Entropy aided Quantum error correction and fault tolerance

Finding high thresholds

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Superoperator notation

- Instead of $\rho = \frac{1}{2}(I + c_X X + c_Y Y + c_Z Z)$

$$\begin{bmatrix} 1 \\ c_X \\ c_Y \\ c_Z \end{bmatrix}$$

- Acted upon by a channel, which has a super-operator

$$\mathcal{N}^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ N_{XI} & N_{XX} & N_{XY} & N_{XZ} \\ N_{YI} & N_{YX} & N_{YY} & N_{YZ} \\ N_{ZI} & N_{ZX} & N_{ZY} & N_{ZZ} \end{bmatrix}$$

Channel map on $[[n, k, d]]$ code

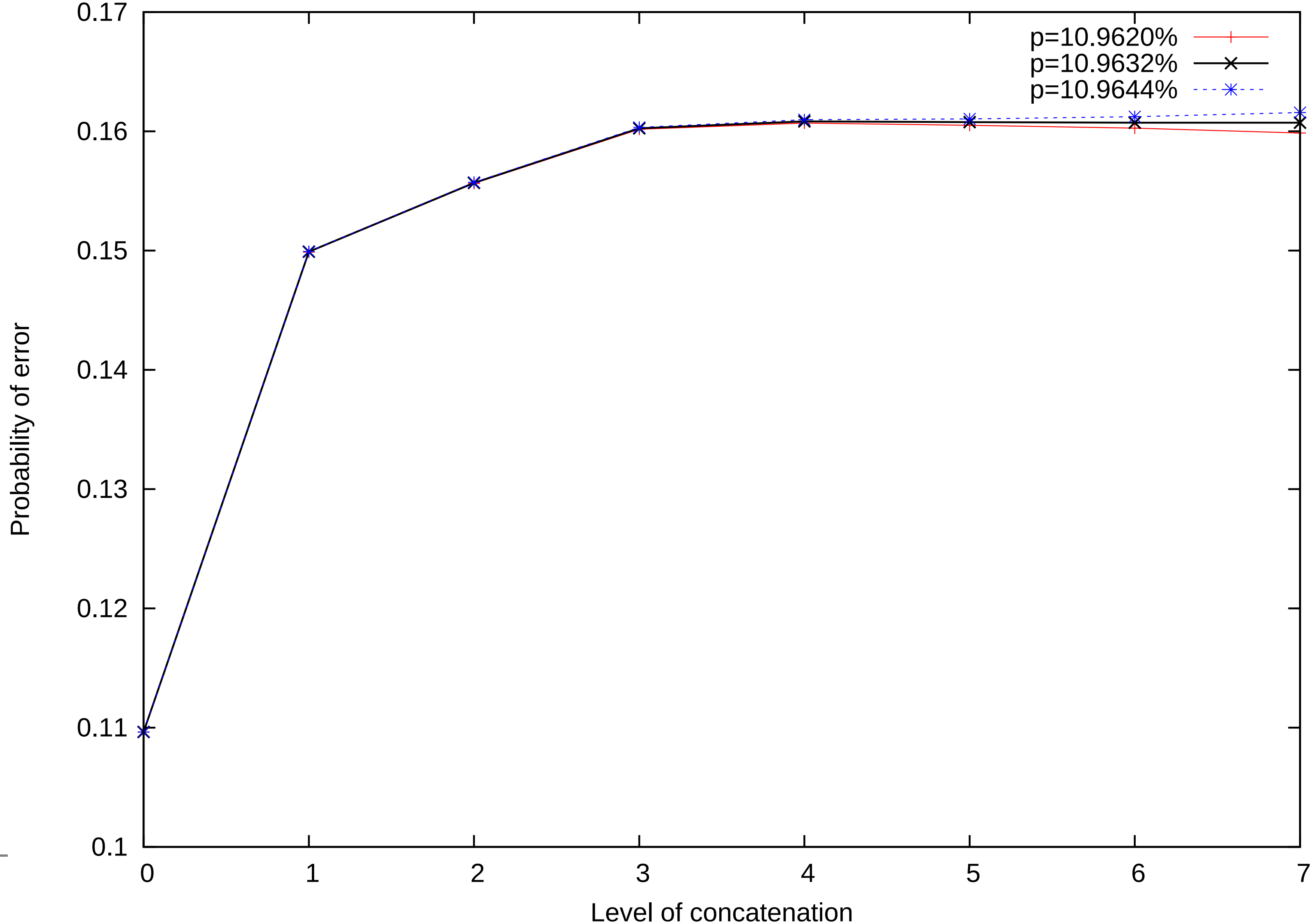
- $\mathcal{E}$ encodes k qubit state into n qubit state
- $\mathcal{N}$ represents the noise in encoded space
- Recovery operators R_β
- $\mathcal{E}^t$ decodes n qubit space into k qubit space
- $$\mathcal{G} = \sum_\beta \mathcal{E}^t \circ \mathcal{O}(R_\beta) \circ \mathcal{N} \circ \mathcal{E}$$

