Protected qubits using Josephson junctions and other superconducting elements.

Goal : all-electric protected* qubits

(Qubits are not elementary devices)

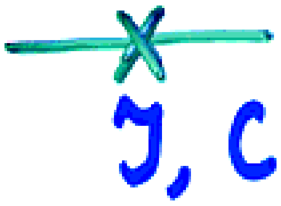
Sub-goal : find a set of basic elements for quantum electric circuits

* Protected =
All unwanted interactions are exponentially suppressed.

"Simple" elements:

Capacitor :

Josephson junction :



Standard

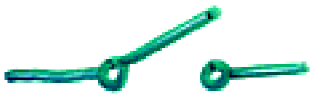
Inductor :

Usually implemented as a chain of Josephson junctions

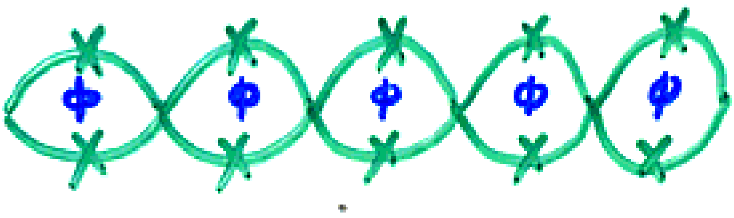
$$J \gg \frac{e^2}{C_2}$$

$$\left(L_{eff} = N \left(\frac{\hbar}{2e} \right)^2 J^{-1} \right)$$

Switch :



(Haviland at al 2000)

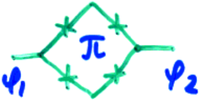


$\phi \approx \pi$ $\Longrightarrow$ transition to an insulating state

(In dimensional units, $\phi \approx \frac{\Phi_0}{2}$)

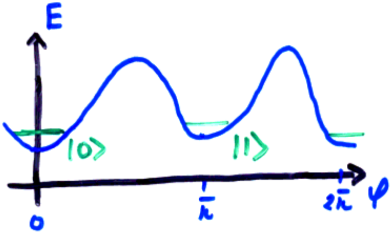
More exotic elements :

1) 0-π - contact (Doucot, Vidal 2002
Ioffe, Feigelman 2002)



$$E = -J_1 \cos(2\varphi) - \underbrace{J_2\, g(\varphi)}_{\text{error term}}$$

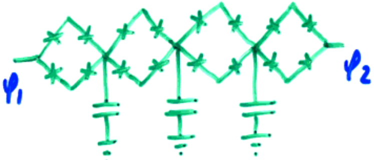
$$J_2 \ll J_1$$



$$\underline{\delta E = E_1 - E_0 = 2 J_2}$$

May depend on the environment ⇒ decoherence

2) Protection (stabilization) $\delta E_{eff} \to 0$



$$J_{1,eff} \sim \frac{1}{N}$$

$$\boxed{\delta E_{eff} \sim \left(\frac{J_2}{\underset{\uparrow\ \text{tunneling amplitude}}{t}}\right)^{N-1}}$$

$|0_L\rangle = |0000\rangle + |1100\rangle + \ldots$
(even number of 1s)

$|1_L\rangle = |1000\rangle + |0100\rangle + \ldots$
(odd number of 1s)

It seems that quantum protection requires a many-body system because a protected state is similar to a quantum code.

Not quite true: One can use a single continuous degree of freedom for quantum encoding (Gottesman, Kitaev, Preskill 2000)

Main conceptual result of the present work:

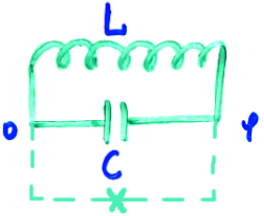
A suffitiently large superconducting inductor provides room for quantum encoding.

(Of course, Josephson junctions are also needed)

Conventions: $\hbar = 1$, $2e = 1$.

Resistance unit: $\frac{\hbar}{(2e)^2} \approx 1\,k\Omega$

In this units, the quantum resistance $\underline{R_q = \frac{h}{(2e)^2} \approx 6\,k\Omega}$ is simply $\underline{2\pi}$.



$$H = \frac{\varphi^2}{2L} + \frac{1}{2C}\underbrace{\left(\frac{\partial}{i\,\partial\varphi}\right)^2}_{q}$$

(charge operator)

$$\langle \varphi^2 \rangle = \frac{1}{2}\sqrt{\frac{L}{C}}$$ (characteristic impedance)

"Superinductor": $\underline{\langle \varphi^2 \rangle \text{ is large}}$

$$\langle \cos\varphi \rangle = e^{-\frac{\langle \varphi^2 \rangle}{2}} = \exp\left(-\frac{1}{4}\sqrt{\frac{L}{C}}\right)$$

$$\boxed{\sqrt{\frac{L}{C}} \gg 4 \quad \left(4\,k\Omega \text{ in the usual units}\right)}$$

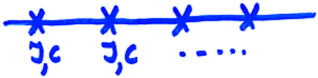
in units of $\frac{\hbar}{(2e)^2}$

How to make a superinductor?

1) **Coil of n loops**: $\sqrt{\frac{L}{C_{geom}}} \sim R_{vac}\sqrt{\frac{\mu}{\varepsilon}}\, n$

$$R_{vac} = \frac{4\pi}{137} \qquad \left(\frac{4\pi}{c} \approx 377\Omega\right)$$

Too many loops are needed

2) 

$L = \frac{N}{J}$

$\sqrt{\frac{L}{C_{chain}}} = \frac{N}{\sqrt{JC}} \gg 4$, but $\sqrt{JC} \gg 1$ is necessary to prevent phase slips.

Too many junctions...

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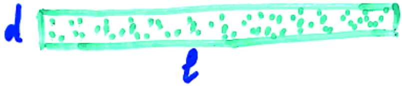
3) **Kinetic inductance**



(Amorphous film)

$$L \sim \frac{R}{\pi\Delta} = \frac{\ell}{d}\,\frac{R_{\square}}{\pi\Delta}$$

normal state resistance

(Assuming that quantum fluctuations are not too strong)

For kinetic inductors, $C_{geom.}$ is not a limiting factor because usually $\frac{1}{\sqrt{L C_{geom}}} > \Delta$.

