x^{(0)} = \sigma_x^{(0)}\,\sigma_z^{(1)}\,S$$

$$S\,\sigma_x^{(N)} = \sigma_z^{(N-1)}\,\sigma_x^{(N)}\,S$$

$$S\,\sigma_x^{(a)} = \sigma_z^{(a-1)}\,\sigma_x^{(a)}\,\sigma_z^{(a+1)}\,S$$

$$H\,\sigma_z^{(a)} = \sigma_x^{(a)}\,H$$

These allow us to produce equivalent operators for different times within the scheme. For example applying the operator

after the first two rounds of the Ising interaction and Hadamards has the same result as having applied $\sigma_x^{(5)}$ initially, since

$$(H\,S)^2\,\sigma_x^{(5)} = \sigma_x^{(3)}\,\sigma_z^{(4)}\,\sigma_x^{(5)}\,\sigma_z^{(6)}\,\sigma_x^{(7)}\,(H\,S)^2$$

State Transfer

For any qubit, *a*:

$$(H\,S)^{N+1}\sigma_x^{(a)}=\sigma_x^{(N-a)}(H\,S)^{N+1}$$ Eq. 1

$$(H\,S)^{N+1}\sigma_z^{(a)}=\sigma_z^{(N-a)}(H\,S)^{N+1}$$ Eq. 2

Using the Eq. 1,

$$(H\,S)^{N+1}|\psi_a\rangle=(\alpha_a+\beta\,\sigma_x^{(N-a)})|\psi\rangle$$

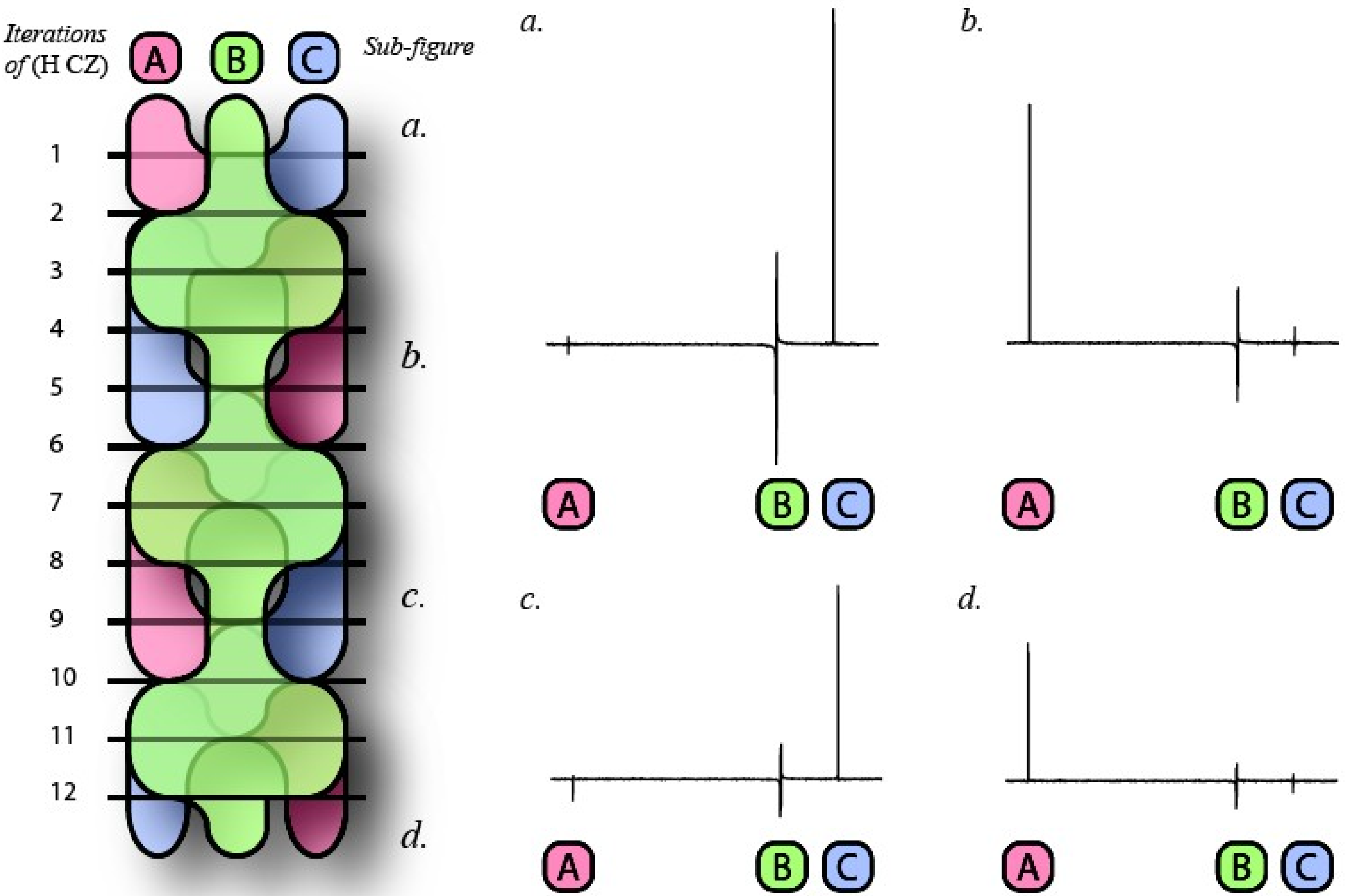
Similarly, by Eq. 2,

$$(H\,S)^{N+1}(\alpha_a+\beta_a\sigma_z^{(a)})|\phi\rangle=(\alpha_a+\beta\,\sigma_z^{(N-a)})|\psi\rangle$$

Since this is true for all *a*, the sites have been inverted, accomplishing a multi-qubit SWAP gate.

