Correction of hybrid quantum-classical information

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Joint work with David Kribs and Achim Kempf

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- The dual channel maps future observables to past observables

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- Sufficiency: $\mathcal{E}^\dagger(P) = \mathcal{E}^\dagger(\mathbb{1}) P = \mathbb{1} P = P$

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- Close in spirit to the original noiseless subsystem paper (Knill, Laflamme, Viola)

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- Note: the same $\mathcal{R}$ still works for a channel with elements $F_i = \sum_j \alpha_{ij} E_j$

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- The structure of the subalgebra selects subspaces and subsystems

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- The amount of quantum information is conditional on the classical information
- Known as hybrid information (Kuperberg) or "quantum system with superselection rules"

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Operator Algebra QEC:

$$\mathcal{A} = \begin{pmatrix} \mathcal{M}_{n_1} \otimes \mathbb{1} & 0 & \cdots & 0 \\ 0 & \mathcal{M}_{n_2} \otimes \mathbb{1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathcal{M}_{n_N} \otimes \mathbb{1} \end{pmatrix} \subseteq \mathcal{B}(\mathcal{H}_C)$$

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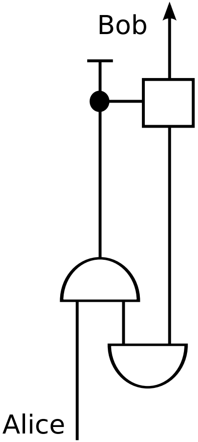

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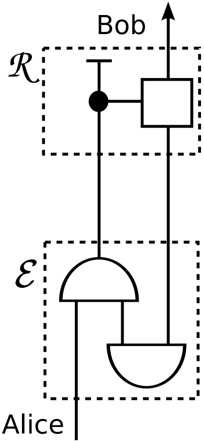

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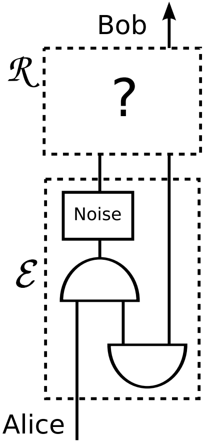

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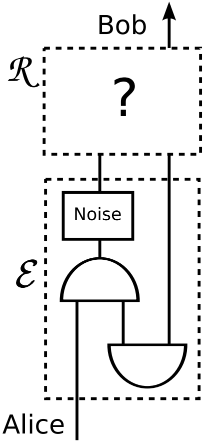

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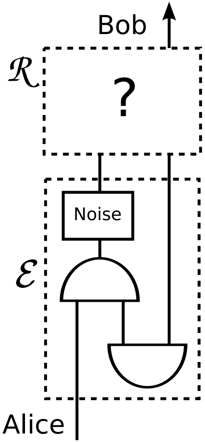

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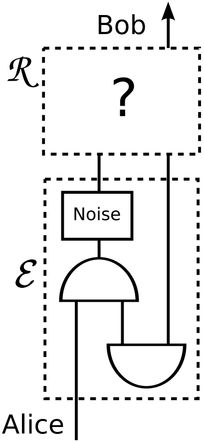

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- $E_{ij}^\dagger E_{kl} \propto \delta_{ik}\sqrt{p_{ij}p_{il}}U_j^\dagger U_l$
- If $p_{ij} \neq 0$ and $p_{il} \neq 0$, then j and l cannot be distinguished

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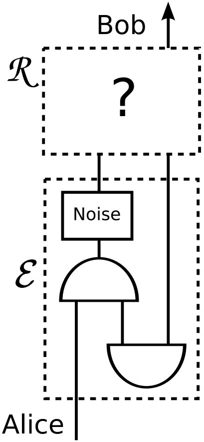

- What happen if there are errors in the classical communication channel for teleportation?
- $\mathcal{E}(\rho) = \frac{1}{N}\sum_i |i\rangle\langle i| \otimes U_i \rho U_i^\dagger$
- $\mathcal{E}(\rho) = \frac{1}{N}\sum_{ij} p_{ij} |i\rangle\langle i| \otimes U_j \rho U_j^\dagger$
- $E_{ij} \propto \sqrt{p_{ij}}|i\rangle \otimes U_j$
- $E_{ij}^\dagger E_{kl} \propto \delta_{ik}\sqrt{p_{ij}p_{il}}U_j^\dagger U_l$
- If $p_{ij} \neq 0$ and $p_{il} \neq 0$, then j and l cannot be distinguished
- Bob can only recover those observables invariant under $U_j^\dagger U_l$

Noisy teleportation

- For instance, consider a qubit: $U_i \in \{\mathbb{1} = \sigma_0, \sigma_x, \sigma_y, \sigma_z\}$

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- Bob can only recover one classical bit, corresponding to a measurement of σ_z on Alice's qubit

- Further reading:
  PRL 98, 100502
  PRA 76, 042303

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Thank you for your attention!