$\langle \varphi^2 \rangle \sim R = \frac{l}{d} R_\square$ It is possible that $R_\square \sim 4\,k\Omega$

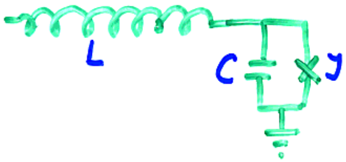
Superinductor requirements:

1) $\frac{l}{d} \gg \frac{4}{R_\square}$

2) $\frac{\xi}{d} \ll \frac{4}{R_\square}$ to prevent phase slips

$\xi \sim 30\,\text{Å}$ in MoGe

Typical application:



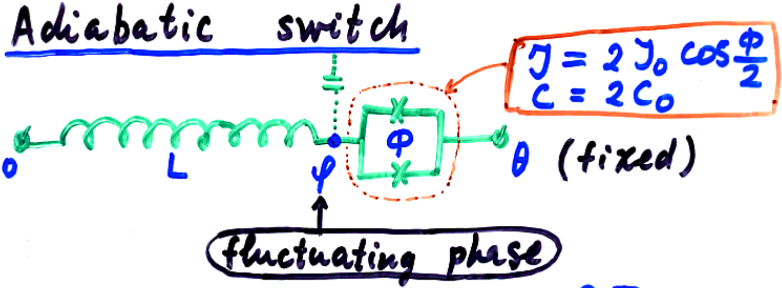
$\mu = \sqrt{JC} \sim 1$
(intermediate quantum regime)

For $Al - Al_2O_3$ junctions $\omega_p = \sqrt{\frac{J}{C}} \sim 20\,GHz$
$\sim 1\,K$

$J \sim C^{-1} \sim \omega_p \Rightarrow C \sim 5 \cdot 10^{-15} F$ in the junction

The geometric capacitance is small if $l < 10\mu m$

Adiabatic switch

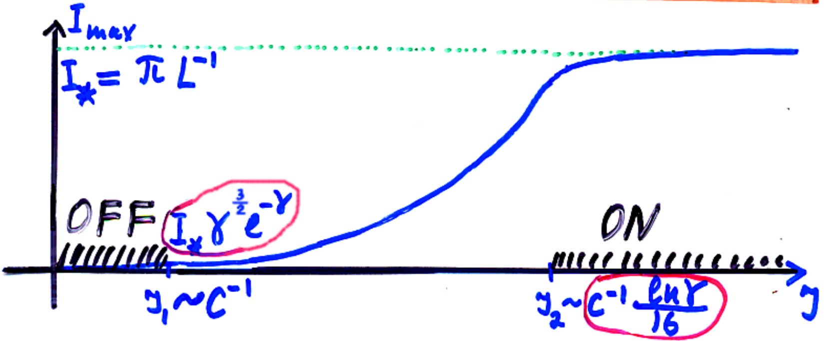


$$\mathcal{J} = 2\mathcal{J}_0 \cos\frac{\Phi}{2}$$
$$C = 2C_0$$

Superconducting current $I(\theta) = \frac{\partial E}{\partial \theta}$

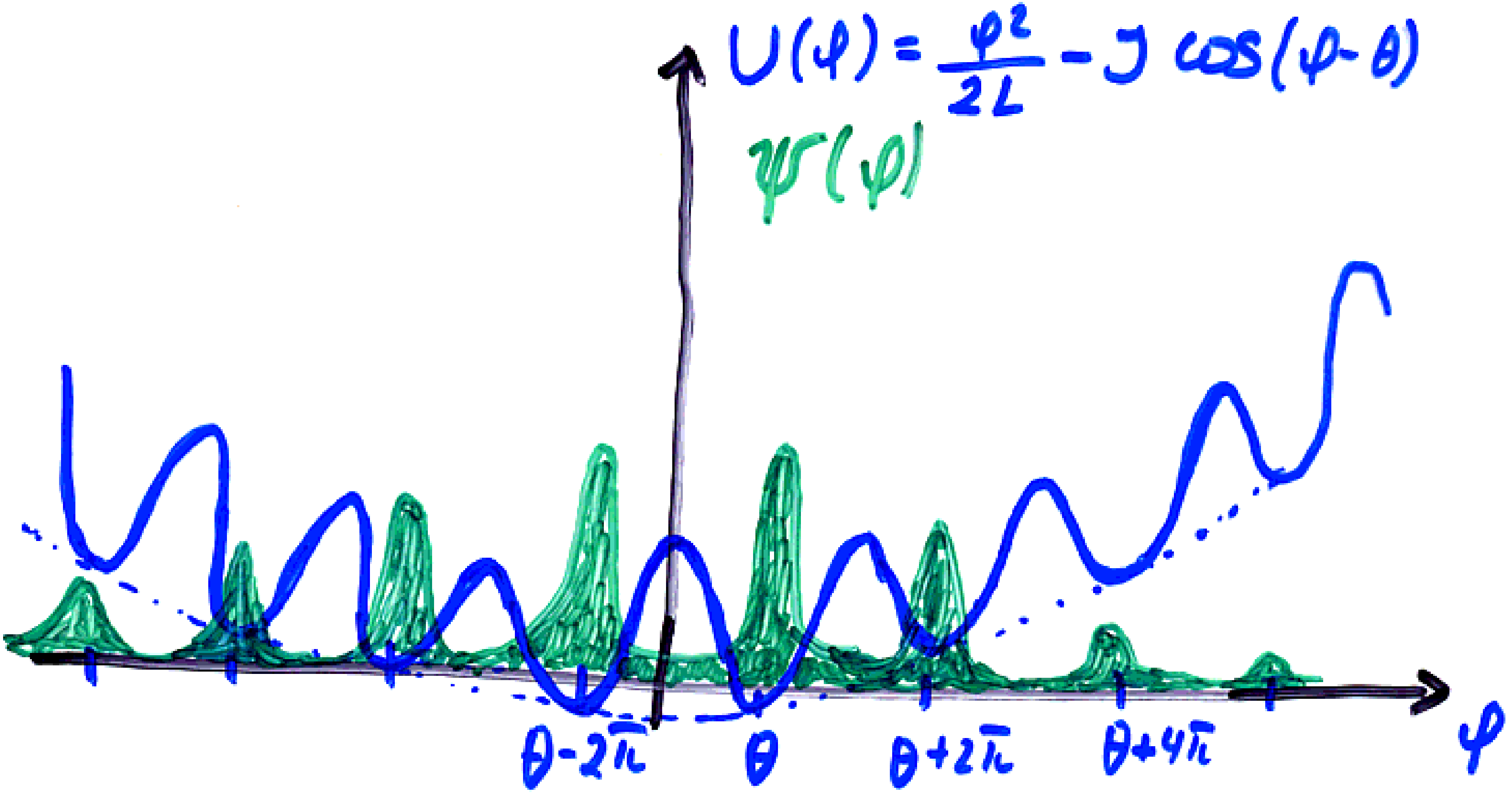
I_{max} change as a function of $\mathcal{J}$ by many orders of magnitude