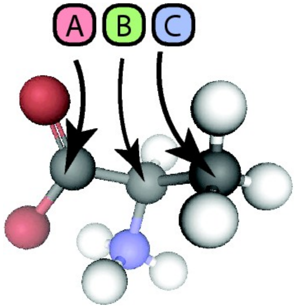
State Transfer

Experimental results



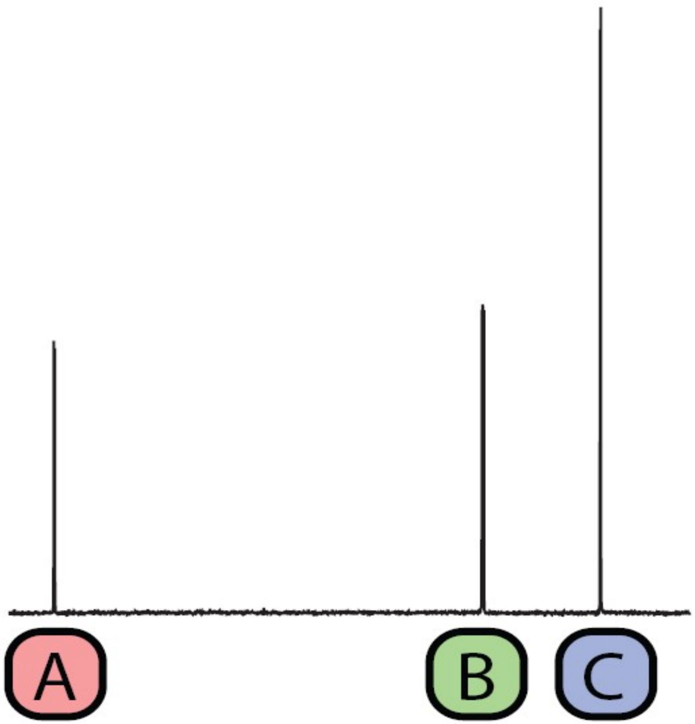
State Transfer

Experimental results



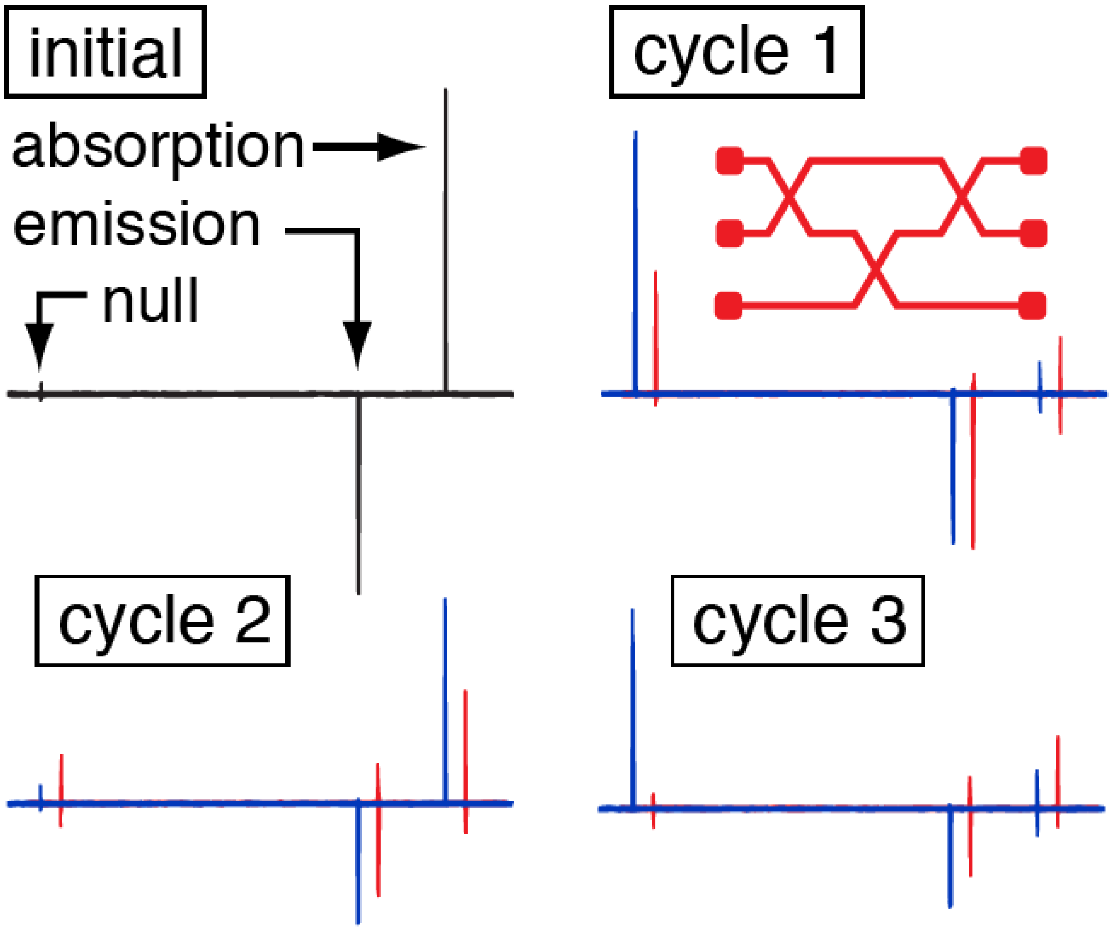
$$\nu_A = +13034.5 \text{ Hz}$$
$$\nu_B = -5869.7 \text{ Hz}$$
$$\nu_C = -11504.2 \text{ Hz}$$

$$J_{AB} = +54.1 \text{ Hz}$$
$$J_{BC} = +35.0 \text{ Hz}$$
$$J_{AC} = -1.3 \text{ Hz}$$