Recovery optimization

- $$\mathcal{G} = \sum_{\beta} \mathcal{G}^{R_\beta} \text{ where } G^{R_\beta} = \mathcal{E}^t \circ \mathcal{O}(R_\beta) \circ \mathcal{N} \circ \mathcal{E}$$
- $\mathcal{G}^{R_\beta}$ is the noise for syndrome β
- Find the optimal recovery operator
- Use the syndrome information from lower levels of the code for higher levels of the code
- Shannon entropy useful for calculations
- Entropy of Pauli errors
  $h(p_I) + h(p_X) + h(p_Y) + h(p_Z)$
  $h(x) = -x \log_2 x$

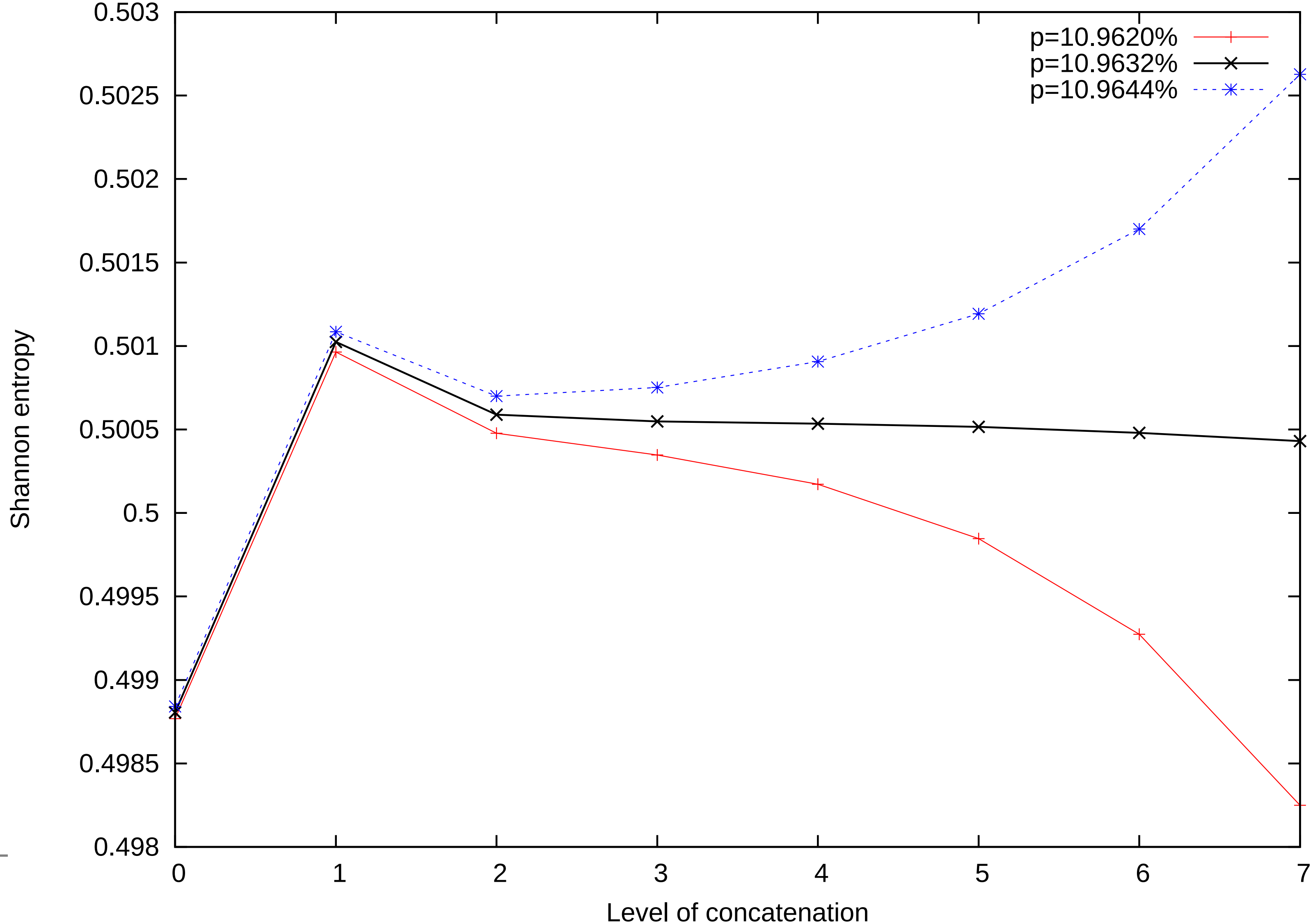
Average error

Doesn't work so well



Average entropy

Works much better



arXiv:quant-ph/0703258

- Correctable noise of Quantum Error Correcting Codes under adaptive concatenation
- Extension of Poulin approach of optimizing recovery operators
- Same noise Pauli noise (p_X, p_Y, p_Z) on each qubit
- Thresholds

Code	(p, p, p)	$(p - p^2, p^2, p - p^2)$
$[[5, 1, 3]]$	6.299(6)%	10.951%
$[[7, 1, 3]]$	6.270(3)%	10.963%
Hashing	6.3097%	11.0028%