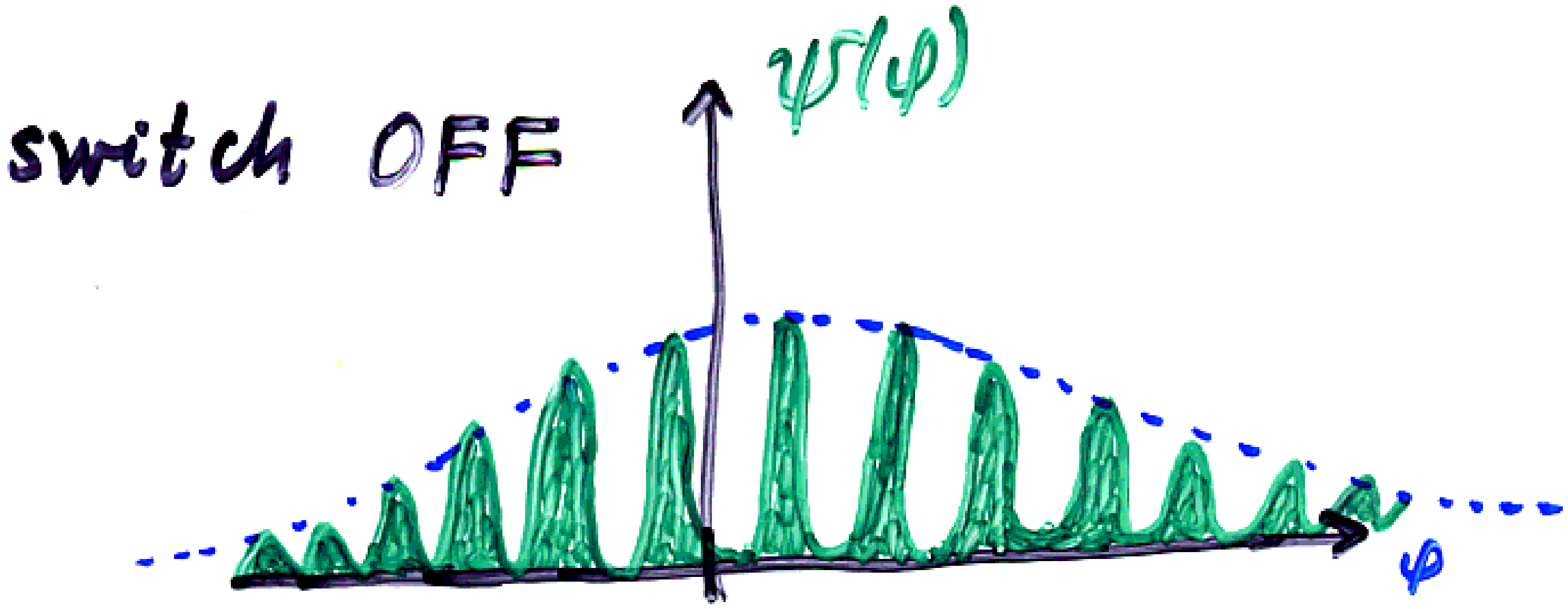
$$\boxed{H = \frac{\varphi^2}{2L} - \mathcal{J}\cos(\varphi - \theta) + \frac{1}{2C}\left(\frac{\partial}{i\partial\varphi} - n_g\right)^2}$$