State Transfer

quantum mirror versus SWAP network



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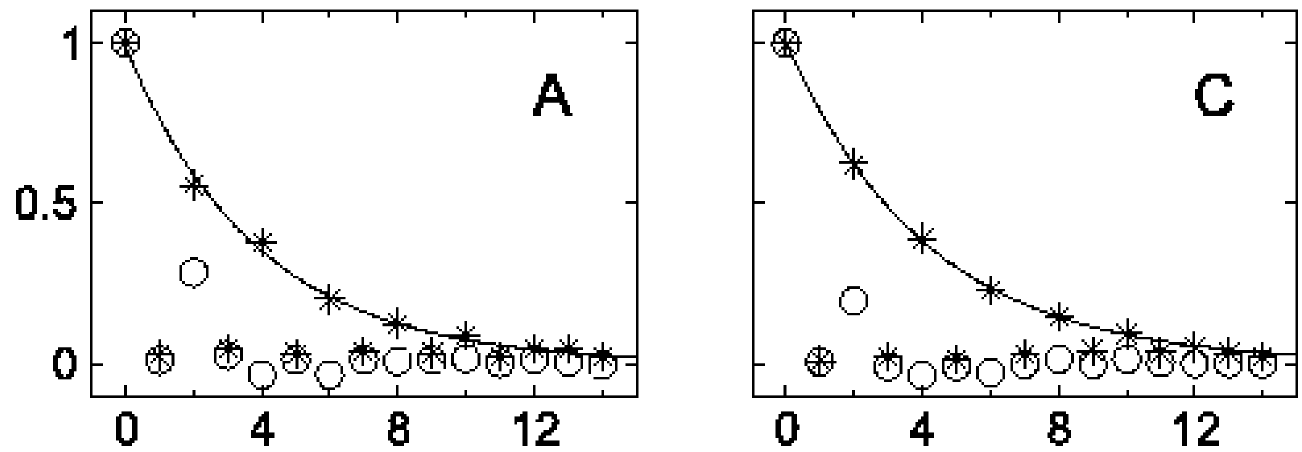
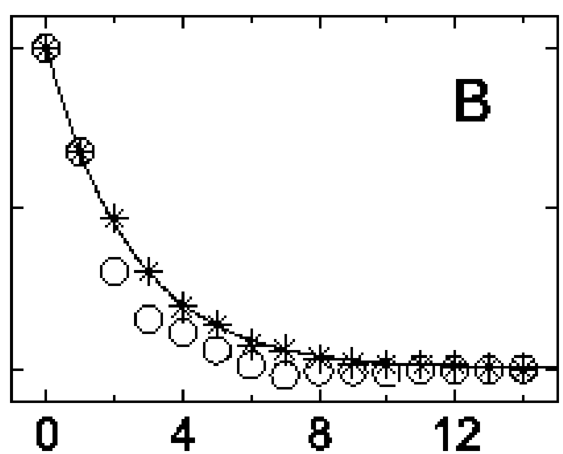




Raw Decay Constants:

A: 3.9 Cycles

B: 2.5 Cycles

C: 4.2 Cycles

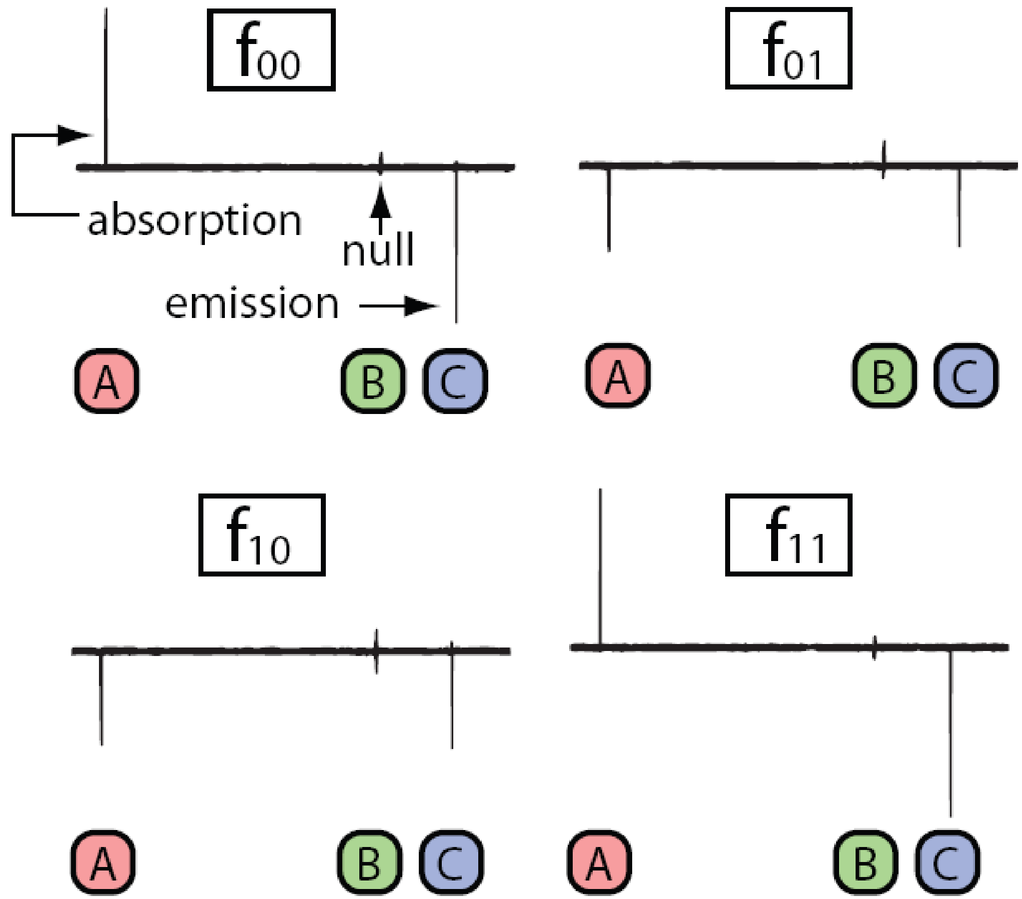
Less RF inhomogineities etc.:

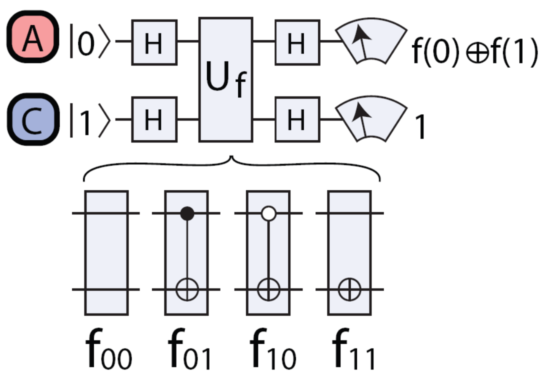
~13 cycles

Equivalent to transport across approximately 52 spins

Two Qubit Gates

Deutsch Algorithm





Decreasing Overhead

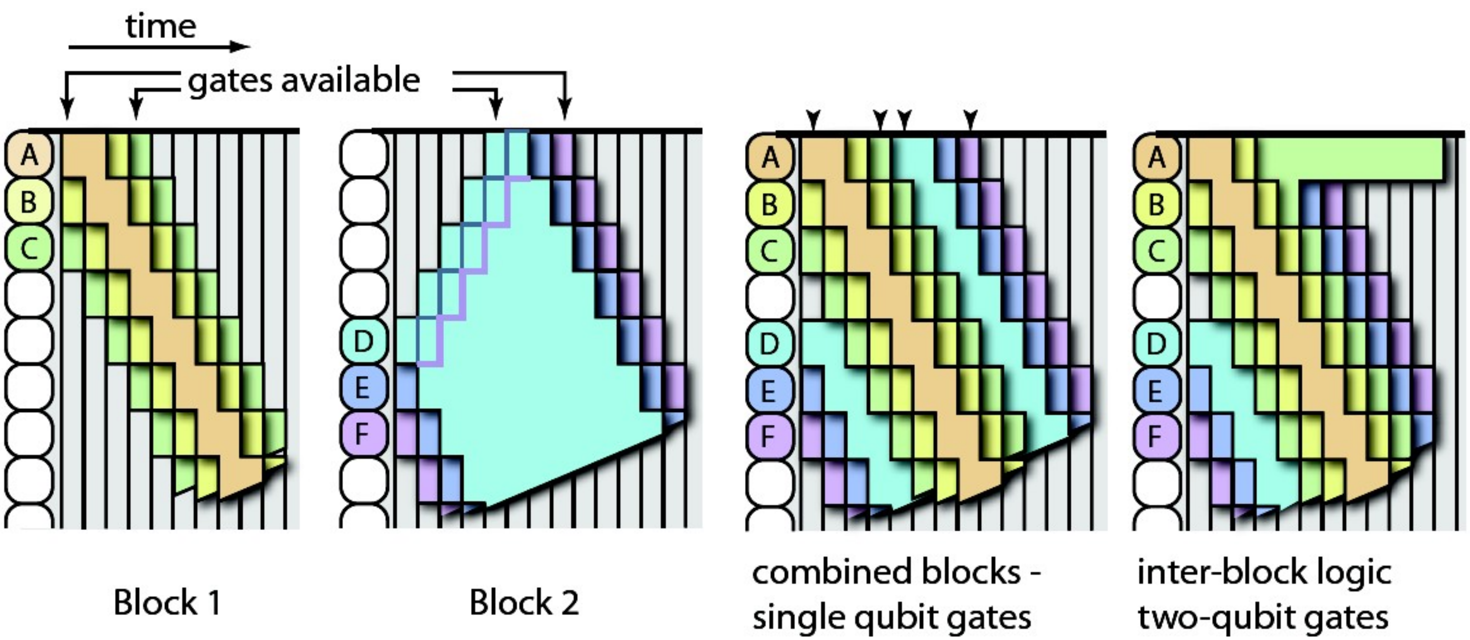
This scheme has an overhead on all operations which scales linearly with N.

This can be reduced, however, since

- All qubits can be rotated in a single mirror inversion cycle
