Correction of hybrid quantum-classical information

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Joint work with David Kribs and Achim Kempf

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The stochastic Heisenberg picture

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Correctable observables form algebras

Algebras represent hybrid information

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- The dual channel maps future observables to past observables
Conserved propositions

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Sufficiency: $\mathcal{E}^\dagger(P) = \mathcal{E}^\dagger(1) P = 1 P = P$
Correctable observables form algebras
Algebras represent hybrid information
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- Close in spirit to the original noiseless subsystem paper (Knill, Laflamme, Viola)
Correctable observables

- $R$ is **correctable** if there is $R$ such that $(R \circ E)^\dagger(P) = P$

Note: the whole algebra can be corrected with a unique channel $R$

Note: the same $R$ still works for a channel with elements $F_i = \sum_j \alpha_{ij} E_j$
Correctable observables

- R is **correctable** if there is \( \mathcal{R} \) such that \( (\mathcal{R} \circ \mathcal{E})^\dagger(P) = P \)
- Then \( [P, R_k E_i] = 0 \)
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- In fact this is **sufficient** for \( \mathcal{R} \) to exist
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**Example**

- Correctable observables
- \( R \) is correctable if there is \( R \) such that \((R \circ \mathcal{E})^+(P) = P\)
- Then \([P, R_k E_i] = 0\) hence \([P, E_i^+ E_j] = 0\) indeed
  \[
  PE_i^+ E_j = E_i^+ E_j P
  \]
- In fact this is sufficient for \( R \) to exist
- **Hence set of all correctable observables span the \( ^+ \)-algebra**

\[
\mathcal{A} = \{ X \mid [X, E_i^+ E_j] = 0 \text{ for all } i, j \}
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  \mathcal{A} = \{ X \mid [X, E_i^\dagger E_j] = 0 \text{ for all } i, j \}
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- Note: the whole algebra can be corrected with a **unique channel** \( \mathcal{R} \)
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- **Hence set of all correctable observables span the** $\dagger$-**algebra**

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- Note: the whole algebra can be corrected with a **unique** channel $R$
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- $\dagger$-algebras have a simple structure
Correctable observables form algebras

Algebras represent hybrid information

Example

- †-algebras have a simple structure
- They are direct-sum of full matrix algebras

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- Inside bigger matrices there may be redundancy

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- The structure of the subalgebra selects subspaces and subsystems
Hybrid quantum-classical information

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Correctable observables form algebras

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Example:

Hybrid quantum-classical information

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  - If the bit is 1 then we have also two qubits: $\mathcal{M}_4$
- The amount of quantum information is conditional on the classical information
- Known as hybrid information (Kuperberg) or “quantum system with superselection rules”
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Comparison to QEC, OQEC
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Example

Comparison to QEC, OQEC

Standard QEC:

\[ \mathcal{A} = \mathcal{B}(\mathcal{H}_C) \]
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Subsystem QEC (Operator QEC):

\[ \mathcal{A} = \mathcal{M}_n \otimes \mathbf{1} \subseteq \mathcal{B}(\mathcal{H}_C) \]
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Subsystem QEC (Operator QEC):

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Operator Algebra QEC:

\[ \mathcal{A} = \begin{pmatrix} \mathcal{M}_{n_1} \otimes 1 & 0 & \cdots & 0 \\ 0 & \mathcal{M}_{n_2} \otimes 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathcal{M}_{n_N} \otimes 1 \end{pmatrix} \subseteq \mathcal{B}(\mathcal{H}_C) \]
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What happen if there are errors in the classical communication channel for teleportation?
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\[ \mathcal{E}(\rho) = \frac{1}{N} \sum_{ij} p_{ij} |i\rangle \langle i| \otimes U_j \rho U_j^\dagger \]
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\[ E_{ij} \propto \sqrt{p_{ij}} |i\rangle \otimes U_j \]
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**Noisy teleportation**

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- $\mathcal{E}(\rho) = \frac{1}{N} \sum_{ij} p_{ij} |i\rangle \langle i| \otimes U_j \rho U_j^\dagger$
- $E_{ij} \propto \sqrt{p_{ij}} |i\rangle \otimes U_j$
- $E_{ij}^\dagger E_{kl} \propto \delta_{ik} \sqrt{p_{ij} p_{il}} U_j^\dagger U_l$
- If $p_{ij} \neq 0$ and $p_{il} \neq 0$, then $j$ and $l$ cannot be distinguished
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Noisy teleportation

What happen if there are errors in the classical communication channel for teleportation?

- \( E(\rho) = \frac{1}{N} \sum_i |i\rangle \langle i| \otimes U_i \rho U_i^\dagger \)
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- \( E_{ij} \propto \sqrt{p_{ij}} |i\rangle \otimes U_j \)
- \( E_{ij}^\dagger E_{kl} \propto \delta_{ik} \sqrt{p_{ij} p_{il}} U_j^\dagger U_l \)
- If \( p_{ij} \neq 0 \) and \( p_{il} \neq 0 \), then \( j \) and \( l \) cannot be distinguished
- Bob can only recover those observables invariant under \( U_j^\dagger U_l \)
Noisy teleportation

- For instance, consider a qubit: $U_i \in \{1 = \sigma_0, \sigma_x, \sigma_y, \sigma_z\}$
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- Then the correctable \( \mathcal{A} \) must commute with \( \sigma_0 \sigma_z = \sigma_z \):

\[
\mathcal{A} = \text{Alg}(\sigma_z) = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \text{ for all } a, b \right\}
\]
For instance, consider a qubit: $U_i \in \{1 = \sigma_0, \sigma_x, \sigma_y, \sigma_z\}$

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Then the correctable $\mathcal{A}$ must commute with $\sigma_0 \sigma_z = \sigma_z$:

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Bob can only recover one classical bit, corresponding to a measurement of $\sigma_z$ on Alice’s qubit
Further reading:
PRL 98, 100502
PRA 76, 042303
Correctable observables form algebras

Algebras represent hybrid information

Example

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PRL 98, 100502
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Thank you for your attention!