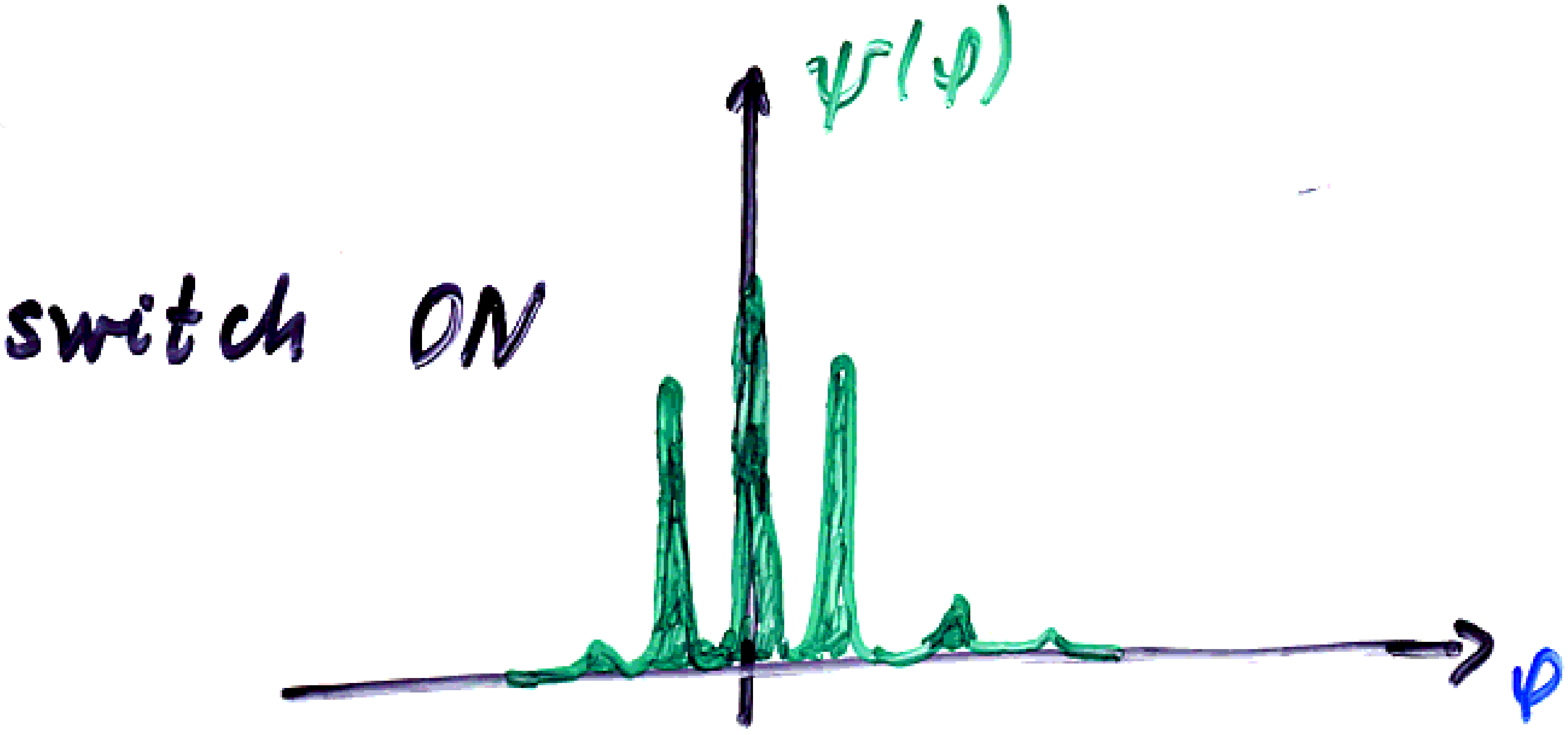


$$\boxed{\gamma = \frac{1}{4}\sqrt{\frac{L}{C}}}$$

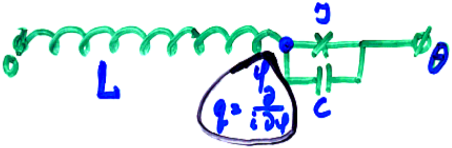
Phase slips suppressed


$U(\varphi) = \frac{\varphi^2}{2L} - J\cos(\varphi - \theta)$
$\psi(\varphi)$
$\theta - 2\pi$
θ
$\theta + 2\pi$
$\theta + 4\pi$
φ


switch OFF
$\psi(\varphi)$
φ


switch ON
$\psi(\varphi)$
φ

The physics is very simple:



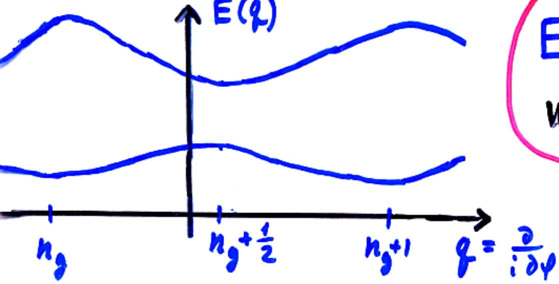
1) If $L \to \infty$ then

$\varphi \approx \theta + 2\pi n$

$$H \approx -J\cos(\varphi-\theta) + \frac{1}{2C}\left(\frac{\partial}{i\partial\varphi} - n_g\right)^2$$

φ varies from $-\infty$ to $+\infty$

Band structure:



$$\boxed{\begin{array}{l} E = \frac{1}{C} f(\mu, q-n_g) \\ \text{where } \underline{\mu = JC} \end{array}}$$

2) <u>Reformulating the problem in the charge basis:</u>

$$\boxed{H_{eff} = E(q) + \frac{1}{2L}\left(\frac{\partial}{i\partial q}\right)^2}$$

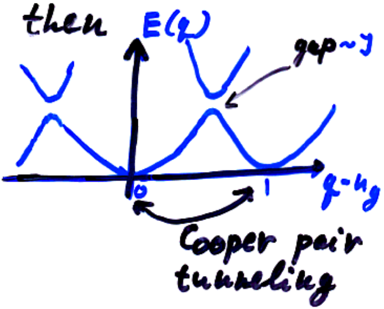
If $\mu = JC \gg 1$ then $E \approx -\frac{t}{C}\cos(2\pi q)$

where $\boxed{t \sim \mu^{3/4} e^{-8\sqrt{\mu}}}$ ← phase slip amplitude $(\times C)$

Switch is "ON" if $\boxed{C^{-1} t \ll L^{-1}}$

If $\mu = JC \ll 1$ then

$$E(q) \approx \frac{(q-n_g)^2}{2C}$$



Critical current $I_{max} \sim$ tunneling amplitude between $q = n_g$ and $q = n_g + 1$

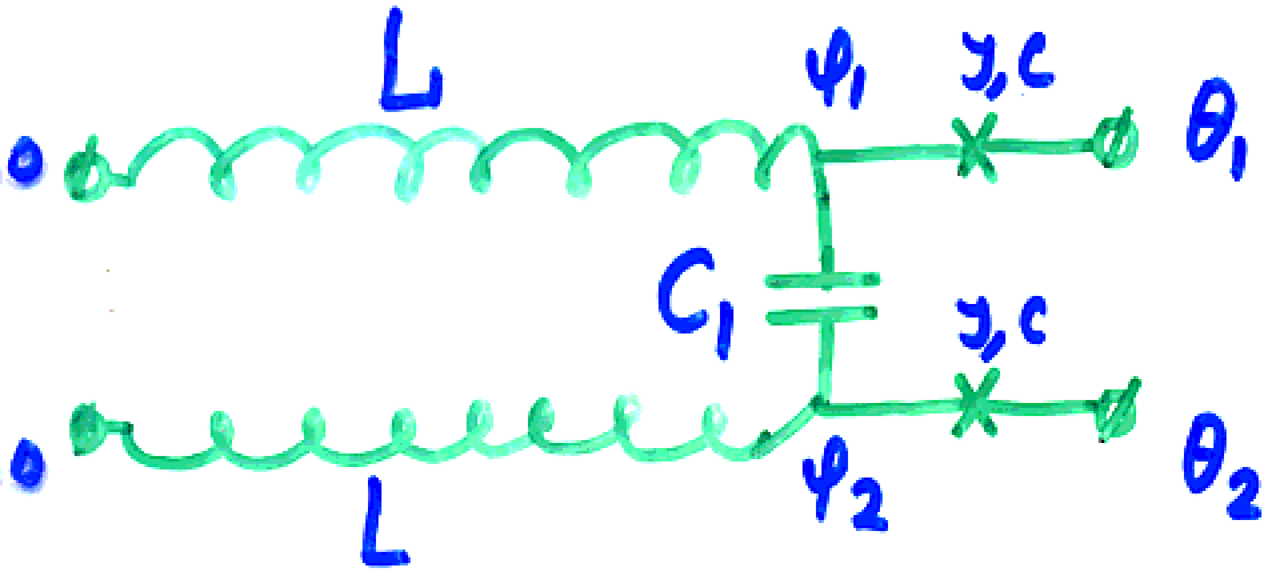
$$I_{max} \sim L^{-1} \gamma^{3/2} e^{-\gamma},$$

where $\gamma = \frac{1}{4}\sqrt{\frac{L}{C}}$

Caveat: Large numeric factors may be involved.