- Hashing rate is when the Shannon entropy is 1

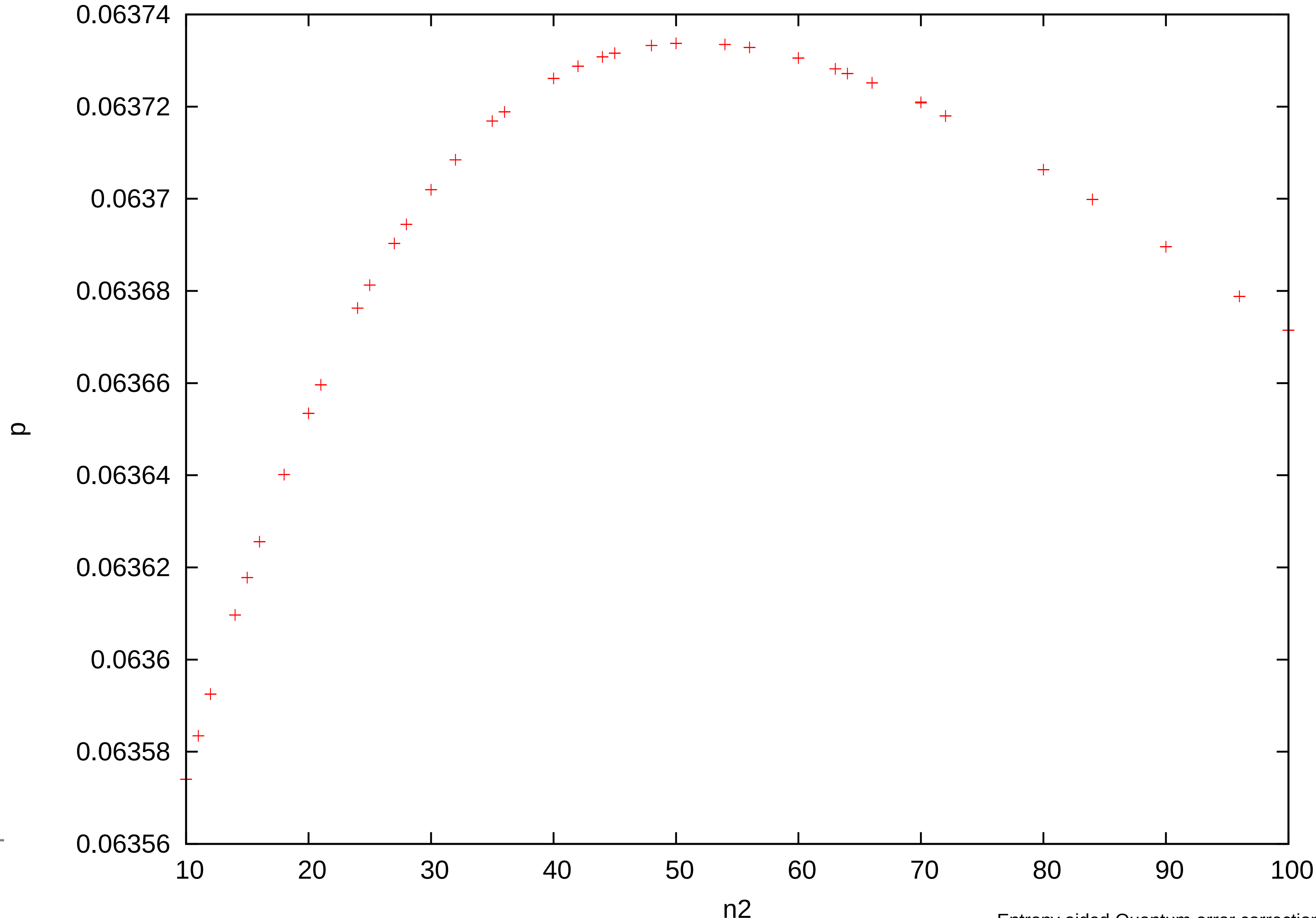
Depolarizing noise that is correctable

Correctable Fidelity $(1 - p_X - p_Y - p_Z)$

- 0.81071 Random coding (hashing rate)
- 0.80964 5 qubit bit flip (Shor, Smolin)
- 0.80944 5 qubit bit flip, 5 qubit phase flip (DiVincenzo, Shor, Smolin)
- 0.80912 5 qubit bit flip, 16 qubit phase flip (Smith, Smolin)
- 0.80880 5 qubit bit flip, 50 qubit phase flip (Fern, Whaley)
- 0.80870 5 qubit bit flip, 56 qubit phase flip, repeated $[[5, 1, 3]]$ (Fern, Whaley)

5 in n_2 code thresholds

5 qubit bit flip concatenated with n_2 qubit phase flip



Knill's work

- Knill, Nature, 2005, quant-ph/0410199
- `CNOT` has probability p of depolarizing error
- Probability $\frac{4p}{15}$ of measurement error
- Massive post-selection of ancilla qubits
- Use quantum teleportation to bring in logical data qubits
- Knill gets a threshold of "above 3%" if no restriction on # of ancillas

Entropy based fault tolerant calculations

- Assume independent noise
- Generalized (multiqubit) teleportation to perform `CNOT`
- Find total noise from teleportation
- Degeneracies lower the threshold
- Correct estimated threshold for degeneracies
- Entropy of correlated noise useful

Fault tolerant thresholds

- No restrictions on overhead - extreme overhead
- Upper bounds
- 6.78% $[[7, 1, 3]]$ CSS Code
- 6.88% $[[23, 1, 7]]$ CSS Code
- 6.90% Hashing rate - Conjectured upper bound
- If no measurement errors 8.25%

Papers

- J. Fern, J. Kempe, S. Simic, S.Sastry. Generalized Performance of Concatenated Quantum Codes – A Dynamical Systems Approach. IEEE Trans. Autom. Control. 51, 448 (2006), `quant-ph/0409084`
- J. Fern Correctable noise of Quantum Error Correcting Codes under adaptive concatenation to appear Physical Review A, `quant-ph/0703258`
- J. Fern, K.B. Whaley New lower bounds on the non-zero capacity of Pauli Channels `arXiv:0708.1597`
- J. Fern An upper bound on quantum fault tolerant thresholds (in preparation)