- Any number of CZs, or Controlled Phase operations with the same controlling qubit can be performed in a single mirror inversion cycle.

Dense Packing

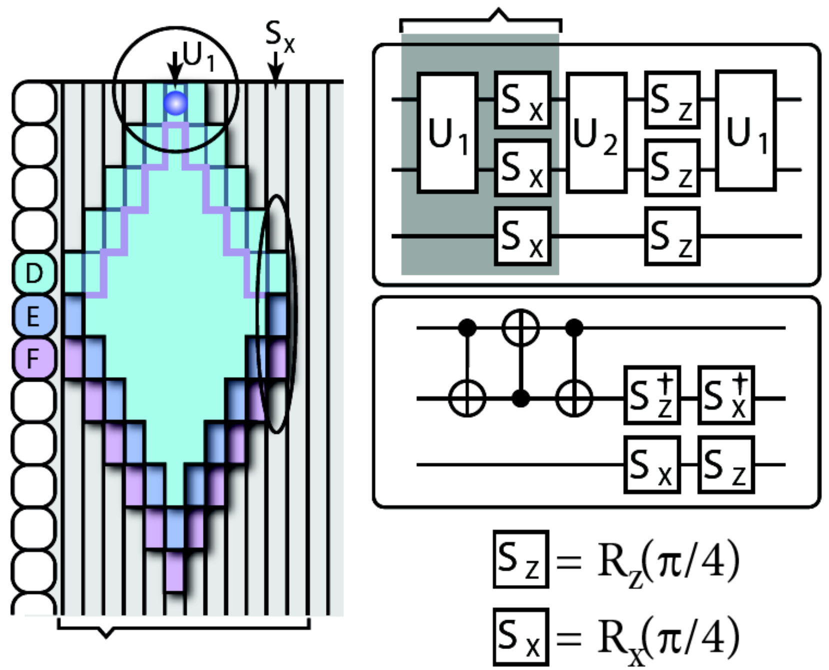
- Qubits can be packed into blocks
- Blocks are separated by a buffer state
- Qubits at the edge of blocks can undergo gates as described earlier