DC transformer with 1:1 current ratio

(current mirror)



$4\mathcal{J}C \sim 1$

$\gamma = \frac{1}{8}\sqrt{\frac{L}{C}} \gg 1$

$C_1 \gtrsim C \ln\gamma$

$$H = \frac{\varphi_+^2}{L} + \frac{\varphi_-^2}{4L} + \frac{q_+^2}{4C} + \frac{q_-^2}{2C_1+C} - 2\mathcal{J}\cos\frac{\varphi_- - \theta_1 + \theta_2}{2} \times$$
$$\times \cos\left(\varphi_+ - \frac{\theta_1 - \theta_2}{2}\right)$$

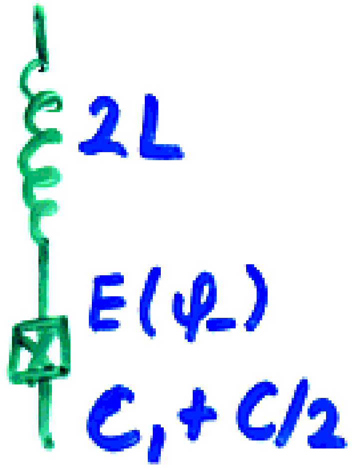
where $\underbrace{\varphi_+ = \frac{\varphi_1 + \varphi_2}{2}}_{\text{fast}}$, $\underbrace{\varphi_- = \varphi_1 - \varphi_2}_{\text{slow}}$

Equivalent circuits:

For φ_+ :

$I_{max} \sim L^{-1}\gamma e^{-\gamma}$
(very small)

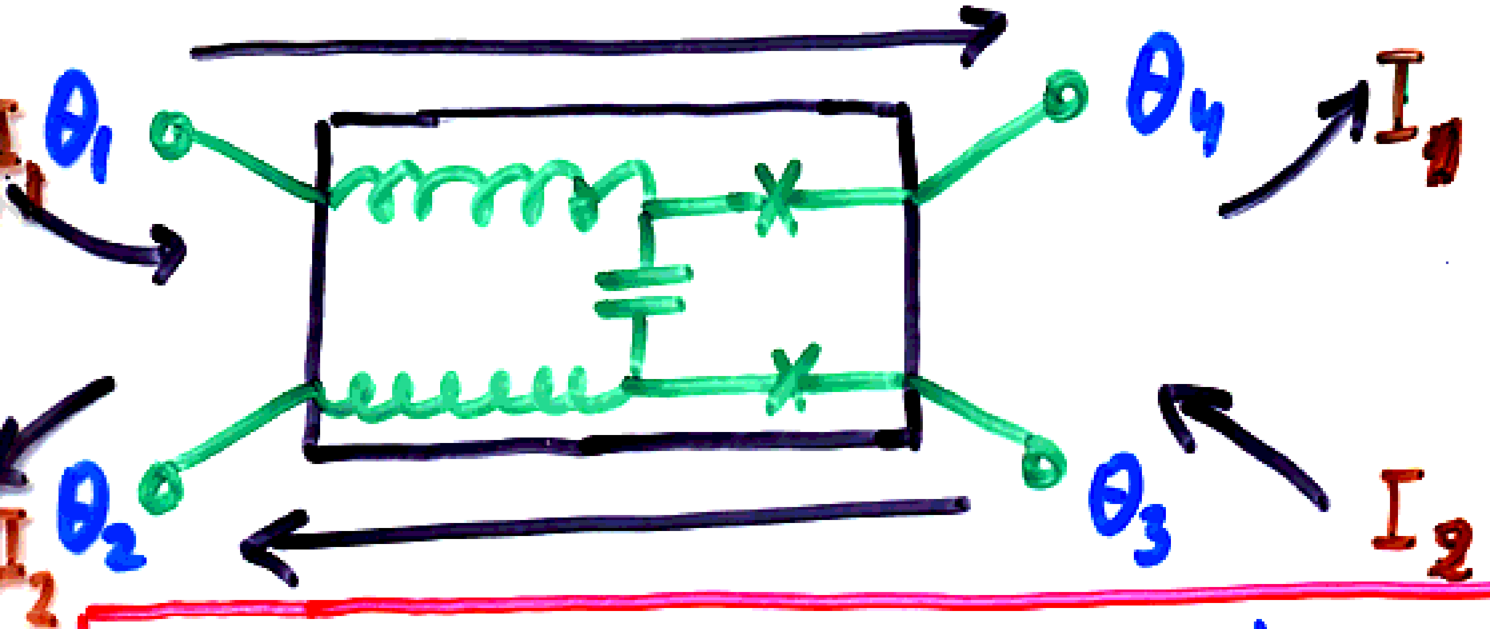
For φ_- :



$I_{max} \sim L^{-1}$
(sufficiently large)

$$E(\theta_1,\theta_2) = \underbrace{F(\theta_1-\theta_2)}_{\min_n \frac{1}{4L}(\theta_1-\theta_2-2\pi n)^2} + \underbrace{g(\theta_1,\theta_2)}_{\sim L^{-1}\gamma e^{-\gamma}}$$

More general situation:

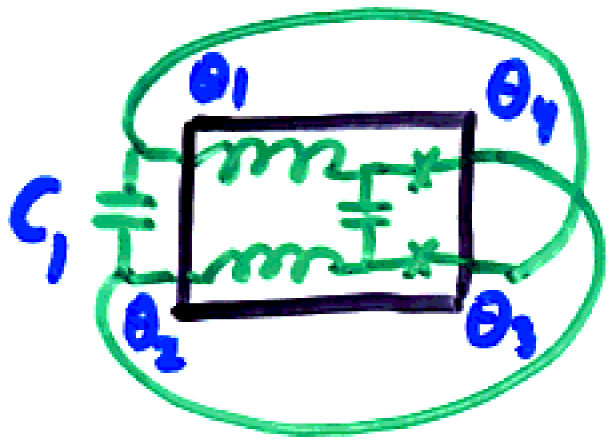


$$\boxed{E = F(\theta_1-\theta_2+\theta_3-\theta_4) + g(\theta_4-\theta_1,\ \theta_3-\theta_2)}$$

$$I_1 \approx I_2 : \begin{cases} I_1 - I_2 = \frac{\partial g}{\partial \theta_1} + \frac{\partial g}{\partial \theta_2} \sim L^{-1}\gamma e \\ I_1 \approx I_2 < \boxed{I_{max} \approx \frac{\pi}{2L}} \end{cases}$$

No dissipation $\Rightarrow V_{14} \approx V_{23}$

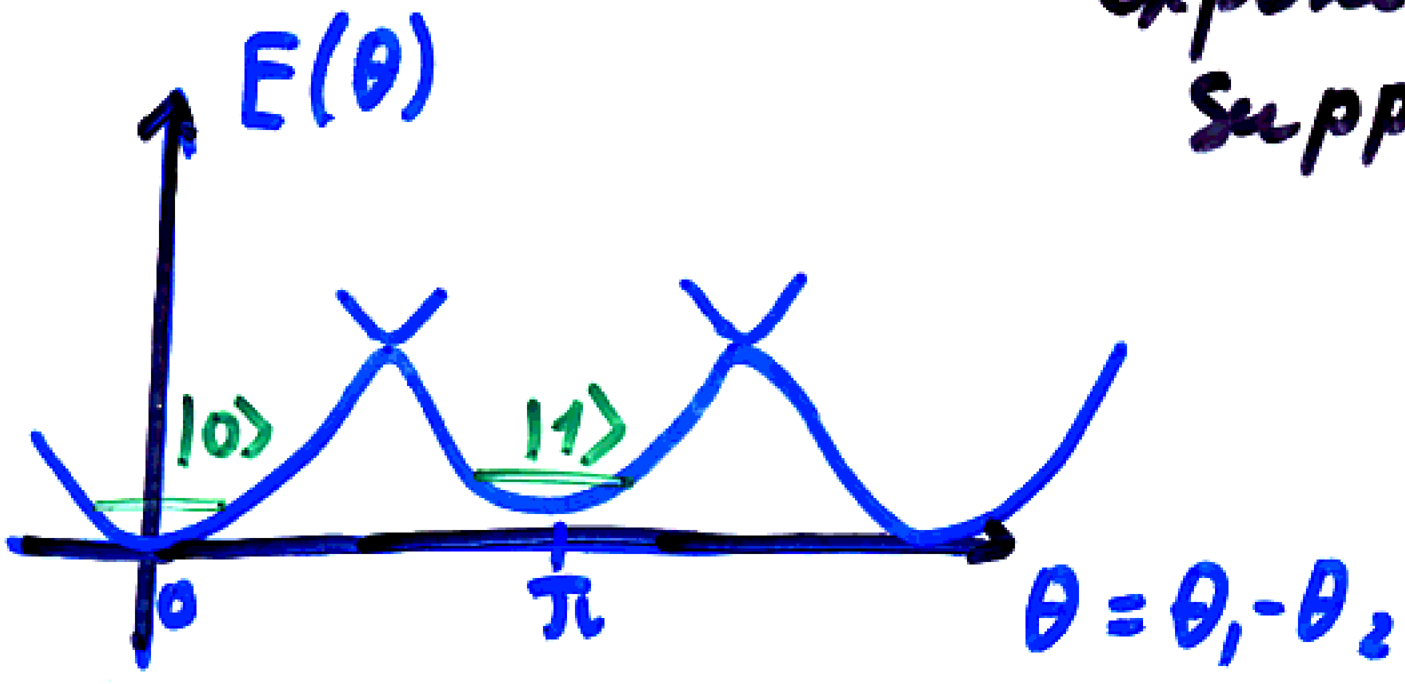
Turning a current mirror into a qubit



$\theta_1 = \theta_3$

$\theta_2 = \theta_4$

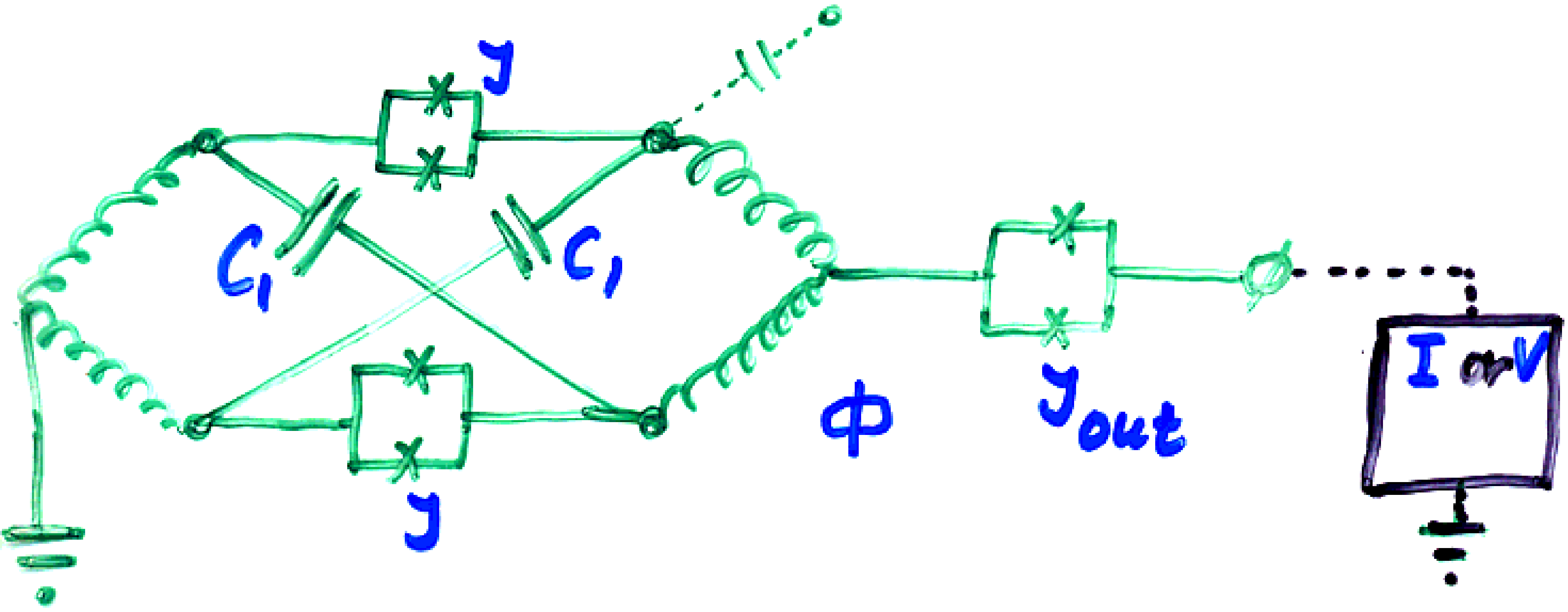
$$E = F(2(\theta_1 - \theta_2)) + \underbrace{h(\theta_1 - \theta_2)}_{\text{exponentially suppressed}}$$