Dense Packing

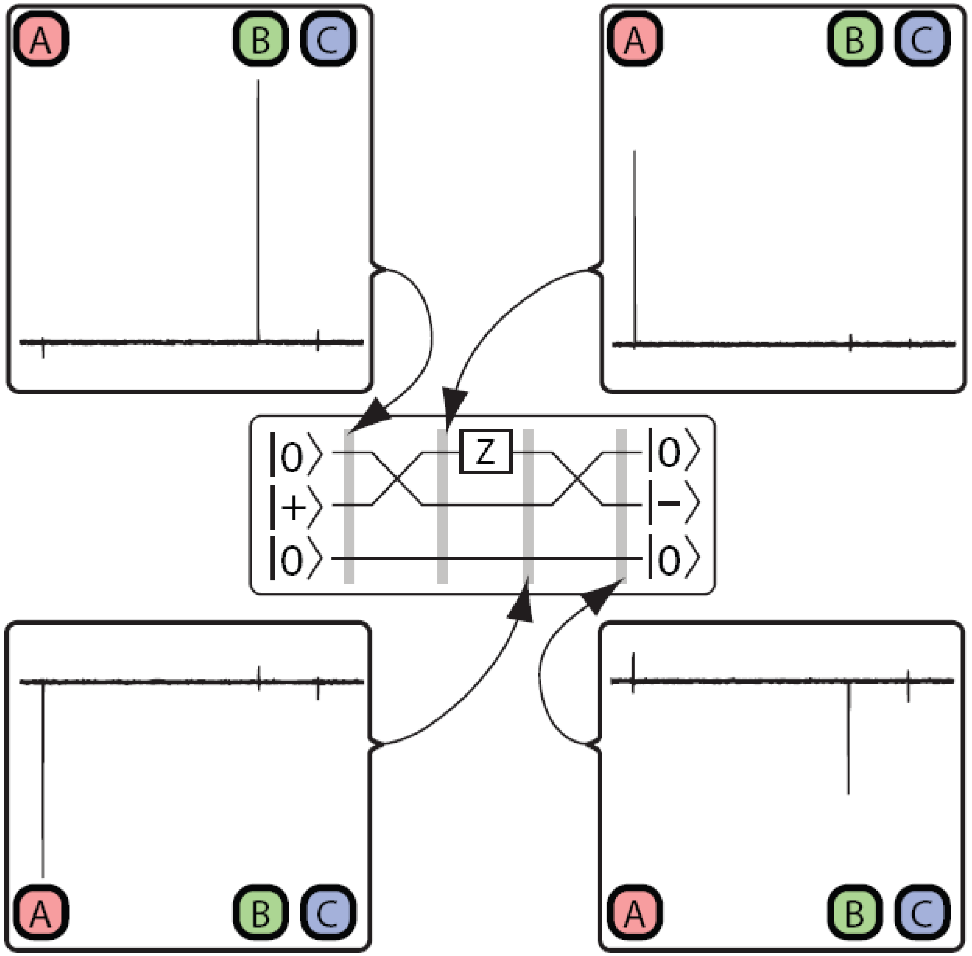
•Qubits on the interior of a block must be swapped to the edge before logic gates can be performed on them

•Trade off between time and space: Gates take O(Nxm) for a block size of m → chain length = (N + N/m-1)



Dense Packing

Single Qubit Rotation



Dense Packing

Deutsch-Jozsa Algorithm