To prevent tunneling $|0\rangle \longleftrightarrow |1\rangle$, we need $\boxed{JC_1 \gg 1}$

No phase slips in the inductors

Complete qubit



1) Quiet state :

J ON

J_{out} OFF

2) Phase measurement :

$|0\rangle$ vs $|1\rangle$

J ON

J_{out} ON

$$I_{out} \sim \frac{1}{L}$$

3) Dual measurement

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}} \text{ vs } \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

J OFF

J_{out} OFF

$$V_{out} \sim \frac{1}{C_1}$$

Quantum gates

Universal set:

1) Measurement in the $|0\rangle, |1\rangle$ basis

2) Measurement in the $|+\rangle, |-\rangle$ basis $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $|-\rangle = \frac{|0\rangle - |1\rangle}{2}$

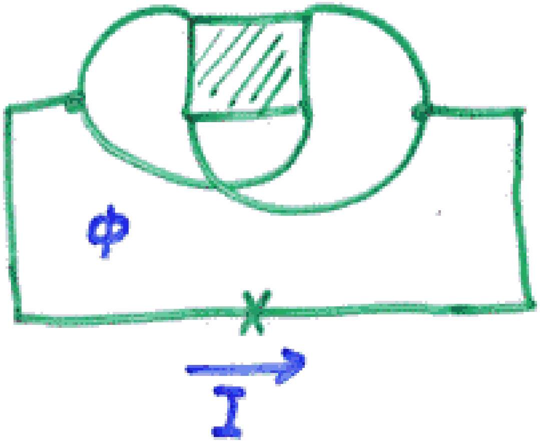
3) $\exp(i\frac{\pi}{4}\sigma^z)$, $\exp(i\frac{\pi}{4}\sigma_1^z\sigma_2^z)$ with high precision (protected)

4) $\exp(i\frac{\pi}{8}\sigma^z)$ with low (~30%) precision (unprotected)

Implementation of the measurements.

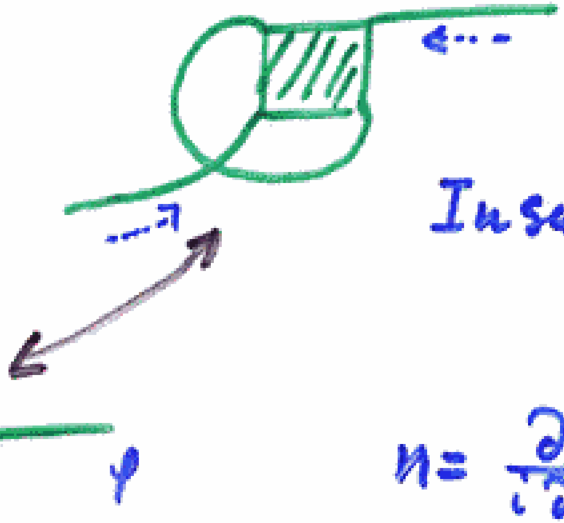
$\underbrace{|0\rangle, |1\rangle}$

Measuring the phase difference (0 or π)



$\underbrace{|+\rangle \quad |-\rangle}$ – more interesting

Consider this setup:



$$n = \frac{\partial}{i\,\partial\varphi}$$

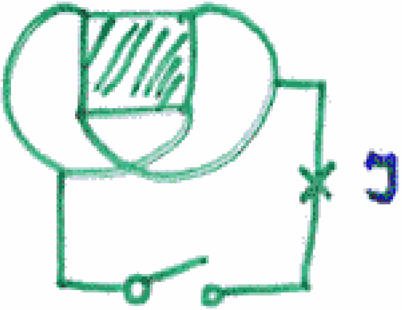
$$H = \frac{1}{2C_{eff}}\left(n - n_0 - \hat{\alpha}\right)^2$$

$$\hat{\alpha} = \frac{1+\sigma^x}{4}$$

$$|+\rangle \Rightarrow \underbrace{\alpha = 0}$$

$$|-\rangle \Rightarrow \underbrace{\alpha = \frac{1}{2}}$$

Unprotected $\exp(i\frac{\pi}{8}\sigma^z)$:

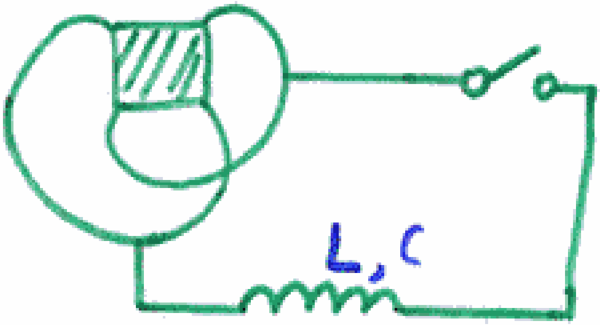


Connect for time interval Δt

$$|\psi\rangle \mapsto \exp(i\, J \Delta t\, \sigma^z)\,|\psi\rangle$$

Protected $\exp(-i\frac{\pi}{4}\sigma^z)$

Cf. Gottesma
Kitaev, Preskil
2000

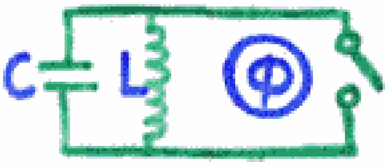


Connect for

$$\Delta t = \frac{L}{\pi}$$

$$\underline{|\psi\rangle \mapsto \exp(-i\frac{\Phi^2}{2L}\Delta t)|\psi}$$

Effective circuit:



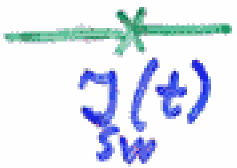
$$\underline{R=\sqrt{\frac{L}{C}} \gg 1}$$

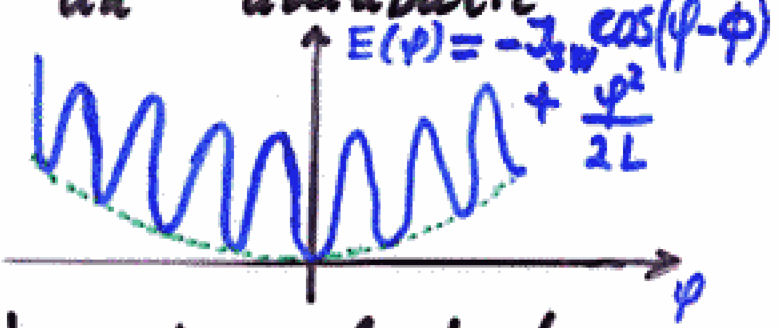
$\Phi = 0$ ($|0\rangle$) $|\psi\rangle \mapsto |\psi\rangle$

or

$\Phi = \pi$ ($|1\rangle$) $|\psi\rangle \mapsto -i|\psi$

Actually need an adiabatic switch:

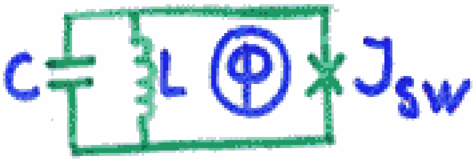




$J_{sw} \lesssim \frac{1}{C}$ — phase is unlocked

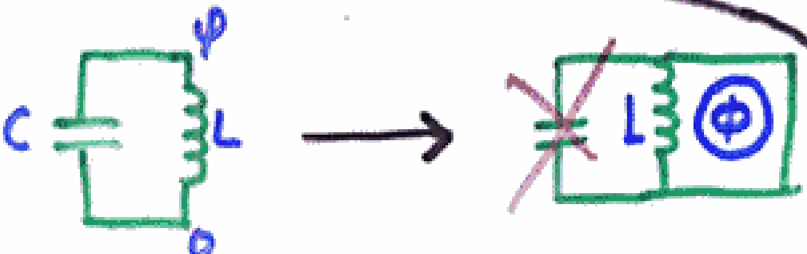
$J_{sw} \gg \frac{1}{C}$ — phase is locked

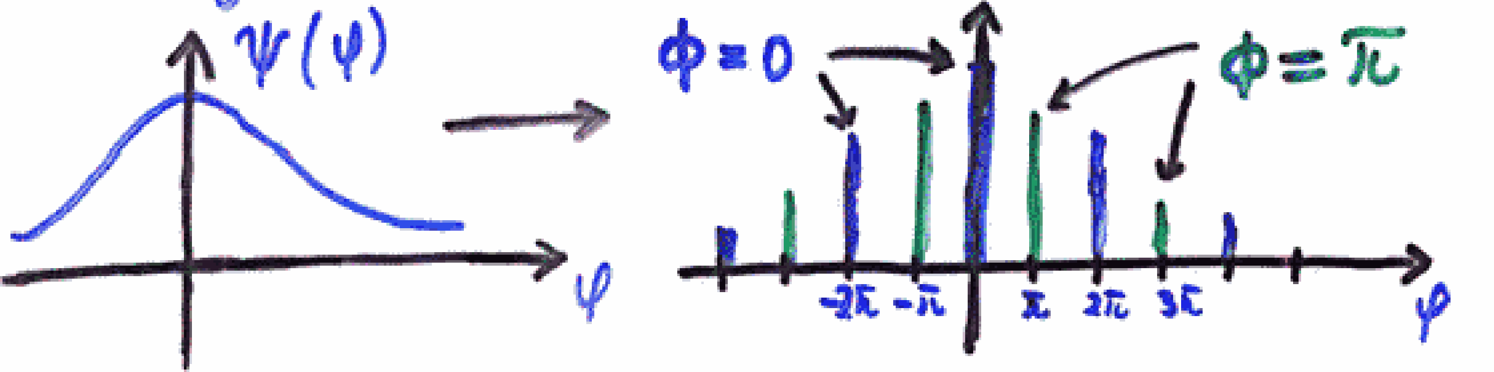
Transition time : $C \ll \tau \ll 2\pi\sqrt{LC}$

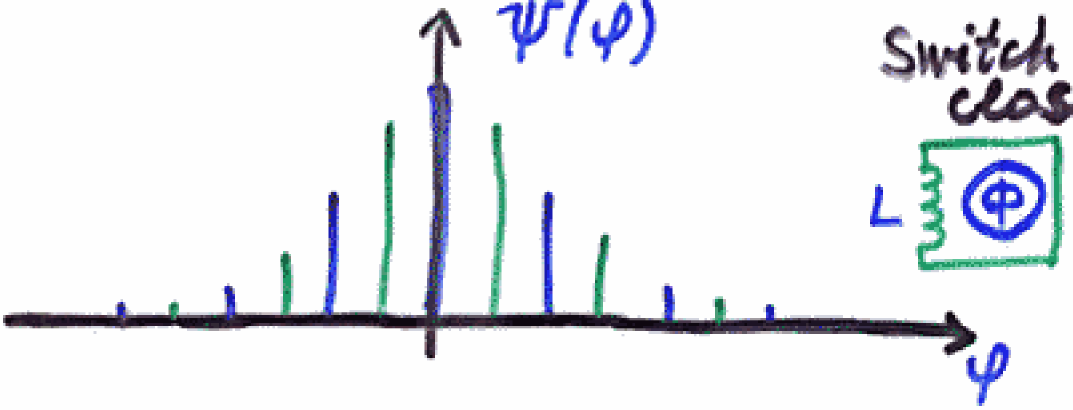


not to create oscillations in the J_{sw}-C circuit

to freeze the zero oscillations in the L-C circuit







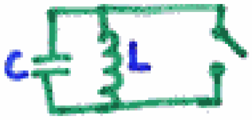
Switch is closed

$\varphi = \Phi + 2\pi K$

$H = \frac{\varphi^2}{2L}$

Each blue peak is multiplied by $\exp\left(-i \underbrace{\frac{(2\pi)^2}{L}\Delta t}_{2\pi} \underset{\uparrow \text{peak \#}}{K^2}\right) = 1$

Each green peak is multiplied by $\exp\left(-i \underbrace{\frac{(2\pi)^2}{L}\Delta t}_{2\pi} (K+\frac{1}{2})^2\right) = -i$

If Δt is not exact, then excitations will be created in the L-C circuit,  but the qubit will remain (almost) undisturbed.

Conclusions

1) An all-electric QC is possible theoretically

2) A lot of fun for theorists; experimental prospects are not clear yet

3) In short term (mid term), it would be great to implement a quantum transformer as an analog device.